

Mechanical Engineering 185 Introduction to Continuum Mechanics

Course Content: This course presents a broad introduction to the fundamentals of mechanics of continuous media. In particular, it covers the kinematics of finite deformations, the concept of stress, the conservation of mass and the balance of linear momentum, angular momentum and energy. The course also discusses certain mechanical constitutive equations, such as those of nonlinear elastic solids and Newtonian viscous fluids. The importance of invariance of material description under superposed rigid body motions is emphasized.

Course Objectives: By the end of this course, students should:

- Feel comfortable with Cartesian vector and tensor analysis in three dimensions;
- Understand the basic kinematic relations for a body subject to finite deformation;
- Understand the relationship between surface tractions and stresses within a body;
- Appreciate the significance of the basic balance laws (for mass, linear momentum and angular momentum);
- Understand the assumptions made in obtaining constitutive relations between stress and the various kinematic quantities;
- Understand the restrictions on constitutive equations that can be achieved through use of invariance requirements.

Prerequisites

- Physics 7A (Mechanics).
- Math 53 & 54 (Linear Algebra, Differential Equations, Vector Calculus).
- Some familiarity with elementary theories of solid and fluid mechanics (as found in typical undergraduate courses such as ME C85, ME106, and ME108).

Text: *Introduction to Continuum Mechanics*, P. Papadopoulos (on bCourses). Reading assignments will be given for most classes, in which case the reading must be done BEFORE that class. Time will be spent in class discussing the reading, not necessarily lecturing on the same material that you will have just read.

Homework: Homework problems will be assigned approximately every week and will account for 20% of the course grade. Major results must be clearly identified.

Exams: There will be two in-class midterm exams, each worth 20% of the course grade, and a 3-hour final exam worth 40% of the course grade.

COURSE OUTLINE

1. Introduction
2. Mathematical Preliminaries
 - Vectors and vector spaces
 - Tensors in a three-dimensional Euclidean space
 - The Einstein summation convention
 - The relationship between matrices and tensors
 - A review of eigenvalues and eigenvectors in three dimensions
 - Vector and tensor calculus (divergence, gradient, curl)

3. Kinematics of Finite Deformation

- Bodies, configurations and motions
- Material time derivative, material curves and material surfaces
- Path lines and stream lines
- Mass and mass density
- Line elements, deformation gradient and stretch
- Cauchy-Green stretch tensors, and finite strain tensors
- The polar decomposition of the deformation gradient: stretch and rotation
- Principal stretches and directions (an eigenvalue problem)
- The Rodrigues formula for the rotation tensor
- Deformation of elements of area and volume
- Velocity gradient, rate of deformation and spin (vorticity) tensors

4. Basic Physical Principles

- Reynolds' transport theorem and the localization theorem
- Conservation of mass
- Balance of linear momentum and balance of angular momentum
- The stress vector and stress tensor (Cauchy's Lemma and Cauchy's tetrahedron)
- Stress components - simple states of stress
- Balance laws and stresses in the reference configuration
- Balance of energy

5. Constitutive Theories

- Invariance requirements and general considerations
- a) Superposed rigid body motions
- b) Transformation of kinematic quantities under superposed rigid body motions
- c) Transformation of tractions and stresses under superposed rigid body motions
- Inviscid fluid
- Viscous fluid
- Nonlinear elastic solid
- Linear elastic solid
- Infinitesimal deformation
- The Gateaux differential
- Linearization of kinematic quantities