Alexander Paulin

apaulin@berkeley.edu

Department of Mathematics
796 Evans Hall
University of California, Berkeley

Calculus 1B (Spring 2025)

Overview

Welcome to Math 1B: Calculus!

In Math 1A or elsewhere, you studied functions of a single variable, limits, and continuity. You learned about derivatives, which describe how functions change, and which can be used to help find maxima and minima of functions. You also learned about integrals which describe the aggregate behavior of a function over an interval, such as the area under a curve or the average of a varying quantity. The derivative and the integral are tied together in the fundamental theorem of calculus, one version of which relates the integral of the derivative of a function over an interval to the values of the function at the endpoints of the interval.

In this course we will continue the study of calculus in three parts as follows:

- The first part of the course is about techniques of integration (sections 7.1 to 7.8 of the book). As you already know, differentiation is relatively straightforward: if you know the derivatives of elementary functions, and rules such as the product rule and the chain rule, then you can differentiate just about any function you will ever come across. Integration, on the other hand, is hard. Sometimes it is even impossible to integrate a given function explicitly in terms of known functions. We will introduce a collection of useful techniques with which you can integrate many functions. The hard part is to figure out which technique(s) to use in a given situation. For integrals which we cannot evaluate explicitly, we will learn how to find good approximations to the answer.

- The second part of the course is about sequences and series (chapter 11 of the book). This can be regarded as the general theory of approximating things. This part of the course is subtle and involves new ways of thinking. It may be a lot harder than the first part, especially if you have seen some of the first part before.

- The third part of the course is an introduction to ordinary differential equations (chapters 9 and 17 of the book). Here one tries to understand a function, given an equation involving the function and its derivatives. ("Ordinary" means that we consider functions of a single variable. Functions of several variables enter into "partial" differential equations, which you can learn about in a more advanced course.) The theory of differential equations is perhaps the most interesting part of calculus, is the subject of much present-day research, and has many real-world applications. Our study of differential equations will make use of most of the calculus we have done so far.

I hope you enjoy the class!

Alex

Lectures

Lectures are where I will introduce and explore the material. Throughout the semester, I'll do my best to make you part of the process, giving you the chance to try problems yourself and ask questions. If there's something you feel unsure of, don't feel nervous about putting your hand up. If you are thinking it, then so are lots of other people in the room. I'm here to guide you towards proper understanding, not deliver dry uninterupted monologues. The more like a back and forth conversation, the better!

LEC 001: 8.00am-9.30am TT, Wheeler Auditorium

LEC 002: 11.00am-12.30pm TT, Wheeler Auditorium

Both lectures will be identical and you are welcome to attend either, although it would be best to attend your offical section for the first few weeks. As is tradition at Berkeley, the lecture will formally begin ten minutes after the official start time. That gives you enough time to get across campus if you need to. Attendance at lecture is not mandatory, although I *strongly* advise that you go.

All lectures will come with pre-recorded videos and lecture notes. You can find these in the Schedule and Course Materials tab below. In terms of content, these will be identical to in-person lectures.

I strongly recommend against using the pre-videos as a total replacement for in-person lectures

In-person lectures offer a more focused learning experience, allowing you to ask and hear questions in real time. While videos are of course great for reviewing material at your own pace, it's challenging to stay fully engaged during an initial viewing. The temptation to skip through sections will lead to missing important details, something that's less likely to occur when we are all together in person.

> *"Don't fall behind, keep a steady pace so you understand what everyone's talking about in the next lecture/discussion section. Even then, reviewing right before your next Math 1B class always helps refresh your memory and help you better build on concepts."*
>
> 1B student, Fall 2022

Here's some advice about getting the most out of lectures and the pre-recorded videos.

- **Pay attention to the details.** These lectures will likely be in more depth than what you are used to. Simply grasping the basic idea of a concept won't suffice. Your goal should be to comprehend each concept in granular detail. Every point covered will be crucial to the topic, so if something isn't clear, don't ignore it. Ask questions in class, or jot down your questions to discuss them during office hours or in the discussion section later on. Understanding these nuances is key to mastering the material.

- **Take your own notes.** I'll be writing everything out in real-time during lecture. If you are following along with me doing the same, it'll really help you focus. A good tip is to highlight points that weren't clear to you so you can specifically revist them later.

- **Come back to the videos after the in-person lecture.** Video lectures offer great flexibility for learning. Being able to revisit them allows you to reinforce your understanding of complex topics or clarify any confusion you may have. This ability to review at your own pace can significantly enhance comprehension and retention of the material. Make use of it, but don't let it completely replace in-person lectures.

- **Be very cautious of other math resources online.** This course is structured to be as self-contained as possible. If you find yourself struggling with a concept, revisit the corresponding video lecture rather than turning to random

online sources. Such sources almost always lack sufficient detail and can lead to a false sense of understanding. Additionally, avoid relying on tools like ChatGPT. They won't contribute to your learning, especially since AI is still terrible at mathematics.

- **Make sure you understand the examples.** The exercises in the worksheets and homework are closely related to examples I go through in the lecture. Realy make sure you really understand the approach I'm taking to solving problems in the lectures.

- **Don't skip any lectures.** Even if the topic is familiar to you, there will be subtleties that you have likely not seen before. Think of it like reading a story. If you skip a paragraph you'll get lost very quickly.

You can find links to the lecture videos and notes in the Schedule and Course Materials tab below.

Discussion Section

Every week you will have three hours of discussion session led by a GSI or UGSI (graduate or undergraduate student instructor). These sessions provide an excellent opportunity to explore the material as a group, with each section consisting of around 40 students. In these interactive discussions, you will collaborate with your peers to tackle challenging problems, enabling you to truly grasp and excel in the subject matter. Here are some tips to get the most out of these sessions.

- **Mastering Mathematics is a challenge and most challenges are best approached as a group.** Regard your discussion section not only as an opportunity to understand the course material better but also as a place to connect with classmates for collaborative work outside of class. Having other people to bounce ideas off is incredibly valuable. The more perspectives on a difficult problem the better.

- **Don't feel apprehensive about participating.** Many of of us get nervous speaking in public, doubly so when it comes to mathematics. You may think that only you are struggling with a concept, but that is never ever true. If something's confusing, ask questions. If the instructor asks a question, try answering it. It can be scary at first but over time it won't be and the payoff will be enormous in the long run

- **Be active in group work.** Your will frequently be split into smaller groups where you'll work on problems more collaboratively. Be an active participant in these. Even if you're not completely confident in your approach, still share it.

- **Be there.** This is probably the most important of all. Even if you feel that you aren't getting much out discussion section, you are. It's three whole hours a weeks where you are completely focused on the class. Ask yourself the following question: Am I realy going to spend three hours a week going through the worksheets on my own? For me the answer would be no! That's a lot of time to lose and it adds up over the course of a semester. Not attending section is almost always a direct indicator of heading for a bad grade.

You can only attend the section you are officially enrolled in. If you have enrollment questions contact: enrollment@math.berkeley.edu

Attendance at discussion section is not mandatory, although I *strongly* advise that you go as I've explained above.

> *"Take advantage of discussion and office hours to practice more problems. You really do most of the learning in homework and discussion, just attending lectures isn't enough!"*
>
> 1B student, Spring 2020

Here is **contact information** for all GSIs and UGSIs.

You'll be following fixed worksheets in your discussions. They'll be posted in the Schedule and Course Materials

tab below.

In general, if you have any queries or issues relating to the course, the first person to contact directly is your GSI.

Office Hours

In addition to lectures and discussion section, you will have access to daily office hours. Office hours are your chance to talk to me or a GSI/UGSI. They are a really good way to get to know your instructors better and get help with any aspect of the course. These are also a space for you to work in small study groups at your own pace if you'd like.

In office hours you can talk about any aspect of the course (and beyond). If you've spent time on a homework problem and have stalled, come to office hours; If you're unsure about your academic trajectory, come to office hours; If you want to learn what mathematics research is about, come to office hours; If you're struggling with the course and don't know what to do, come to office hours; If you think you might need a reference from a professor in the future, come to office hours; If you just want a chat, come to office hours.

> *"Go to as many office hours as you can! I only started going after doing badly on the second midterm. I should have been attending from the start. It would have made a big difference."*
>
> 1B student, Spring 2019

I'll be having office hours 2pm-4pm on Tuesday, Wednesday, and Friday. They will be at my office (796 Evans Hall), or very close by if there is insufficient space. That that's the case there will be a note on the door with the appropriate room number. As a default GSI/UGSI office hours are in-person, although some will be conducted online via Zoom.

Here is a link with the full office hour schedule.

Review Materials

Mathematics is an extremely constructive subject. Every new concept builds on the ones that came before it. If there are gaps in your foundational knowledge, this is very likely to make things challenging fast.

This class assumes a solid grasp of a first curse in single variable calculus (Math 1A or the equivalent), which in turn assumes a solid grasp of the main topics in precalculus and the subjects that come before it.

Here are three resources I've designed to help you review these more foundational topics.

- **Precalculus Cheatsheet** This is the most basic of all. A lighting quick review of the key topics required before starting a first calculus class.
- **Precalculus Essentials** This is a comprehensive online precalculus course I designed several years ago. It explains everything in greater detail that other online resources and is a good place to turn if your struggling with specific topics.
- **Math 1A: Calculus** This is a complete first course in calculus. You'll find comprehensive videos and lecture notes here. The most important thing to be up to speed on is integration, especially substitution.

You'll also find a review video in the Schedule and Course Materials tab below. It's important that you look at this before the first day of class.

Textbook

The textbook for this course is: **Stewart, *Single Variable Calculus: Early Transcendentals for UC Berkeley***, 8th edition (ISBN: 9781305765276, Cengage).

This is a custom edition containing chapters 1-9, 11 and 17 of Stewart's "Calculus: Early Transcendentals", 8th edition. The regular edition is also fine, it just contains extra chapters covered in math 53. The 7th edition is also acceptable, but you will need to watch for differences in the numbering of assigned homework problems. In chronological order, we'll cover material in chapters 7, 11, 9, 17

Even though it's probably something you aren't used to doing, I **strongly** recommend reading the textbook throughout the semester. It provides more detail than can possibly be covered in lectures (either in-person or recorded), and is a habit you want to get into now. Future courses will not have videos to fall back on and reading the textbook will be absolutely vital. It's also where the homework exercises are taken form. If you read the examples in the main text it'll be much more obvious how to approach them.

> *" Reading the textbook alongside the lectures really helps. When you're stuck on a homework problem try to find a similar example in the book. "*
>
> 1B student, Spring 2018

Homework

There will be weekly homework assignments, each covering topics from the previous week. These will be due each Wedesday at 11.59pm on Gradescope (an online platform to submit assignments). You can access Gradescope from this **link**. Here are **instructions** for how to upload your work. **If you have issues submitting your work, contact your GSI/UGSI.**

The homework exercises will be from the textbook. Here is a link to the homework schedule. You can also link to this directly from the Schedule and Course Materials tab below.

Homework Solutions will be made available shortly after submission in the Schedule and Course Materials Tab.

Each assignment will be graded on a combination of completeness and correctness. More specifically you will get three points for submitting solutions to all exercises. In addition, each week a randomly selected question will be graded (out of three points) for correctness. Your two lowest scores will be dropped.

You may discuss the homework problems with your classmates, but **you must write your solutions on your own.** Make use of discussion sections and office hours if you need assistance, but in the end, you should still write up your own solutions.

Lectures are where new concepts are introduced, but homework is where much of the real learning happens. It's where you internalize the abstract ideas and discover for yourself how they can be used to solve problems. From my experience the main distinction between those who succeed in mathematics classes and those who don't is how they treat the homework. Here's some advice about how to approach it.

- **Be organized.** Don't leave things for the last moment. You'll may struggle to complete the homework assignment if you start on the night before it is due. Work consistently in small installments.
- **As you progress through the homework you'll notice it increasing in difficulty.** The more difficult problems are often the most important, giving you the opportunity to really master the material. Make sure you attempt them. Almost every problem will be very similar to one done in the video lectures or the worksheets. Consult those if you're unsure of where to start.
- **Seek help, but only after seriously thinking about a problem on your own.** If you're struggling with a challenging problem you should spend at least 30 minutes on your own thinking about it carefully. Even if you fail to make a breakthrough, this is still more worthwhile than giving up after a couple of minutes and talking to peer or looking up a random source on the internet. Knowing what doesn't work is just as important knowing what does. The questions will be very closely related to the lectures and textbook. Those should be where you look for

help first. In future courses you'll be solving problems that require days (or weeks) of dedicated thought. Now is the time to hone this skill and get comfortable with not being able to immediately solve a problem.

"Take the homework seriously. It will really help on exams as practice. In addition, doing more problems on the homework for subjects you are weak in will help as well"

1B student, Spring 2019

Projects and Quizzes

In addition to homework, roughly every week, you will have either a quiz or project.

Quizzes will take place on Thursday (LEC001) or Friday (LEC002), roughly every two week in discussion section. They will last about 15 minutes and be closely related to the homework submitted that week. Your lowest score will be dropped from your grade.

Projects will give you the opportunity to explore certain topics in more depth, perhaps exploring interesting real-world applications, or more subtle questions what we don't have time to cover in lectures. They will be due on Friday, roughly every two weeks on Gradescope. Here are **instructions** on how to upload your work. Your lowest score will be dropped from your grade.

The solutions to the projects will be available after submission in the Schedule and Course Materials tab below.

What	When
Quiz 1	Week 2
Project 1 - Probability and Calculus	Week 3
Quiz 2	Week 4
Project 2 - Fractals	Week 6
Quiz 3	Week 7
Project 3 - The Abel and Dirichlet Tests	Week 8
Quiz 4	Week 9
Project 4 - Direction Fields and Equilibia	Week 12
Quiz 5	Week 13
Project 5 - Vibrations and Resonance	Week 15

There will be no make-up quizzes, unless there are exceptional circumstances.

Exams

There will be two (1-hour) midterm exams and one (3-hour) final exam.

Following the course policy (see below), you can miss one midterm, for whatever reason, without penalty. On the other hand, missing both midterms will seriously harm your grade and make it much more difficult to pass the course.

As per university policy, if you do not sit the final exam, you automatically fail the course. Please check the dates now to make sure that you have no unavoidable conflicts.

Here is the exam schedule.

- **Midterm 1: Thursday February 20, in class (Wheeler Auditorium)**
- **Midterm 2: Thursday April 3, in class (Wheeler Auditorium)**
- **Final Exam: LEC 001 Thursday May 15, 7pm(sharp)-10pm (Location TBD); LEC 002 Thursday May 15, 8am(sharp)-11am (Location TBD).**

Calculators are NOT be allowed for the exams. That's the policy for all university math classes. That may seem scary, but it means the exams are designed not to require challenging mental arithmetic. Thats a good thing!

Cheatsheets are not be allowed for the exams. Making one yourself is a very good way of preparing, but it can't be taken into the test.

To obtain full credit for an exam question, you must obtain the correct answer and give a correct and readable derivation or justification of the answer. Unjustified correct answers will be regarded very suspiciously and will receive little or no credit. The graders are looking for demonstration that you understand the material. To maximize credit, cross out incorrect work. We will be scanning all exams so you will get them back electronically.

After each midterm, there will be a brief window when you can request a regrade. If you are unsure about making a regrade request consult myself or your GSI beforehand. Regrade requests may result in a lowering of your grade. As per university policy, final exams cannot be regraded.

All special exam accomodations for DSP students will be arranged by us and not the DSP proctoring service. If you are a DSP student with exam accomodations we will contact you directly the week before an exam.

It goes without saying that **cheating** is unacceptable. Any student caught cheating will be reported to higher authorities for disciplinary action.

There will be no make-up exams, unless there are truly exceptional circumstances.

Here is my basic advice on how to do well in exams.

- **Take a moment to look through the whole exam at the start.** We can all agree that exams are no fun. I have always been very anxious taking exams and something that always helped was looking through the exam for a few minutes at the start. Find a question you know how to approach and tackle that first. It'll help settle your nerves.

- **Success is in the details.** It's not enough that you have a broad superficial understanding of a topic. You must appreciate the subtleties and how they turn up in problems.

- **Diagnose and solve systematically.** Once you've identified what type of problem you are dealing with, think about what's in your toolkit. How does one approach such a problem in a methodical way? You want to be familiar and comfortable with the standard procedures to deal with different problems. This is a skill for life and is the essence of analytic problem solving.

- **The practice exams are the single most important component of exam preparation.** I'm going to provide you with four practice exams for each test. These will be pretty challenging. You should be aiming for mastery of every question in the practice exams before the real thing. The main exam will be close variations of these questions and if you've not spent serious time with them it is going to be much harder. Ideally, by the time you take the main exam you want to be able to write perfect solutions to the practice questions unprompted. You should be looking at these at least one week before the main exam. **Solutions to practice exams will be provided one week before the main exams.**

  " Spend a lot of time studying the practice exams. Don't leave it till the day before to look at them. They are hard, but the real exams are basically the same. "

Lastly, I know that many of you find exams acutely stressful (I know I do). If you are struggling (before or during the exam) do come and speak to me or a GSI. We are here to help.

Schedule and Course Materials

This is your one-stop-shop for all the course materials: videos, notes, worksheets, etc. As the semester progresses each of the links below will go live. For example, video lectures will be available directly after the LEC002 in-person lecture is complete.

When	What	Where
Week 1 (1/20 - 1/24)	First Calculus Course Review	
	1. Integration by Parts	7.1
	2. Rational Integrals	7.4
Week 2 (1/27 - 1/31)	3. Trigonometric Integrals	7.2
	4. Trigonometric Substitution	7.3
	Assignments - Homework 1 (1/29) and Quiz (1/30 or 1/31)	
Week 3 (2/3 - 2/7)	5. Strategies of Integration	7.5
	6. Approximate Integration	7.7
	Assignments - Homework 2 (2/5) and Project (2/7)	
Week 4 (2/10 - 2/14)	7. Errors and Approximate Integration	7.7
	8. Improper Integrals	7.8
	Assignments - Homework 3 (2/12) and Quiz (2/13 or 2/14)	
Week 5 (2/17 - 2/21)	9. Sequences and Series	11.1, 11.2
	Midterm 1 (In class, Thursday 2/20)	
	Assignments - Homework 4 (2/21)	
Week 6 (2/24 - 2/28)	10. Divergence and Integral Tests	11.2, 11.3
	11. Comparison and Alternating Series Tests	11.4, 11.5
	Assignments - Homework 5 (2/26) and Project (2/28)	
		11.6

Week 7 (3/3 - 3/7)	12. The Ratio and Root Tests	
	13. Strategies for Series Testing	11.7
	Assignments - Homework 6 (3/5) and Quiz (3/6 or 3/7)	
Week 8 (3/10 - 3/14)	14. Power Series	11.8
	15. Representing Functions as Power Series	11.9
	Assignments - Homework 7 (3/12) and Project (3/14)	
Week 9 (3/17 - 3/21)	16. Taylor Series	11.10
	17. Errors and Taylor Polynomials	11.10
	Power Series Review Video	
	Assignments - Homework 8 (3/19) and Quiz (3/20 or 3/21)	
Week 10 (3/24 - 3/28)	**Spring Break!**	
Week 11 (3/31 - 4/4)	18. Differential Equations and Direction Fields	9.1, 9.2
	Midterm 2 (In class, Thursday 4/3)	
	Assignments - Homework 9 (4/4)	
Week 12 (4/7 - 4/11)	19. Separable Equations	9.3
	20. Population Growth and Linear First-Order Equations	9.4, 9.5
	Assignments - Homework 10 (4/9) and Project (4/11)	
Week 13 (4/14 - 4/18)	21. Complex Numbers	
	22. Linear Second-Order Homogeneous Equations	17.1
	Assignments - Homework 11 (4/16) and Quiz (4/17 or 4/18)	
Week 14 (4/21 - 4/25)	23. Linear Second-Order Non-homogeneous Equations	17.2
	Assignments - Homework 12 (4/23)	
Week 15 (4/28 - 5/2)	24. Power Series Solutions to Differential Equations	17.4
	Something Fun	

	Assignments - Homework 13 (4/30) and Project (5/2)	
Week 16 (5/5 - 5/9)	RRR week	
Week 17 (5/12 - 5/16)	Final Exams (5/15)	

Grading and Course Policy

Grades are calculated as follows:

Homework	10%
Projects and Quizzes	10%
First Midterm	20%
Second Midterm	20%
Final Exam	40%

If your final exam score is higher than your lowest midterm exam score, then it will replace it, thus accounting for 60% of your grade. This means you can miss a midterm for any reason whatsoever and it will not necessarily adversely effect your grade.

Your final letter grade will ultimately be decided by your ability to demonstrate a crisp understanding of the material and the ability to apply it to a diverse set of problems. Broadly speaking I will be looking for the following criteria for each letter grade:

- A-/A/A+: A clear demonstration that the central concepts have been fully understood; Computational techniques (and their many subtleties) have been mastered and can be applied accurately to a diverse problem set; A strong understanding of how the abstract concepts can be applied to many real world applications.
- B-/B/B+: Demonstration that the central concepts have been reasonably understood, but perhaps with minor misunderstandings; Core computational techniques have been reaonably understood (but generally not key subtleties) and can be applied fairly accurately to a fairly large problem set; Reasonable understanding of how the abstract concepts can be applied to some real world applications.
- C-/C/C+: Demonstration that the central concepts have been vaguely understood, but with major misunderstandings; Core computational techniques have been poorly understood and can be a applied accurately only in the most standard examples; Weak understanding of how the abstract concepts can be applied to even basic real world applications.

To be as fair as possible, I will also take into account the historic average of the class. This means that if I set an exam which is very difficult it will be taken into account in the final letter grades.

Please note: incomplete grades, according to univlersity policy, can be given only if *unanticipated events beyond your control* (e.g. a medical emergency) make it impossible for you to complete the course, *and if you are otherwise passing* (with a C- or above).

External Support

Enrollment: For question about enrollment contact Marsha Snow, or email enrollment@math.berkeley.edu.

The Student Learning Center provides support for this class, including full adjunct courses, review sessions for exams, and drop-in tutoring. This is a truly fantastic resource. I definitely recommend you take advantage of it.