Course Announcement - Fall 2011

Math 255: Algebraic Curves

Instructor: Bernd Sturmfels

Office hours: Wednesdays 8:00-10:00 and by appointment
Contact: bernd at math, 925 Evans

Time and Place: Tuesdays and Thursdays, 12:30-2:00pm, 87 Evans Hall

Prerequisites: Abstract Algebra at the level of Math 250A. Ideally, also Undergraduate Algebraic Geometry (Math 143) and Commutative Algebra (Math 250B). Experience in working with Fields, Rings, Modules, Ideals, and their Grobner Bases.

Text Books: The following two books are required for this class:
William **Fulton**: Algebraic Curves. An Introduction to Algebraic Geometry, Reprint of 1969 original, Addison-Wesley, 1989.
Frances **Kirwan**: Complex Algebraic Curves, London Mathematical Society Student Texts, 23, Cambridge University Press, 1992.

Syllabus: In September and October, we shall cover the core material on curves from the two text books: local properties, plane curves, morphism and rational maps, Riemann surfaces, differentials, Puiseux series, resolution of singularities, and the Riemann-Roch Theorem. In November, we shall discuss selected topics, with emphasis on 19th century geometry, algorithms, moduli spaces, and tropical curves.

Term Papers: Students wrote term papers on topics of their choice related to algebraic curves.
Here are links that show their excellent accomplishments:
Emily Berger: Chip Firing and Riemann-Roch
Sarah Brodsky: Inflection Points of Real and Tropical Curves
Ka Laam Chan: Resolution of Singularities
Shelly Manber: Algebraic Geometry Codes
James McIvor: A First Glimpse of Deformation Theory
Maria Monks: Duality of Plane Curves
Ralph Morrison: Tropical Intersections: Where they go wrong and where they go right
Qingchun Ren: A Formula for the Cayley-Bacharach Theorem
Jose Rodriguez: How to E-Mail a Riemann Surface
Zvi Rosen: Graph Curves
Eugenia Rosu: Reduction of Ternary Forms
Shamil Shakirov: Invariants of Quartic Curves
Alexander Shapiro: Harnack Curves
Gus Schrader: Faye's Trisecant Identity
Victoria Wood: Parametrizing Rational Curves
Raki Yatchak: The Degree of a Severi Variety

Consultant: Melody Chan will help with the course. Questions can be directed to either her or me.

Further Reading: Here is a selection of recommended resources on algebraic curves:
Lecture Notes from the Math 255 class taught by **Hendrik Lenstra** in the Fall of **1995**.
Egbert Brieskorn and Horst Knorrer: Plane Algebraic Curves, Birkhauser Verlag, Basel, 1986.
Joe Harris and Ian Morrison: Moduli of Curves, Graduate Texts in Mathematics, 187, Springer 1998.

George Salmon, Arthur Cayley: A Treatise on the Higher Plane Curves, Elibron Classics, original from 1852.
Rafael Sendra, Franz Winkler and Sonia Perez-Diaz: Rational Algebraic Curves - A Computer Algebra Approach, Springer, 2008.
Ernesto Girondo and Gabino González-Diez: Introduction to Compact Riemann Surfaces and Dessins d'Enfants, Cambridge University Press, 2011.

Schedule:
August 25: Foundations [Kirwan, Chapter 2]
August 30: Bezout's Theorem [Kirwan, Section 3.1]
September 1: Bezout's Theorem [Kirwan, Section 3.1]
September 6: Points of inflection and cubic curves [Kirwan, Section 3.2]
September 8: The degree-genus formula [Kirwan, Section 4.1]
September 13: No Class: please consider attending the POLYMAKE seminar
September 15: Branched covers of the line [Kirwan, Sections 4.2-4.3]
September 20: The Weierstrass p-function [Kirwan, Section 5.1]
September 22: Riemann surfaces [Kirwan, Section 5.2]
September 27: Holomorphic differentials [Kirwan, Section 6.1]
September 29: Abel's Theorem [Kirwan, Section 6.2]
October 4: Melody Chan: The Riemann-Roch Theorem [Kirwan, Section 6.3]
October 6: Melody Chan: The Riemann-Roch Theorem [Kirwan, Section 6.3]
October 11: Local rings, DVRs, Multiplicities [Fulton, Sections 2.4, 2.5, 3.1, 3.2]
October 13: Linear Systems, Multiple Points, Noether's Theorem [Fulton, Sections 5.2, 5.4, 5.5]
October 18: Curves in Projective Space [Fulton, Chapters 4 and 6]
October 20: Charley Crissman: Introduction to Moduli of Curves
October 25: Resolution of Singularities [Fulton, Chapter 7]
October 27: Shaowei Lin: Computing Resolutions -- How and Why
November 1: Divisors and their Sections [Fulton 8.1-8.2]
November 3: Riemann's Theorem, Derivations, Differentials [Fulton 8.3-8.4]
November 8: Canonical Divisors and Riemann-Roch revisited [Fulton 8.5-8.6]
November 10: David Eisenbud: The Most Interesting Embeddings of a Curve
November 15: Shamil Shakirov: Invariants of Quartic Curves
Jose Rodriguez: How to E-mail a Riemann Surface
November 17: Qingchun Ren: A Formula for the Cayley-Bacharach Theorem
Emily Berger: The Riemann-Roch Theorem for Graphs
November 22: Maria Monks: Duality of Plane Curves
Zvi Rosen: Graph Curves
November 29: Rika Yatchak: Severi Varieties
Sarah Brodsky: Inflection Points of Real and Tropical Plane Curves
December 1: Shelly Manber: Algebraic Geometry Codes
Alexander Shapiro: Harnack Curves.
December 5: [10am-noon, 939 Evans]
Gus Schrader: Faye's Trisecant Identity
Ralph Morrison: Tropical Intersections -- Where They Go Wrong, and Where They Go Right
Eugenia Rosu: Stoll's Reduction Theory for Ternary Forms

Homework: In the first eight weeks there will be five assignments, posted here in pdf format:
Homework 1 is due Thursday, September 1.
Homework 2 is due Tuesday, September 13.
Homework 3 is due Tuesday, September 27.
Homework 4 is due Tuesday, October 11.
Homework 5 is due Thursday, November 3.

Term paper deadlines:
Thursday, October 27: Project proposal is due

Thursday, December 1: Final term paper is due

Grading: The course grade will be based on both the homework and the term paper.