Department of Mechanical Engineering **Fall Semester 2015**
University of California, Berkeley **J. Casey**

ME 185: Introduction to Continuum Mechanics

TuTh 3:30-5:00, 3113 Etcheverry Hall

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SYLLABUS

0 INTRODUCTION

0.1 The continuum: a mathematical model. Phenomenological theories. Purely mechanical versus thermodynamical theories. Material behavior and constitutive equations: examples from elasticity, fluid mechanics, rigid body mechanics. Historical remarks.
0.2 The six primitive concepts of the purely mechanical theory: body, space, time, mass, force, torque. One, two, and three-dimensional bodies, and combinations of them.
0.3 Coordinate systems. Reference frames in classical mechanics.
0.4 The three balance laws: mass, linear momentum, angular momentum. Remark on Newton's Laws versus Euler's Laws.
0.5 Scalars, vectors, tensors. Euclidean vector spaces. Index notation. Tensor product and direct notation. Multilinear mappings and determinants.
0.6 Derivatives and differentials.

1 KINEMATICS

1.1 Body, configurations, motions. Convected curvilinear and Cartesian coordinates.
1.2 Particle velocity. The deformation gradient.
1.3 Material, referential, and spatial descriptions of fields.
1.4 The material derivative.
1.5 Material transport of lines, surfaces and volumes.
1.5 Deformation of a line element. The Cauchy-Green deformation tensors and their properties. The stretch tensors. Maximal stretching. Method of Lagrange multipliers. Strain tensors.
1.7 Deformation of area and volume elements.
1.8 The polar decomposition theorem.
1.9 Representation of the rotation tensor. Application to rigid body kinematics.

1.10 The velocity gradient. The rate of deformation tensor (or stretching tensor), and the vorticity tensor and vector. Material derivative of deformation measures.
1.11 Streamlines and vortex-lines. Circulation. Vortex tubes.
1.12 Rigid motions superposed on a given motion of a deformable body. Objective

fields. Objective rates of vectors and second-order tensors. Interpretation of upper and lower convected rates.

1.13 Linearization of kinematical measures. Fréchet derivative. Infinitesimal strain and rotation.

2 BALANCE LAWS

2.1 The divergence theorem and some applications.
2.2 The transport theorem.
2.3 Mass and mass density. Traction and body forces. Torques.
2.4 Conservation of mass: Material, referential, and spatial integral statements. Point forms of conservation law.
2.5 Balance of linear momentum: Integral statements.
2.6 Balance of angular momentum: Integral statements.
2.7 Rigid body dynamics.
2.8 Applications of the balance laws in integral form (e.g.: material flowing through pipes).
2.9 The Cauchy stress tensor. Existence theorem. Properties of the stress tensor.
2.10 Cauchy's first and second laws.
2.11 Application: the expansion of the universe under Newtonian gravitation.
2.12 Kelvin's kinematical theorem and Helmholtz's vorticity theorems.
2.13 Piola transforms. Piola-Kirchhoff stress tensors. Referential form of Euler's laws and Cauchy's laws.
2.14 Invariance requirements under superposed rigid motions.
2.15 The work-energy theorem.
2.16 Remarks on thermodynamical concepts, and on the first and second laws of thermodynamics.
2.17 On the derivation of Euler's Laws from the energy equation.

3 CONSTITUTIVE RELATIONS

3.1 Concept of a *material* in continuum mechanics. Examples of constitutive relations. Solids and fluids. Constrained and unconstrained materials.
3.2 Local action, "equipresence," materials with memory.
3.3 Restrictions due to invariance requirements.
3.4 Material symmetry.
3.5 Ideal fluids. Incompressibility.
3.6 Viscous fluids. Linear viscous fluids. Navier-Stokes equations.
3.7 Elastic solids. Examples of simple deformations. Uniqueness and non-uniqueness of solutions to boundary value problems.
3.8 Linearly elastic solids. Bending and torsion of bars. Saint Venant's principle.
3.9 Linear viscoelasticity.

SELECTED REFERENCES

Continuum Mechanics

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CM2. P. Chadwick. *Continuum Mechanics.* Dover Publications, Inc., Mineola, New York, 1999.
CM3. P. Papadopoulos. *ME 185* (Classnotes). Berkeley, 2008.
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FM1. L. Prandtl & O.G. Tietjens. *Fundamentals of Hydro- and Aerodynamics.* Dover Publications Inc., New York, 1957.
FM2. J. Lighthill. *An Informal Introduction to Theoretical Fluid Mechanics.*Oxford University Press, Oxford, 1996.
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Solid Mechanics

SM1. J. Bell. The experimental foundations of solid mechanics. *Encyclopedia of Physics,*Vol. VIa/1 (Eds.: S. Flügge and C. Truesdell), pp. 1-778. Springer-Verlag, Berlin, 1973.
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SM3. R.W. Ogden. *Nonlinear Elasticity with Application to Material Modelling.* AMAS Lecture Notes, **6**. Institute of Fundamental Technological Research, Warsaw, 2003.
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GRADING SCHEME

Homework: 40%
Midterm Exams: 20%
Final Exam: 40%

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