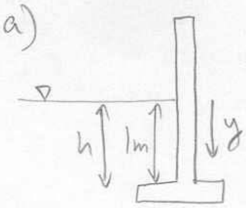
CE 100 Midterm #1 Solutions

10/10/2005

Problem 1)

a)



w = length into page

$$F_H = \gamma h_c A = \gamma(\tfrac{1}{2}h)(h \cdot w)$$

$$F_H = (9.81\,\text{m/s}^2 \cdot 1000\,\text{kg/m}^3)(\tfrac{1}{2} \cdot 1\text{m})(1\text{m} \cdot 100\,\text{m})$$

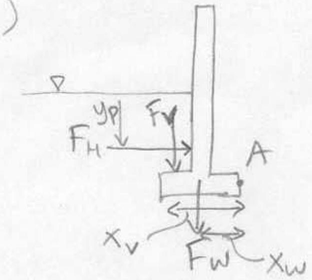
$$\underline{F_H = 490.5\,\text{kN}}$$

$$F_V = P(y=1\text{m})A = \gamma h A$$

$$F_V = (9.81\,\text{m/s}^2 \cdot 1000\,\text{kg/m}^3)(1\,\text{m})(0.25\,\text{m} \cdot 100\,\text{m})$$

$$\underline{F_V = 245.3\,\text{kN}}$$

b)



$$\Sigma M_A = 0$$

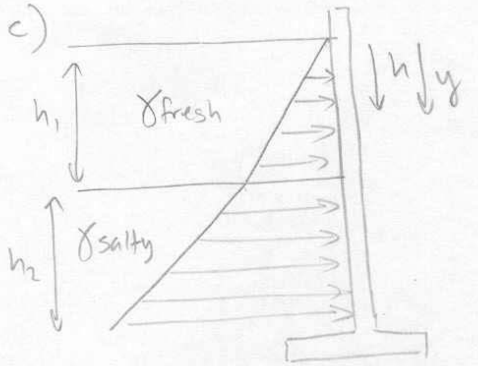
$$F_H y_P - F_v x_v - F_W x_W = 0$$

$$F_W = \frac{1}{x_W}(F_H y_P - F_V x_V)$$

$$F_W = \frac{1}{0.25\,\text{m}}\left(490.5\,\text{kN} \cdot \tfrac{1}{3}\,\text{m} - 245.25\,\text{kN} \cdot 0.375\,\text{m}\right)$$

$$\underline{F_W = 286.1\,\text{kN}}$$

c)



$h_1 = 1m, \; h_2 = 1m$

$P_1(y) = \gamma_f y$ (fresh water layer)

$P_2(y) = \gamma_f h_1 + \gamma_s (y - h_1)$ (salt water layer)

Check: $P_1(0) = 0, \; P_1(h_1) = \gamma_f h_1$

$P_2(h_1) = \gamma_f h_1$

$P_2(h_1 + h_2) = \gamma_f h_1 + \gamma_s h_2$

$F_{H_1} = \gamma_f h_c A = (9.8 \cdot 1000)(\tfrac{1}{2})(1 \cdot 100) = \underline{490.5 \text{ kN}}$ (same as before)

$$F_{H_2} = w \int_{h_1}^{h_1+h_2} \gamma_f h_1 + \gamma_s (h - h_1) \, dy$$

$$= \gamma_f h_1 h_2 w + \gamma_s \frac{(y-h_1)^2}{2} w \Big]_{h_1}^{h_1+h_2}$$

$$= \gamma_f h_1 h_2 w + \gamma_s \frac{h_2^2}{2} w = (1000 \cdot 9.81)(1)(1)(100) + (1030 \cdot 9.81)(\tfrac{1}{2})(100)$$

$F_{H_2} = \underline{1486.2 \text{ kN}}$

$\underline{F_{H_{total}} = 1976.7 \text{ kN}}$

$F_V = P_2(h_1 + h_2) \cdot A$

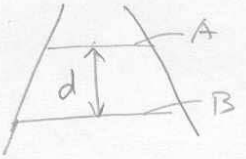
$= (\gamma_f h_1 + \gamma_s h_2)(0.25 \cdot 100 m)$

$= (1000 \cdot 9.81 \cdot 1 + 1030 \cdot 9.81 \cdot 1)(0.25 \cdot 100)$

$F_V = \underline{497.9 \text{ kN}}$

Problem 2)

a) Use material derivative in 1-D



$$\frac{Du}{Dt} = \underset{\substack{0\\ \text{steady}}}{\cancel{\frac{\partial u}{\partial t}}} + u\frac{\partial u}{\partial x} = \underbrace{\left(\frac{V_A + V_B}{2}\right)}_{\substack{\text{average}\\ \text{velocity}\\ \text{betw. A \& B}}} \frac{V_B - V_A}{d}$$

$$Q \doteq 2.5 \text{ gpm} = 2.5 \frac{\text{Gal}}{\text{min}} \cdot \frac{3.79\text{L}}{1\text{Gal}} \cdot \frac{1\text{ min}}{60\text{ s}} \cdot \frac{1\text{m}^3}{1000\text{L}} = 0.000158 \text{ m}^3/\text{s}$$

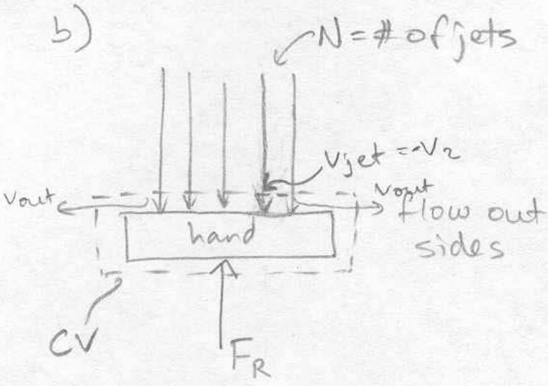
$Q = V_A A_A = V_B A_B$ $A_A = 0.00126 \text{ m}^2$, $A_B = 0.00503 \text{ m}^2$

$d = 0.03 \text{ m}$

$V_A = 0.125 \text{ m/s}$ $V_B = 0.0314 \text{ m/s}$

$$\frac{Du}{Dt} = \left(\frac{0.125 + 0.0314}{2}\right)\left(\frac{0.0314 - 0.125}{0.03}\right) = \underline{-0.244 \text{ m/s}^2}$$

b)



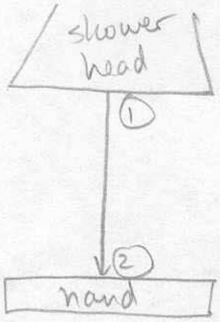
$$\sum F_z = \sum_{out} \dot{m} V - \sum_{in} \dot{m} V$$

$$F_R = 0 - \dot{m} V_{jet}$$

$$\dot{m} = \rho Q = \rho V_{jet} A_{jet} \cdot N$$

$$\dot{m} = (1000 \text{ kg/m}^3)(0.000158 \text{ m}^3/\text{s}) = 0.158 \text{ kg/s}$$

Find V_{jet}: use Bernoulli equation from ① to ②



$$\underset{0}{\cancel{P_1}} + \tfrac{1}{2}\rho V_1^2 + \gamma z_1 = \underset{0}{\cancel{P_2}} + \tfrac{1}{2}\rho V_2^2 + \gamma z_2$$

pressure in jet is atmospheric

$$Q = V_1 \underset{\substack{\uparrow \\ \text{area of} \\ \text{hole at shower head}}}{A_h} N = 0.000158 \text{ m}^3/\text{s}$$

$$V_1 = \frac{Q}{A_h N}$$

Solve for V_2:

$$\tfrac{1}{2}\rho V_2^2 = \tfrac{1}{2}\rho V_1^2 + \gamma(z_1 - z_2) \qquad (\gamma = \rho g)$$

$$V_2 = \sqrt{V_1^2 + 2g(z_1 - z_2)} = \sqrt{\frac{Q^2}{A_h^2 N^2} + 2g(z_1 - z_2)}$$

For $N = 30$, $V_2 = \sqrt{\dfrac{(0.000158 \text{ m}^3/\text{s})^2}{(3.14\times10^{-6} \text{ m}^2 \cdot 30)^2} + 2(9.81)(0.25 \text{ m})} = 2.78 \text{ m/s}$

$N = 50$, $V_2 = 2.43 \text{ m/s}$

$V_{jet} = -V_2 \Rightarrow F_{R_{30}} = +\dot{m} V_{2,30} = 0.439 \text{ N}$

$F_{R_{50}} = \dot{m} V_{2,50} = 0.384 \text{ N}$

Force felt on hand opposite: $F_H = -F_R \Rightarrow \boxed{F_{H_{30}} = -0.44 \text{N}}\ \boxed{F_{H_{50}} = -0.38 \text{N}}$