Math 185: Introduction to complex analysis

UC Berkeley, Spring 2016

Instructor

Michael Hutchings
hutching@math.berkeley.edu
Office phone: 510-642-4329.
Office: 923 Evans.

Tentative office hours: Thursday 2:00-3:30.

Piazza

In addition to my office hours, I have set up a piazza page for the course here. Please feel free to ask questions about the course content there.

Head GSI

The head GSI for Math 185 (all sections) is Anh Nguyen. She will hold office hours from 4-6 pm on MTWThF in 961 Evans. Her annoucements and schedule changes (including some homework hints) are posted here.

Textbook

The textbook for this course is Theodore Gamelin, *Complex analysis*. If you connect to the internet through the UCB network or proxy server, then you can (legally!) download the book for free from link.springer.com. Please be sure to also download the errata. I will sometimes do things differently from the book, or in a different order. I may also occasionally post supplemental notes here. In any case, the course will cover most of the first part of the book, some of the second part, and possibly a bit of the third part.

There are many other complex analysis books available. The classic text by Ahlfors is more challenging and a bit old-fashioned, but very good. If you like pictures, check out *Visual Complex Analysis* by Tristan Needham. Some other nice books at an advanced undergraduate to beginning graduate level are *Complex function theory* by Sarason, *Complex analysis* by Lang, *Functions of one complex variable I* by Conway, *Complex analysis* by Stein-Shakarchi, and *Basic Complex Analysis: A Comprehensive Course in Analysis, Part 2A* by Barry Simon. All these books cover the same basic topics that are covered our course, but with different styles, and most of them include additional more advanced topics which vary from one book to the next.

Homework

Homework is due on most Tuesdays at the beginning of class. You can either bring it to class or slide it under my office door. (If it doesn't fit under the door, please be more concise!) Homework assignments will be posted below at least a week before they are due. No late homeworks will be accepted for any reason, so that we can go over the homework right after it is handed in (which is when people are most eager to see solutions to troublesome problems). However it is OK if you miss the deadline once or twice, because your lowest two homework scores will be dropped.

When preparing your homework, please keep the following in mind:

1) You are encouraged to discuss the homework problems with your classmates. Perhaps the best way to learn is to think hard about a problem on your own until you get really stuck or solve it, then ask someone else how they thought about it. However, when it comes time to write down your solutions to hand in, you must do this **by yourself**, in your own words, without looking at someone else's paper. In addition, you must **acknowledge any collaboration**.

2) All answers should be written in **complete, grammatically correct English sentences** which explain the logic of what you are doing, with mathematical symbols and equations interspersed as appropriate. Results of calculations and answers to true/false questions etc. should always be justified. Proofs should be complete and detailed. Avoid phrases such as "it is easy to see that"; often this means "I don't feel like explaining that", and what follows is actually a tricky point that needs justifiction, or even false. You can of course cite theorems that we have already proved in class or from the book.

Now here are the assignments. Below, "x.y" means exercise y on page x of the book.

- HW#1, due 2/2. See piazza for some solutions.
- HW#2, due 2/9. See piazza for solutions to the problems that are not in the book.
- HW#3, due 2/16.
- HW#4, due 2/23: 82.3, 84.1, 106.1, 106.2, 106.4, 106.5, 110.5, 111.1.
- HW#5, due 3/8.
- HW#6, due 3/15: 143.2, 143.5, 147.3, 148.7, 149.13, 153.3, 157.1acegi, 158.8.
- HW#7, due 3/29: 170.1, 170.2, 176.1acegi, 176.3, 181.1acd, 182.3, 198.1acegi.
- HW#8, due 4/5: Fun with integrals! 198.3ace, 202.4, 202.8, 205.2, 205.3, 207.1, 207.2.
- HW#9, due 4/19: 211.2, 211.4, 228.3, 228.4, 228.8, 230.2, 230.4, 230.7, 230.8.
- HW#10, due 4/26: 234.1, 235.4, 245.1, 245.3, 245.7, 257.1, 257.2, 257.3,258.4.
- HW#11: Try these review problems to see how much you have learned. This will not be graded, but I can go over the solutions to the problems.

Exams and grading

There will be in-class midterms on Thursday 2/25 and Thursday 4/7. The final exam is on Thursday 5/12.

There will be no makeup exams. However you can miss one midterm without penalty, as explained in the grading policy below.

There is no regrading unless there is an egregious error such as adding up the points incorrectly. Every effort is made to grade all exams according to the same standards, so regrading one student's exam would be unfair to everyone else.

The course grade will be determined as follows: homework 25%, midterms 25% each, final 50%, lowest exam score -25%. **The homework score will not be dropped.** All grades will be curved to a uniform scale before being averaged.

Syllabus

The following is the core syllabus, listed in the order in which it is presented in the book, which is not always the order in which I will cover it. I may discuss some additional topics as time permits. I will also skip the last couple of sections of some chapters of the book (which tend to contain more optional or specialized material).

Math 104 or equivalent is a prerequisite; I may briefly review some of this material as needed, but assume that you are generally comfortable with basic real analysis.

- Complex numbers, exponential and trigonometric functions (chapter I)
- Holomorphic functions, a.k.a. analytic functions, a.k.a. conformal maps (chapter II)
- Harmonic functions, review of line integrals (chapter III)
- Complex integration (chapter IV)
- Power series (chapter V)
- Isolated singularities and Laurent series (chapter VI)
- Residues and evaluation of definite integrals (chapter VII)
- Topological considerations: winding numbers, counting zeroes, simply connected domains (chapter VIII)
- If time permits, a bit about conformal mapping from chapters IX and XI.

What we actually did in class

- (Tuesday 1/19) Review of the algebra of complex numbers, and the geometric interpretation of this algebra. See Gamelin sections I.1 and I.2.
- (Thursday 1/21) More about the algebra and geometry of complex numbers. Proof that there is no continuous square root function. Limits and continuity in the context of complex numbers. Stereographic projection and the Riemann sphere. See Gamelin sections I.3, I.4, and II.1.
- (Tuesday 1/26) Review of different notions of regularity for real functions (which, for complex functions, are miraculously equivalent). Real derivatives versus complex derivatives. The Cauchy-Riemann equations. See Gamelin sections II.2 and II.3.
- (Thursday 1/28) More about complex differentiation, including two versions of the chain rule. Introduction to power series and radius of convergence. Definition of the exponential function as a power series. (I am explaining this material in a somewhat different order than the book. See these notes for clarification of the first few lectures.)
- (Tuesday 2/2) Properties of the exponential function. The principal branch of the logarithm. The inverse function theorem and the derivative of the logarithm. See Gamelin sections I.5, I.6, and II.4.
- (Thursday 2/4) Harmonic functions and harmonic conjugates. Conformal maps. Started on linear fractional transformations. See the last few sections of Gamelin chapter II.
- (Tuesday 2/9) More about linear fractional transformations. Started to review line integrals. (So far this is just like the line integral of Pdx + Qdy in Math 53, except that now we allow P and Q to be complex-valued.) See Gamelin sections II.7 and III.1.
- (Thursday 2/11) More review of line integrals and Green's theorem. Application: the mean value property for harmonic functions. See Gamelin sections III.1, III.2, and III.4. Here are some notes on line integrals to clarify what I said in class. (This is all standard stuff, but I explained some of it differently from the book.)
- (Tuesday 2/16) Line integrals involving dz. Cauchy's theorem. Started on the Cauchy Integral Formula. See the above notes, and the first three sections of Gamelin, Chapter IV.
- (Thursday 2/18) Details of the Cauchy Integral Formula. Corollary: a holomorphic function is infinitely (complex) differentiable. Liuoville's theorem and application to the "fundamental theorem of algebra". Also, the maximum principle. See Gamelin sections IV.4, IV.5, and III.5.
- (Tuesday 2/23) Review for the midterm.
- (Thursday 2/25) Midterm #1. To be covered: the lectures up to and including 2/16 (not including the Cauchy Integral Formula), the corresponding sections of the book (listed above) and posted notes (when needed for clarification), and the first four homework assignments.

- (Tuesday 3/1) Finished discussing the maximum principle and the Cauchy integral formula. Discussed how every holomorphic function can be expanded as a power series, and the radius of convergence is "the distance to the nearest singularity or branch point". See the first four sections of chapter V. We will explain more about power series next time.
- (Thursday 3/3) Manipulation of power series. Zeroes of holomorphic functions. See sections V.6 and V.7.
- (Tuesday 3/8) Laurent series. See section VI.1.
- (Thursday 3/10) Classification of isolated singularities. See section VI.2.
- (Tuesday 3/15) Partial fractions. The residue theorem. See sections VI.4 and VII.1.
- (Thursday 3/17) Using the residue theorem to evaluate definite integrals. See section VII.2.
- (Tuesday 3/29) More tricks for evaluating definite integrals using the residue theorem. See sections VII.3 and VII.4.
- (Thursday 3/31) Fractional residues. The argument principle. See sections VII.5 and VIII.1.
- (Tuesday 4/5) Review for the midterm.
- (Thursday 4/7) Midterm #2. To be covered: the lectures up to and including 3/29, the readings for these lectures listed above, and the first 8 homework assignments. Emphasis will be on material after the first midterm, but anything covered up to 3/29 is fair game.
- (Tuesday 4/12) More about the argument principle. Rouche's theorem. Winding numbers. See sections VIII.1,2,4,6.
- (Thursday 4/14) All about winding number. Among other things I explained how to compute winding number by counting intersections with a ray with signs. This is not in the book; you just had to be there.
- (Tuesday 4/19) Simply connected domains. See section VIII.8.
- (Thursday 4/21) More about simply connected domains and topological issues from Chapter VIII.
- (Tuesday 4/26) The Schwarz lemma and conformal automorphisms of the unit disk. See sections IX.1 and IX.2.
- (Thursday 4/28) More about harmonic functions. See Chapter X.
- (Tuesday 5/3) Optional review session (RRR week)
- (Thursday 5/5) No class (RRR week)
- (Thursday 5/12) Final exam, 7-10pm, location TBA