Midterm #1

Name

Scoring

1		(15)
2		(20)
3		(20)
Total		(55)

Instructions

This exam is closed book. For reference, a formula sheet is attached. You must submit this formula sheet along with your exam. Please write your name at the top of all sheets submitted. Clearly state any assumptions you make beyond what is described in the problem. You may assume small angle approximations are valid. Partial credit may be awarded based on understanding of principles embedded in each problem. Therefore extraneous or irrelevant computations will *decrease* your overall score where the final answer is incorrect. If you need more space to write, use the back of the page.

The duration of the examination is 50 minutes.

Good Luck

Name: morgan

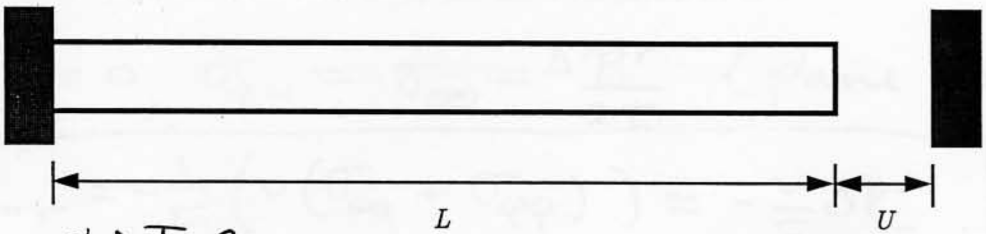
Midterm #1

Problem 1

Consider the bar shown below, having length L and cross-sectional area A. The material has elastic modulus E, yield stress σ_y, and coefficient of thermal expansion α.

a) If this bar is subjected to a uniform increase in temperature ΔT, what is the gap size U such that the bar just reaches its yield stress? 10

b) Given the gap size U computed above, what is the reaction at the right support due to ΔT? 5



$\varepsilon = \alpha \Delta T$. 3

a) $\Delta T = \Delta T_1 + \Delta T_2$ ← development of stress up to yield

ΔT_1: elongation

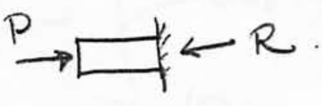
$$\Delta T = \frac{1}{\alpha}\frac{u}{L} + \frac{\sigma_y}{E}\cdot\frac{1}{\alpha}$$

solve for u → $u = \left(\Delta T - \frac{\sigma_y}{\alpha E}\right)\cdot \alpha L$.

$$\boxed{u = \alpha \Delta T L - \frac{\sigma_y L}{E}}$$ 7.

b) Force in bar is $P = \sigma \cdot A$

since bar just yields, $\sigma = \sigma_y$ → $\boxed{R = \sigma_y A}$ 5



F.B.D.

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Problem 2

A spherical balloon having radius r and thickness t is subjected to an <u>increase</u> in internal pressure Δp. You may assume $r \gg t$. The balloon is made of material having elastic modulus E and Possion's ratio ν.

a) At any point on the surface of the balloon, what are the three components of axial strain, $\varepsilon_{rr}, \varepsilon_{\theta\theta}, \varepsilon_{\varphi\varphi}$, due to the pressure change Δp? 10

b) What is the change in stored energy due to the pressure change Δp? (hint: in spherical coordinates, $dV = r^2 t d\theta d\varphi$ for small thickness t.) 6.

a) $\sigma_{rr} = 0, \quad \sigma_{\varphi\varphi} = \sigma_{\theta\theta} = \frac{\Delta p r}{2t}$ (plane stress)

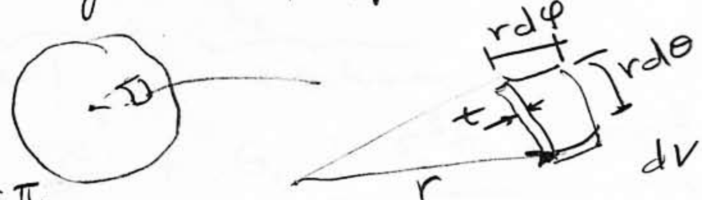
$$\boxed{\begin{aligned} \varepsilon_{rr} &= -\frac{1}{E}\left(\nu\left(\sigma_{\theta\theta} + \sigma_{\varphi\varphi}\right)\right) = -\frac{\nu}{E}\frac{\Delta p r}{t} \\ \varepsilon_{\theta\theta} &= \frac{1}{E}\left[\sigma_{\theta\theta} - \nu\sigma_{\varphi\varphi}\right] = \frac{(1-\nu)}{E}\frac{\Delta p r}{2t} \\ \varepsilon_{\varphi\varphi} &= \varepsilon_{\theta\theta} = \frac{(1-\nu)}{E}\frac{\Delta p r}{2t} \end{aligned}}$$

10.

b) $W_{st} = \int_V \frac{1}{2}\left(\sigma_{\theta\theta}\varepsilon_{\theta\theta} + \sigma_{\varphi\varphi}\varepsilon_{\varphi\varphi}\right) dV$ (note $\sigma_{rr} \approx 0$)

look @ small segment of sphere

see hint



$dV = t \cdot r^2 d\varphi d\theta$.

$$W_{st} = \frac{1}{\cancel{2}} \int_0^{2\pi}\int_0^{\pi} \cancel{2} \cdot \frac{(1-\nu)}{E} \cdot \frac{\Delta p^2 r^2}{4t^2} \cdot t \cdot r^2 d\varphi d\theta.$$

$$\boxed{W_{st} = \frac{\pi^2 (1-\nu)\Delta p^2 r^4}{2tE}}$$

units $[F \cdot L]$ ✓

10.

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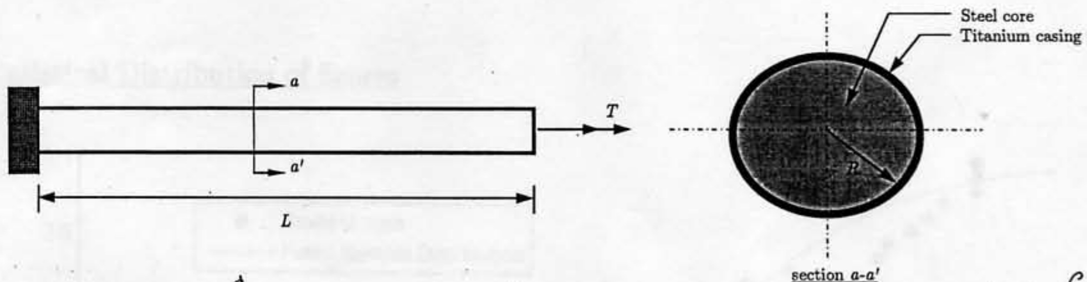
Problem 3

Consider the circular composite shaft shown below, having length L, and cross-section as indicated. The core of the shaft is mild steel having radius R. This mild steel shaft is fitted with a titanium encasement of small thickness t (that is, $R \gg t$.) Mild steel has shear modulus G_s and yield shear stress τ_{ys}. Titanium has shear modulus G_t and yield shear stress τ_{yt}. You may assume elastic-perfectly-plastic material response.

a) Under the applied torque T, what is the twist at the end of the shaft (in radians) assuming elastic behavior?

b) What is the ultimate torque, T_u, that the shaft can carry? Remember, this is greater than the yield torque, T_y (which you are not asked to solve.)



a) $$T = \int_A \tau r \, dA = \int_A G(r)\, r^2 \frac{d\phi}{dz} \cdot dA = \frac{d\phi}{dz} \underbrace{\int_A G(r) r^2 dA}_{(GJ)_{eff}}$$

$$(GJ)_{eff} = \underbrace{\int_{A_s} G_s \cdot r^2 dA}_{\text{steel}} + \underbrace{\int_{A_t} G_t r^2 dA}_{\text{titanium}}$$

$$(GJ)_{eff.} = G_s \cdot \frac{\pi R^4}{2} + G_t \cdot 2\pi R^3 t = \pi R^3 \left(G_s \frac{R}{2} + 2G_t t \right)$$

$$\text{So, } \phi(L) = \frac{TL}{(GJ)_{eff}} = \boxed{\frac{TL}{\pi R^3 (G_s R/2 + 2G_t t)}}$$

b) $$T_u = \tau_{ys} + t \cdot \tau_{yt} = 2\pi \int_0^R \tau_{ys} \cdot r^2 dr + 2 \underbrace{\tau_{yt} \cdot t}_{q} \cdot \underbrace{\pi R^2}_{A_e}$$

$$T_u = \frac{2\pi}{3} \tau_{ys} R^3 + 2\pi R^2 \cdot \tau_{yt} \cdot t \rightarrow \boxed{T_u = 2\pi R^2 \left(\frac{R \tau_{ys}}{3} + \tau_{yt} \cdot t \right)}$$