Course Announcement - Spring 2019

Math 255: Algebraic Curves

Instructor: Bernd Sturmfels

Office hours: Wednesdays 8:00-10:00 and by appointment
Contact: bernd at math, 925 Evans

Time and Place: Tuesdays and Thursdays, 9:30-11, 3 Evans Hall

Prerequisites: Abstract Algebra at the level of Math 250A. Ideally, also Undergraduate Algebraic Geometry (Math 143) and
Commutative Algebra (Math 250B). Experience in working with Fields, Rings, Modules, Ideals, and their
Gröbner Bases.

Text Books: The following two text books will be used in this class:
Frances **Kirwan**: Complex Algebraic Curves, London Mathematical Society Student Texts, 23, Cambridge
University Press, 1992.
William **Fulton**: Algebraic Curves. An Introduction to Algebraic Geometry, Reprint of 1969 original, Addison-Wesley, 1989.

Syllabus: Before Spring Break, we will cover the core material on curves from the two text books: local properties, plane curves,
morphisms and rational maps, Riemann surfaces, differentials, Puiseux series, resolution of singularities, and the Riemann-Roch Theorem.
After Spring Break, students and instructor will present selected topics (e.g. 19th century geometry, algorithms, moduli, and tropical curves).

Term Papers: Students select a topic of their choice related to algebraic curves. They will research that
topic and write a term paper about their findings. Presentations on these projects will take place in April.

Grading: The course grade will be based on the homework (50%) and the term paper (50%).

Consultants: Madeline Brandt and Lynn Chua will help with the course. Questions can be directed to either
them or me.

Further Reading: Here is a selection of recommended resources on algebraic curves:
Lecture Notes from the Math 255 class taught by **Hendrik Lenstra** in the Fall of **1995**.
Egbert Brieskorn and Horst Knorrer: Plane Algebraic Curves, Birkhauser Verlag, Basel, 1986.
Joe Harris and Ian Morrison: Moduli of Curves, Graduate Texts in Mathematics, 187, Springer 1998.
George Salmon, Arthur Cayley: A Treatise on the Higher Plane Curves, Elibron Classics, original from 1852.
Rafael Sendra, Franz Winkler and Sonia Perez-Diaz: Rational Algebraic Curves - A Computer Algebra
Approach, Springer, 2008.
Ernesto Girondo and Gabino González-Diez: Introduction to Compact Riemann Surfaces and Dessins d'Enfants,
Cambridge University Press, 2011.

Schedule:
January 22: Foundations [Kirwan, Chapter 2]
January 24: Bezout's Theorem [Kirwan, Section 3.1]
January 29: Points of inflection and cubic curves [Kirwan, Section 3.2]
January 31: The degree-genus formula [Kirwan, Section 4.1]

February 5: Branched covers of the line [Kirwan, Sections 4.2-4.3]
February 7: Invariant Theory of Plane Curves
February 12: The Weierstrass p-function [Kirwan, Section 5.1]
February 14: Riemann surfaces [Kirwan, Section 5.2]
February 19 [MB]: Holomorphic differentials [Kirwan, Section 6.1]
February 21 [MB]: Abel's Theorem [Kirwan, Section 6.2]
February 26: The Riemann-Roch Theorem [Kirwan, Section 6.3]
February 28: The Riemann-Roch Theorem [Kirwan, Section 6.3]
March 5: Local rings, DVRs, Multiplicities [Fulton, Sections 2.4, 2.5, 3.1, 3.2]
March 7: Linear Systems, Multiple Points, Noether's Theorem [Fulton, Sections 5.2, 5.4, 5.5]
March 12: Varieties, Morphisms, and Rational Maps [Fulton, Chapter 6]
March 14: Resolution of Singularities [Fulton, Chapter 7]
March 19 [LC]: Divisors, Riemann's Theorem, Differentials [Fulton 8.1-8.4]
March 21 [LC]: Canonical Divisors and Riemann-Roch [Fulton 8.5-8.6]
Student Lectures:
April 2: The Plücker Formulas [Yuhan Jiang]
April 2: Inflection Points of Plane Cubics [Tyler Zhu]
April 4: Orthogonal Matrices with Maximal 4-Norm [Zitong Yang]
April 4: Riemann-Roch on Graphs [Frederick Huang]
April 9: Riemann-Roch in the 20th Century [Siyang Liu, Zhongkai Tao]
April 9: Riemann-Roch for Algebraic Surfaces [Nikolay Grantcharov, Sanat Mulay]
April 11: Algebraic Geometry Codes [Siqi Liu, Peter Manohar, Tahsin Saffat]
April 11: Elliptic Curve Cryptography [Andrew Gitlin, Kristina Nelson, Jana Sotakova]
April 16: Completions of Rings and Curve Singularities [Marvin Castellon]
April 16: Chow Rings [Holly Mandel]
April 18: Differential Forms on Riemann Surfaces [Suxuan Chen]
April 18: Belyi's Theorem [Emilio Valle]
April 23: Modular Curves and Modularity Theorems [Zhenghui Li]
April 23: Mordell-Weil Theorems [Grant Posner]
April 23: Nonnegative Polynomials and Sums of Squares [Han Feng, 11:10h in 939 Evans]
April 25: Moduli Spaces of Riemann Surfaces [Ziwen Zhao]
April 25: Moduli Spaces of Stable Maps [Foster Tom]
Workshops:
April 30-May 3: Hyperbolic Polynomials at the Simons Institute
May 6-May 10: Moduli Spaces at MSRI

Homework: There are seven assignments. Click on the date to see solutions:
due January 29: Kirwan 2.2, 2.4, 2.5, 2.7, 2.8, 3.1, 3.6
due February 5: Kirwan 3.3, 3.8, 3.11, 3.13, 3.14, 3.16
due February 12: Kirwan 4.1, 4.2, 4.3, 4.4, 4.5
due February 19: Kirwan 5.4, 5.9, 5.10, 5.12, 5.14, 5.18
due February 26: Kirwan 6.1, 6.3, 6.5, 6.6, 6.7, 6.8
(Problem 6.3 has a typo: one occurrence of "meromorphic" should be "holomorphic")
due March 5: Kirwan 6.10, 6.11, 6.15 and Fulton 2.17, 8.2, 8.6
(Problem 6.15: must assume that the curve has genus one)
due March 12: Fulton 2.25, 2.28, 3.6, 3.14, 5.11, 5.19, 5.21, 5.21, 5.30

Term paper deadlines:
Thursday, March 14: Project proposal is due
Tuesday, May 14: Final term paper is due