Course Announcement - Fall 2015

Math 143: Elementary Algebraic Geometry

Instructor: Bernd Sturmfels

Office hours: Mondays, 3-4:30pm, or grab me after the lecture.
Contact: bernd at math, 925 Evans, phone messages: 642 6550

Lectures: Tuesdays and Thursdays 8:00-9:30am, 6 Evans Hall,

First Day of Class: Thursday, August 27.
Last Day of Class: Thursday, December 3.
Midterm Exam: Thursday, October 15.
Term Papers Due: Thursday, December 17.

Prerequisites: Linear Algebra (Math 110) and Abstract Algebra (Math 113)
Some familiarity with mathematical software (e.g. SAGE, Maple, or Mathematica)

Course text: Ideals, Varieties and Algorithms (3rd edition) by David Cox, John Little, and Donal O'Shea, Springer Undergraduate Texts in Mathematics, 2007.

Syllabus: We will study topics from all nine chapters of the text book:
1. Geometry, Algebra, and Algorithms
2. Gröbner Bases
3. Elimination Theory
4. The Algebra-Geometry Dictionary
5. Polynomial and Rational Functions on a Variety
6. Robotics and Automatic Geometry Theorem Proving
7. Invariant Theory of Finite Groups
8. Projective Algebraic Geometry
9. The Dimension of a Variety

The sections to be covered in each lecture are listed below. Please read these before coming to class.

Grading: There will be weekly homework sets and a midterm exam (in-class). The final is a term paper (take-home).
The grading scheme is: **Homework 35%, Midterm 30%, Term Paper 35%.**

Homework: There will be a weekly homework assignment, to be handed in on Tuesdays at 9:30am, at the end of class.
Late homework will not be accepted. No exceptions. The assignments, posted below, refer to the text book.
No homework after November 10, so you can focus on your term paper.

Final Exam: You will write a term paper on a topic of their choice related to the class. You may work on this by yourself
or in teams of two. Please submit a proposal for your project on Tuesday, October 27. This should fit on one page and contain:
names of author(s), title, sources, and a brief description. The final version of the paper is due on Wednesday, December 16.

DAILY SCHEDULE:
Aug 27 (ER): 1.1 Polynomials and Affine Space, 1.2 Affine Varieties, 1.3 Parametrizations of Affine Varieties
Sep 1: 1.4 Ideals, 1.5 Polynomials of One Variable, 2.1 Introduction to Gröbner Bases
Sep 3: 2.2 Orderings on Monomials, 2.3 A Division Algorithm
Sep 8: 2.4 Monomial Ideals and Dickson's Lemma, 2.5 The Hilbert Basis Theorem and Gröbner Bases
Sep 10: 2.6 Properties of Gröbner Bases, 2.7 Buchberger's Algorithm, 2.8 First Applications
Sep 15 (MM): 3.1 The Elimination and Extension Theorem, 3.2 The Geometry of Elimination
Sep 17 (MM): 3.3 Implicitization, 3.4 Singular Points and Envelopes
Sep 22 (MM): 3.5 Unique Factorization and Resultants, 3.6 Resultants and the Extension Theorem
Sep 24: 4.1 Hilbert's Nullstellensatz, 4.2 Radical Ideals and the Ideal-Variety Correspondence
Sep 29: 4.3 Sums, Products and Intersections of Ideals, 4.4 Zariski Closure and Quotients of Ideals
Oct 1: 4.5 Irreducible Varieties, 4.6 Decomposition of a Variety
Oct 6 (MH): 5.1 Polynomial Mappings, 5.2 Quotients of Polynomial Rings
Oct 8 (MH): 5.3 Computing in K[x]/I, 5.4 The Coordinate Ring of an Affine Variety
Oct 13: Review for the Midterm
Oct 15: MIDTERM EXAM
Oct 20: The software Macaulay2, Discussion of Term Papers
Oct 22: 4.7. Primary Decomposition of Ideals, 9.1 The Variety of a Monomial Ideal
Oct 27: 8.1 The Projective Plane, 8.2 Projective Space and Projective Varieties
Oct 29: 8.3 The Projective Algebra-Geometry Dictionary, 8.4 The Projective Closure of an Affine Variety
Nov 3: 8.5 Projective Elimination Theory, 8.6 The Geometry of Quadric Hypersurfaces
Nov 5: 8.7 Bezout's Theorem, the software Bertini
Nov 10: 9.2 The Complement of a Monomial Ideal, 9.3 The Hilbert Function and the Dimension of a Variety
Nov 12: 9.4 Elementary Properties of Dimension, 9.5 Dimension and Algebraic Independence
Nov 17: 7.1 Symmetric Polynomials, 7.2 Finite Matrix Groups and Rings of Invariants
Nov 19: 7.3 Generators for the Ring of Invariants, 7.4 Relations Among Generators and the Geometry of Orbits
Nov 24: 9.6 Dimension and Nonsingularity, 9.7 The Tangent Cone

Dec 1: **Presentation of Term Papers**
8:10 Liz Ferme: Borel-fixed Ideals
8:30 Marley Ummel: Robotics
8:50 Richard Adelstein: Bezout's Theorem
9:10 Albert Zheng: The Cayley-Bacharach Theorem

Dec 3: **Presentation of Term Papers**
8:10 Joelle Lim: Elliptic Curves
8:30 Julio Soldevilla: N-Fold Integer Programming Games
8:50 Hannah Wheelen: Buchberger's Algorithm in Particle Physics
9:10 Claire Tiffany-Appleton and Meghan McConlogue: Reverse Engineering of Gene Regulatory Networks

Dec 17: **Presentation of Term Papers** (in **939** Evans Hall)
9:00 Nishant Pappireddi: Zerodimensional Varieties via Eigenvalues
9:20 Wei Cheng Ng: Phylogenetic Algebraic Geometry and Linguistics
9:40 Benjamin Chu: Ideals and Neural Codes
COFFEE BREAK
10:20 Vrettos Moulos: Real Algebraic Geometry and Optimization
10:40 Frank Ong: The Positivstellensatz
11:00 Mahrud Sayrafi: Semidefinite Optimization and Nonnegative Polynomials
COFFEE BREAK
11:40 Hui Yu Lu: The Quadratic Line Complex
12:00 Davis Foote: Algebraic Coding Theory
12:20 Sophia Elia: Modeling Surfaces with Surfex
LUNCH BREAK

13:40 Helen Zhenzheng Hu: Applications of Algebraic Geometry in Game Theory
14:00 Bryan Wang: The Polynomial Method in Graph Theory
14:20 Shensheng Chen: Gröbner Bases with a View towards Tropical Geometry
14:40 Edward Kim: The Hopkins-Levitzki Theorem

Homework assignments:
due Sep 1: Section 1.1: # 4; Section 1.2: # 4, 7, 10; Section 1.3: # 4, 6.
due Sep 8: Sec 1.4: # 3,8,12,15;Sec 1.5: # 2,10,12,17; Sec 2.1: # 5; Sec 2.2: # 5,10,12; Sec 2.3: # 6,7.
due Sep 15: Sec 2.4: # 3, 11; Sec 2.5: # 10, 18; Sec 2.6: # 3, 10; Sec 2.7: # 2, 11; Sec 2.8: # 3, 5.
due Sep 22: Sec 3.1: # 4, 5, 9; Sec 3.2: # 3, 5; Sec 3.3: # 9, 14; Sec 3.4: # 9, 12.
due Sep 29: Sec 3.5: # 7, 8, 9; Sec 3.6: # 2, 7; Sec 4.1: # 7, 10; Sec 4.2: # 2, 7.
due Oct 6: Sec 4.3: # 9, 11; Sec 4.4: # 2, 8, 10; Sec 4.5: # 4, 6, 12; Sec 4.6: # 1, 4, 7.
due Oct 13: Sec 5.1: # 2, 9; Sec 5.2: # 5, 11, 16; Sec 5.3: # 5, 10, 13; Sec 5.4: # 9, 15.
due Oct 27: Sec 4.7: # 2, 3, 11, 12; Sec 9.1: 2, 5, 6; submit your term paper proposal.
due Nov 3: Sec 8.1: 9, 11; Sec 8.2: 5, 16, 17, 19; Sec 8.3: 3, 6; Sec 8.4: 5, 11, 12.
due Nov 10: Sec 8.5: 7, 9, 17; Sec 8.7: 7, 8, 9; Sec 8.6: 8, 14, 16.