Course Announcement - UC Berkeley - Fall 2012

Math 249: Combinatorial Commutative Algebra

Instructor: Bernd Sturmfels

Office hours: Before class [MWF, 9:15-10:00am, 740 Evans Hall] or by appointment

Time and Place: Monday, Wednesday and Friday, 10:00-11:00am, 740 Evans Hall

What's Happening in Class:
Friday, August 24: Squarefree monomial ideals (Sections 1.1-1.3)
Monday, August 27: Monomial matrices and Betti numbers (Sections 1.4-1.5)
Wednesday, August 29: Borel-fixed monomial ideals (Chapter 2)
Friday, August 31: Three-dimensional staircases (Sections 3.1-3.4)
Wednesday, September 5: Cellular resolutions (Sections 4.1-4.3)
Friday, September 7: The hull resolution (Sections 4.4-4.5)
Monday, September 10: Alexander duality (Sections 5.1-5.2)
Wednesday, September 12: Duality for Resolutions (Sections 5.3-5.4)
Friday, September 14: No class: please attend the 5-min postdoc lectures at MSRI
Monday, September 17: Projective dimension and regularity (Section 5.5)
Wednesday, September 19: Genericity and the Scarf complex (Section 6.1-6.2)
Friday, September 21: Deformations and bounds (Sections 6.3-6.4)
Monday, September 24: Thomas Kahle: Semigroups and lattice ideals (Section 7.1-7.2)
Wednesday, September 26: No class: please attend the RTG Workshop
Friday, September 28: No class: please attend the RTG Workshop
Monday, October 1: Hilbert bases (Section 7.3)
Wednesday, October 3: Initial ideals of lattice ideals (Section 7.4)
Friday, October 5: Multigradings and K-polynomials (Sections 8.1-8.2)
Monday, October 8: Multidegrees (Section 8.5)
Wednesday, October 10: Betti numbers of lattice ideals (Section 9.1)
Friday, October 12: Dave Bayer: Laurent monomial modules (Section 9.2)
Monday, October 15: Resolutions of lattice ideals (Sections 9.3-9.4)
Wednesday, October 17: Ideals of points in the plane (Section 18.1)
Friday, October 19: Connectedness, smoothness and Haiman's theory (Sections 18.2-18.3)
Monday, October 22: Ehrhart polynomials (Sections 12.1-12.2)
Wednesday, October 24: Brion's formula (Section 12.3)
Friday, October 26: Short rational generating functions (Section 12.4)
Monday, October 29: Flag variety and Plücker relations (Section 14.1-14.2)
Wednesday, October 31: Minors form sagbi basis (Section 14.3) and a spooky Halloween surprise
Friday, November 2: Gelfand-Tsetlin semigroups (Section 14.4)
Monday, November 5: Adam Boocher: Introduction to determinantal ideals
Wednesday, November 7: Pablo Solis: Schubert polynomials (Section 15.5)
Friday, November 9: Alex Fink: Quiver ideals (Section 17.1)
Wednesday, November 14: George Melvin: Modules over the exterior algebra; Zvi Rosen: Determinantal matroids
Friday, November 16: Thanh Vu: Betti numbers of toric ideals; Zach Bowen (Jack Love): Order polytopes
Monday, November 19: Kim Laine: The MinRank problem in cryptography; Sarah Brodsky: Cluster varieties
Wednesday, November 21: Paul Helminck: Antidiagonal initial ideals; Steven Karp: Unimodular Lawrence ideals
Monday, November 26: Caitlin Lownes: Toric varieties; Ralph Morrison: Fast algorithms for computing Gröbner

bases
Wednesday, November 28: Elina Robeva: Miniversal Gröbner bases of binomial ideals; Jose Rodriguez: Excess Intersection and Newton polytopes
Friday, November 30: No class: relax and prepare for the workshop.

Prerequisites: Math 250B or equivalent background in commutative algebra. Exposure to combinatorics, topology and geometry.

Course Work: Eight exercises and one term paper

Exercises: Please turn in **one exercise every Wednesday**, starting on August 29 and ending on October 17. Choose from the book.
Term Paper: Write a term paper on a relevant topic of your choice. Collaborations with guests from MSRI are strongly encouraged.
Paper Deadlines: Proposal due October 12, First Draft due November 14, Final Version due December 7. Topics for the term papers and the finished products will be posted on this webpage (unless you voice objections).
Everyone is invited to attend the workshop at MSRI which will take place December 3-7.

Text Book: Ezra Miller and Bernd Sturmfels, Combinatorial Commutative Algebra, Springer, 2004.

Syllabus: The book has the following **18 chapters**. We plan to discuss a different chapter each week, with the aim of covering most of the book. Your input on selection, order and speed is welcome.

1. Squarefree Monomial Ideals
2. Borel-fixed Monomial Ideals
3. Three-dimensional Staircases
4. Cellular Resolutions
5. Alexander Duality
6. Generic Monomial Ideals
7. Semigroup Algebras
8. Multigraded Polynomial Rings
9. Syzygies of Lattice Ideals
10. Toric Varieties
11. Irreducible and Injective Resolutions
12. Ehrhart Polynomials
13. Local Cohomology
14. Plücker Coordinates
15. Matrix Schubert Varieties
16. Antidiagonal Initial Ideals
17. Minors in Matrix Products
18. Hilbert Schemes of Points