

# Linear Algebra and Differential Equations

## GENERAL COURSE INFORMATION

**Lectures:** MWF 10:10–11, Room 155 Dwinelle

**Course Control Number:** 22496

**Instructor:** [Constantin Teleman](https://bspace.berkeley.edu/portal/index.html) (<https://bspace.berkeley.edu/portal/index.html>), 903 Evans, [e-mail \(mailto:teleman@math.berkeley.edu\)](mailto:teleman@math.berkeley.edu)

**Office Hours:** M11:15-12:30, W11:45-1, Friday by appointment.

**GSI Office Hours:**

- Kuan M3–4, W3–4:30 in 1042 Evans;
- Lim M9–10, 11–12 in 1020;
- Luecke W3–4, Th10–11 in 854;
- Nelson WTh1–2 in 1060;
- Oltman M12–2 in 840;
- Padgett M1–3 in 1060;
- Shankar TuTh 2:30–3:30 in 739;
- Stefanich W9–10, 12–1 in 737;
- Yan M2–3, W3–4 in 747

**Prerequisites:** Math 1B.

**Topics covered:**

Basics of linear equations. Matrix algebra. Vectors, vector spaces and Linear transformations. Determinants. Eigenvalues and eigenvectors. Diagonalization. Inner product spaces. Projections. Least squares. Linear ordinary differential equations. Fourier series.

**Textbook:**

Custom UC Berkeley Edition, combining in a single volume parts of:

David Lay, *Linear Algebra*, 5th ed., and

Nagle, Saff & Snider, *Fundamentals of differential equations and boundary value problems*, 9th ed.

The schedule of lectures, homework assignments and additional reading is found in the syllabus.

## HOUSE RULES

**Class attendance:** Highly recommended; the only way to stay current with the material, which is essential. There is evidence that taking notes in real time helps your understanding. Questions during lecture are welcome: if you think you spot a mistake, cannot read a formula or find a comment difficult to digest, your colleagues may well be grateful to you for raising that. Still, the limitations of

the lecture medium might mean that complex points are better addressed in individual or small group discussion; please do take advantage of office hours.

You may find that you do not follow everything in lecture. This is not unusual, especially upon first exposure to the material. Write things down, working out a personal code to flag things you are unsure about, or which you want to review later. Reading the relevant textbook sections in advance is a great way to improve your uptake in class. Re-read the book after lecture, and refer to it again when doing your homework.

**Class behavior:** Please display reasonable courtesy to your peers. Late arrivals and early departures, texting, email and other use of social media can be quite distracting for your neighbors.

**Section attendance:** Mandatory. Your section grade will suffer if you go AWOL.

## ASSESSMENT

**Homework:** Assigned for every lecture; please keep up, as it is essential for following the material, which builds up quite quickly. We are experimenting with electronic submission and assessment. For now, the homework comes in several portions, all assigned for each lecture:

- WebWork, due by midnight on the day of the next class meeting. (WebWork link in the left column).
- A written assignment (gold highlighted problems on the syllabus), due in section every week, normally on Tuesdays. These will be marked based on completion and not reviewed individually
- A thinking assignment (**green problems** on the syllabus), to be discussed in section. Your GSI may call upon you for comments, so do give them some thought before the section meeting.
- Additional recommended problems (**black** on the syllabus), which also serve for exam review. No submission is needed, but you are encouraged to discuss them.

**Quizzes:** Every Thursday in section (exam weeks excepted).

**Exams:** In-class, CLOSED book unless otherwise instructed, NO calculators or electronic devices permitted.

- Exam 1, in class, Friday 4 October
- Exam 2, in class, Wednesday 6 November
- Final, Monday 16 December, 8–11am, Location: TBA.

**Grading Formula:** 40% Homework+Quizzes+Section; 20% each Exam; 40% Final.

The lowest 20% exam component (the Final has two) will be discarded, as will be the two lowest quiz scores. We will assign a letter grade to each item above; these will be averaged into your final grade, weighted as above. (In other words: the exams and section grades are curved separately and there is no final re-curving.)

In borderline cases, we will use the numerical scores to decide the final grade.

**Important: No make-up examinations or quizzes** are offered. PLEASE mark these dates in your diary. Any scheduling issues with the midterms should be discussed with me or with your GSI **no later than Wednesday, September 4.**

If you must miss an exam due to force majeure, we expect advance notice: please tell or e-mail your GSI. The normal solution is that the missed exam will become the one omitted from the formula. If you skip an exam without notice, you can expect the failing grade to be included in your tally.

If you miss the final exam, absent arrangements, you should expect to fail the course.

Cheating on exams results in automatic failure, in addition to other disciplinary action. Plus, it's really really embarrassing for everyone.

**Collaboration:** You are encouraged to work in groups on the **written** homework problems, but the submission must be your own writing. The online homework (WebWork) must be your own doing.

### Discussion sections:

| Section | Days/Times             | Location       | Instructor       | Class |
|---------|------------------------|----------------|------------------|-------|
| 201 DIS | TuTh 05:00PM - 06:29PM | Dwinelle 206   | Isabelle Shankar | 22314 |
| 202 DIS | TuTh 08:00AM - 09:29AM | Evans 75       | Dong Gyu Lim     | 22315 |
| 203 DIS | TuTh 05:00PM - 06:29PM | Evans 7        | Izak Oltman      | 22413 |
| 204 DIS | TuTh 08:00AM - 09:29AM | Dwinelle 251   | Kristina Nelson  | 22414 |
| 205 DIS | TuTh 11:00AM - 12:29PM | Hearst Gym 242 | Kiran Luecke     | 22415 |
| 206 DIS | TuTh 11:00AM - 12:29PM | Evans 87       | German Stefanich | 22416 |
| 207 DIS | TuTh 09:30AM - 10:59AM | Dwinelle 228   | Dong Gyu Lim     | 22417 |
| 208 DIS | TuTh 12:30PM - 01:59PM | Dwinelle 229   | Kiran Luecke     | 22418 |
| 209 DIS | TuTh 12:30PM - 01:59PM | Cory 237       | German Stefanich | 22419 |
| 210 DIS | TuTh 03:30PM - 04:59PM | Dwinelle 229   | Isabelle Shankar | 22420 |
| 211 DIS | TuTh 02:00PM - 03:29PM | Evans 70       | Adele Padgett    | 22421 |
| 212 DIS | TuTh 03:30PM - 04:59PM | Kroeber 115    | Kristina Nelson  | 24533 |
| 213 DIS | TuTh 03:30PM - 04:59PM | Wheeler 220    | Xiaohan Yan      | 22433 |
| 214 DIS | TuTh 02:00PM - 03:29PM | Barrows 80     | Xiaohan Yan      | 22434 |
| 215 DIS | TuTh 03:30PM - 04:59PM | Evans 736      | Adele Padgett    | 22435 |
| 216 DIS | TuTh 06:30PM - 07:59PM | Dwinelle 247   | Izak Oltman      | 22693 |
| 217 DIS | TuTh 06:30PM - 07:59PM | Dwinelle 130   | Jeffrey Kuan     | 22694 |
| 218 DIS | TuTh 05:00PM - 06:29PM | Evans 87       | Jeffrey Kuan     | 25758 |

# Course Syllabus

## [Jump to Today](#)

### Mathematics 54-2, Fall 2019 Lecture schedule and Homework assignments

**Please remember to check this page regularly for changes,  
especially before starting your homework assignment!**

This version: 16 December

Section numbers for lectures 1–26 refer to Lay; thereafter, to Nagle, Saff&Snider.

The Berkeley edition combines them in a single volume.


The right strategy is to read the sections *before* the lecture, and then consult them again thereafter and as you go through your homework assignment. Additional relevant reading is linked to the relevant lecture.

Some non-examinable topics (starred) may be dropped if we are pressed for time.

Homework assignments are listed with the relevant lecture. Remember:

- There is a WebWork assignment for (almost) every lecture, accessed through the WebWork link
- **Gold highlighted problems** are due in section, on the **Tuesday of the following week**; your GSI will confirm.
- **Green boldface** questions will be discussed in section: think about them ahead of time. (You need not submit answers.) **Similar questions will appear on the tests**, possibly in T/F or Multiple Choice format.
- **Black problems** are optional; no submission is needed, but you are encouraged to discuss them and refer to them for exam review.

|    | Date    | Topic                                                                                                | Book Sections                                           | Homework                                                                                                    |
|----|---------|------------------------------------------------------------------------------------------------------|---------------------------------------------------------|-------------------------------------------------------------------------------------------------------------|
| 1  | W28 Aug | Linear systems. Matrices. Row-reduction.                                                             | 1.1, 1.2; <a href="#">Reading</a>                       | <b>1.1:</b> Odds 3-13, 15; 20, <b>23, 24</b> ; <b>1.2:</b> 1, 3, 5, 6                                       |
| 2  | F30 Aug | Echelon forms. Parametric solution.                                                                  | 1.2                                                     | <b>1.2:</b> 7, 9, 11; <b>21-24</b> , 25, 26; <b>1.3:</b> 5, 11, 12, <b>24</b> ;<br><b>1.5:</b> <b>29-32</b> |
| 3  | W4 Sep  | Vector and Matrix equations. Parametric vector solution.                                             | 1.3 (omit Span for now), 1.4, 1.5                       | <b>1.4:</b> 1, 5, 7, 9, 17, <b>24, 31, 34</b> ; <b>1.5</b> (postponed)                                      |
| 4  | F6 Sep  | Span and linear independence.                                                                        | 1.3 (Span), 1.7                                         | <b>1.5:</b> <b>9, 10, 14, 23, 24, 29-32</b> , 36; <b>1.7:</b> 1-7, 9, 11, 17, <b>21, 22</b> , 23;           |
| 5  | M9 Sep  | Subspaces of $\mathbf{R}^n$ . Basis and dimension.<br>Bases of special subspaces of $\mathbf{R}^n$ . | 2.8, 2.9, 4.2 (up to Kernel and Range )                 | <b>2.8:</b> Odds 1-11; <b>2.9:</b> 3, 5, <b>15</b> ; <b>4.2:</b> 1, 3, 5, 17, 23, <b>25abdf</b>             |
| 6  | W11 Sep | Linear transformations                                                                               | 1.8, 1.9                                                | <b>1.8:</b> 1, 3, 9, 11, 17; <b>1.9:</b> Odds 1-7, 11, 15, <b>23, 24</b>                                    |
| 7  | F13 Sep | Matrix algebra.                                                                                      | 2.1                                                     | <b>2.1:</b> Odds 1-7, 9, <b>15, 16</b> , 17, <b>23, 24</b> , 27;<br><b>2.3:</b> <b>11</b> , 21, 24, 30      |
| 8  | M16 Sep | Invertible matrices and solutions to linear systems                                                  | 2.2, 2.3                                                | <b>2.2:</b> 1, 3, <b>9</b> , 11, 21; <b>2.3:</b> Odds 3-7, <b>12, 13, 15</b> , 18;                          |
| 9  | W18 Sep | Abstract vector spaces and subspaces                                                                 | 4.1 , 4.2 (Kernel and Range)                            | <b>4.1:</b> 1, 3, 9, 11, 17, <b>23, 24, 27</b> ; <b>4.2:</b> 7, 9, <b>25, 26</b> , 30                       |
| 10 | F20 Sep | Bases and coordinates                                                                                | 4.3, 4.4 (also review 2.8, 2.9) <a href="#">Reading</a> | <b>4.3:</b> 1, 3, 9, 11, 15, <b>21, 22</b> , 32, 33; <b>4.4:</b> 3, <b>15, 16</b> , 17                      |
| 11 | M23 Sep | Dimension and rank                                                                                   | 4.5, 4.6                                                | <b>4.5:</b> Odds 1-7, <b>19, 20</b> , 26, 27; <b>4.6:</b> 1, 3, 5, 9, <b>10-15</b> , 33                     |
| 12 | W25 Sep | Change of basis and change of coordinates                                                            | 4.7                                                     | <b>4.7:</b> 1, 3, 5, 11, 12, 13; Supplementary <b>1</b> , 10, 11                                            |
| 13 | F27 Sep | Determinants                                                                                         | 3.1, 3.2                                                | <b>3.1:</b> 1, 5, 9, 13, 31; <b>3.2:</b> 1, 3, 5, 7, 11, 19, 21, <b>27, 28</b>                              |
| 14 | M30 Sep | Applications: Cramer's rule and volumes                                                              | 3.3; <a href="#">Reading</a>                            | <b>3.2:</b> 31, <b>39</b> ; <b>3.3:</b> 3, 5, 7, 11, 24, Supplementary <b>1</b>                             |
|    | W2 Oct  | Review                                                                                               | <a href="#">Quick review questions</a>                  | Extra practice problems recommended                                                                         |
|    | F4 Oct  | <b>EXAM #1. In class at the usual time.</b>                                                          | <a href="#">midterm1ans</a>                             | Covers Lectures 1–13                                                                                        |

|    |         |                                                       |                                                                                                                          |                                                                                              |
|----|---------|-------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------|
| 15 | M7 Oct  | Eigenvalues, eigenvectors. Dynamical systems          | 5.1; 5.6 Examples 1-4                                                                                                    | <b>5.1:</b> 1, 3, 9, 11, 17, <b>21, 22;</b>                                                  |
|    | W9 Oct  | Class cancelled                                       |                                                                                                                          |                                                                                              |
|    | F11 Oct | Class cancelled                                       |                                                                                                                          |                                                                                              |
| 16 | M14 Oct | Characteristic equation. Diagonalization              | 5.2, 5.3                                                                                                                 | <b>5.2:</b> Odds 1-9, 17, <b>21, 22</b> <b>5.3:</b> 1, 2, 3, 9, 11, <b>21, 22;</b>           |
| 17 | W16 Oct | Geometry of Diagonalization                           | 5.4                                                                                                                      | <b>5.4:</b> 1, 3, 9, 17;                                                                     |
| 18 | F18 Oct | Complex eigenvalues and rotations                     | 5.5; <b>Reading</b>                                                                                                      | <b>5.5:</b> Odds 1-5, 7, 9, 11, 13, <b>23.</b>                                               |
| 19 | M21 Oct | Inner product, length, angles, orthogonality.         | 6.1, 6.2                                                                                                                 | <b>6.1:</b> 1,5,7,9, 13, 17, <b>19, 20,</b> 22, 24; <b>6.2:</b> 3, 9, 17, 21, <b>23, 24;</b> |
| 20 | W23 Oct | Projections. Best approximations                      | 6.3                                                                                                                      | <b>6.3:</b> 1, 3, 5, 7, 11, 17, <b>21, 22;</b>                                               |
| 21 | F25 Oct | The Gram-Schmidt process. QR factorization*           | 6.4                                                                                                                      | <b>6.4:</b> Odds 1-9, <b>17, 18ab,</b> 22                                                    |
| 22 | M28 Oct | Least Squares and Normal equations                    | 6.5, <b>Lecture</b> and Recordings <b>1, 2</b> and <b>3</b>                                                              | <b>6.5:</b> 1, 3, 7, 13, <b>17, 18;</b>                                                      |
| 23 | W30 Oct | Applications of Least Squares                         | 6.6                                                                                                                      | <b>6.6:</b> 1, 3, 5, 7a. <b>Optional: 6.7:</b> 1, 3, 5, 7, 13, 19, 22;                       |
| 24 | F1 Nov  | Spectral Theorem for Symmetric matrices               | 7.1                                                                                                                      | <b>7.1:</b> 7, 11, 13, 17, 24, <b>25, 26</b>                                                 |
|    | M4 Nov  | Review                                                | Chapts. 5,6, 7.1                                                                                                         | Supplementary <b>1</b>                                                                       |
|    | W6 Nov  | <b>EXAM #2.</b> In class at the usual time.           | <b>midterm2ans</b>                                                                                                       | Covers Lectures 15 — 24                                                                      |
| 25 | F8 Nov  | Singular Value Decomposition*                         | 7.4                                                                                                                      | <b>7.2:</b> 3, 5, 9, 19, <b>21, 24</b> or <b>7.4:</b> 1, 3, 5, 7, 9                          |
| 26 | W13 Nov | Introduction. 1st and 2nd order equations. Linearity  | 4.1, 4.2 <b>Reading I Complex Exp</b>                                                                                    | <b>4.2:</b> Odds 5-15, 26;                                                                   |
| 27 | F15 Nov | Generalities on differential equations                | 4.3 <b>Reading II</b>                                                                                                    | <b>4.3:</b> 1, 3, 9, 11, 33abc                                                               |
| 28 | M18 Nov | Method of Undetermined coefficients                   | 4.4, 4.5; <b>Reading III</b>                                                                                             | <b>4.4:</b> Odds 1-13, 16; <b>4.5:</b> 1, 3, 5, 7, <b>9-15</b>                               |
| 29 | W20 Nov | First-order linear equations. Reduction of order      | <b>Reading IV&amp;V</b>                                                                                                  |                                                                                              |
| 30 | F22 Nov | Variable coefficients. Variation of parameters        | 4.6, 4.7 after Example 3                                                                                                 | <b>4.6:</b> 1, 3, 9; <b>4.7:</b> 32, 33, 47                                                  |
| 31 | M25 Nov | Matrix methods for ODEs                               | 9.1 - 9.4                                                                                                                | <b>9.4:</b> 1, 3, 9, 11, 17, 21, 24, 27                                                      |
| 32 | M2 Dec  | The eigenvector method (constant-coefficients)        | 9.5, 9.6 <b>Reading VII&amp;VIII</b>                                                                                     | <b>9.5:</b> 13, 15, 19, 21, 31, 33; <b>9.6:</b> 1, 3, 9                                      |
| 33 | W4 Dec  | Matrix exponential. Inhomogeneous systems             | 9.8;                                                                                                                     | <b>9.7:</b> 3, 5, 11, 13, 25; <b>9.8:</b> 7, 9, 24, 25                                       |
| 34 | F6 Dec  | Fourier series of periodic functions*                 | 10.3 <b>Fourier Reading</b>                                                                                              | <b>10.3:</b> 9, 11, 17, 19, 28, 34, 35                                                       |
|    | M9 Dec  | <b>Optional:</b> Fourier series and the heat equation | <b>Reading and more Reading</b>                                                                                          |                                                                                              |
|    | W11 Dec | No class meeting, ask GSI about review                |                                                                                                                          |                                                                                              |
|    | M16 Dec | <b>FINAL EXAM, 8–11am,</b> RSF Field House            | <b>Prep info</b>  <b>FinalWithKey</b> | Comprehensive (all the material)                                                             |

## Course Summary:

| Date | Details |
|------|---------|
|      |         |