University of California Berkeley
Department of Civil and Environmental Engineering

CEE 122: Design of Steel Structures
Fall 2008

Midterm 2:
Beams and Beam-Columns

11/18/2008, 502 Davis Hall, 2 hours

Name ___________________________________

Problem	Points	Maximum
1		25
2		25
3		25
4		25
total		100

Honor Pledge:

I have neither give nor received aid during this examination, nor have I concealed any violation of the Honor Code.

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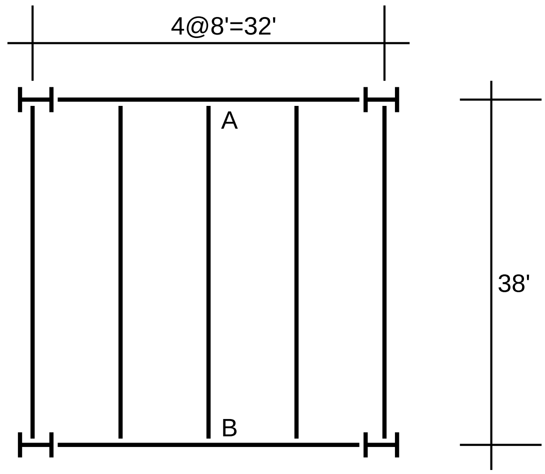
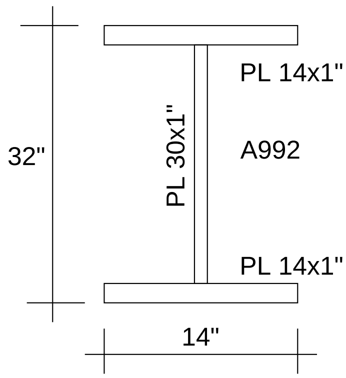
Problem 1: (25%)

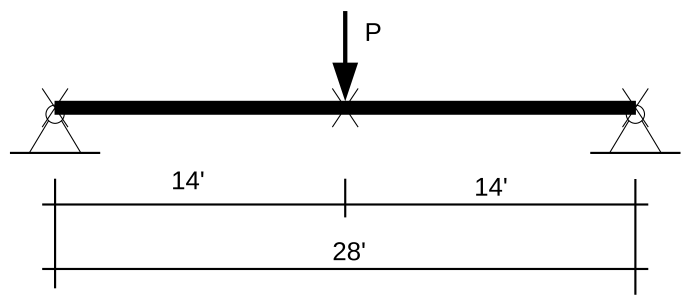
Select the lightest available A992 W-section for beam AB in the floor system shown below using AISC LRFD provisions. The conventional reinforced concrete floor slab is 6 inches thick. The self-weight of the beam must be taken into account. The uniform live load is 110 psf. There are no other loads. The maximum allowable deflection is L/240. Assume that the floor slab provides continuous lateral bracing for the beams.



Problem 2: (25%)

Determine the maximum value of service load P that can be carried by this beam using AISC LRFD considering 1) bending; 2) shear strength and 3) deflection limit of L/360. The load P is 25% dead load and 75% live load. Disregard the self weight. The beam is braced at the supports and at the mid-span point only. Beam section is built-up of A992 steel plates, as shown. Use the **User Note** in section F2, page 16.1-48 to calculate L_r for this doubly symmetric section with rectangular flanges. In this user note h is the clear distance between flanges.



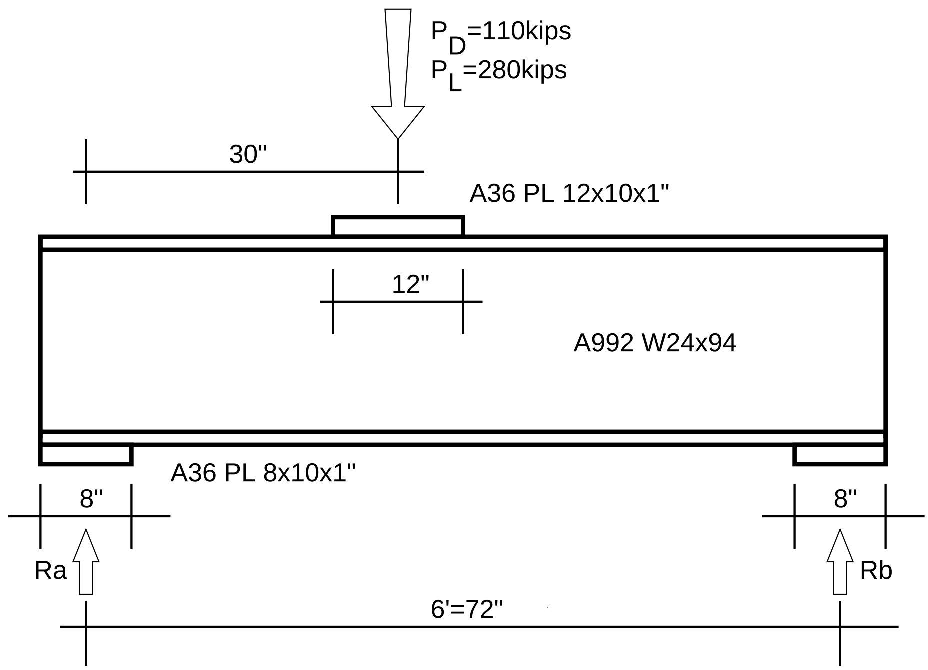


Problem 3: (25%)

An 6-foot long A992 W24x94 underpins a column located 30” from the left support, as shown. The column carries dead and live axial loads, as shown. The bearing plates under the column and the beam are made of A36 steel and have dimensions as shown. Check if this beam satisfies the AISC LRFD provisions for:

1. Bending
2. Shear
3. Web yielding and web crippling under the column
4. Web yielding and web crippling at the support

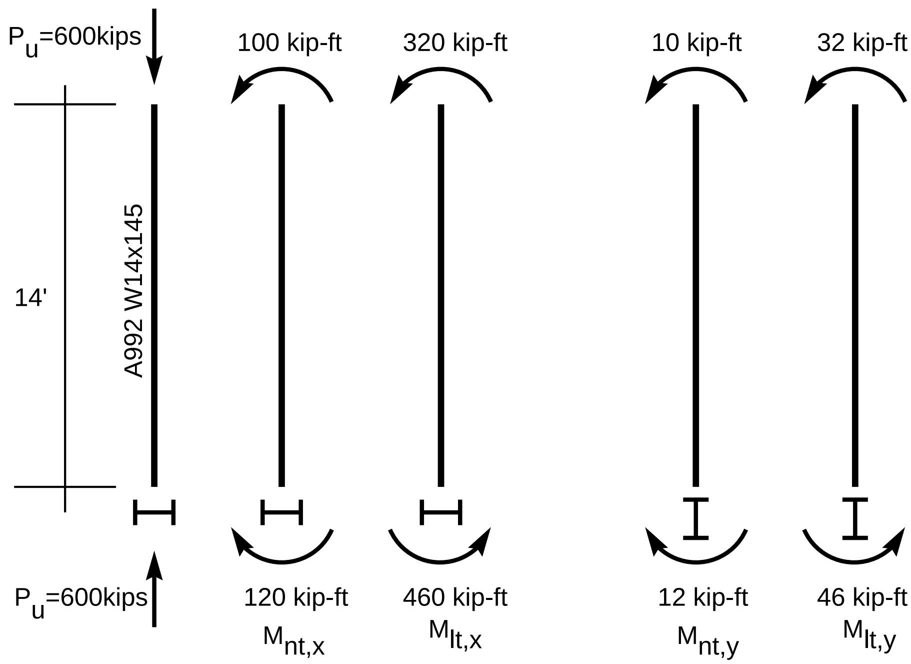
You do not have to re-design in case the beam does not satisfy the provisions. You can do the statics assuming the column force and the reactions are point loads.



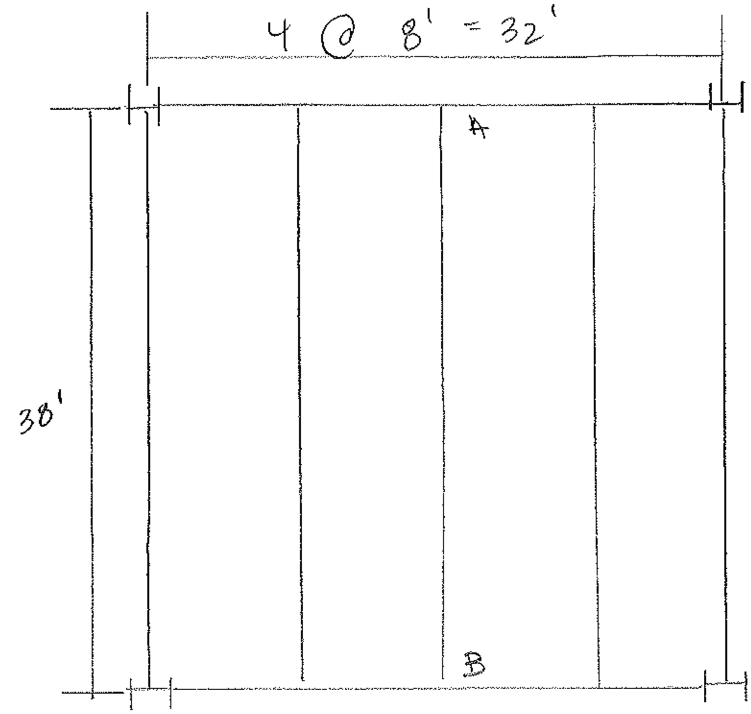
Problem 4: (25%)

Is an A992 W14x145 W-section is used as a 14-foot tall column in a building frame loaded as shown adequate?

This frame is not braced in its plane, making $K_x = 1.7$. The frame is braced out-of-plane, making $K_y = 1.0$. First-order analysis for factored gravity and lateral loads gives bending moments M_{nt} and M_{lt} about the beam-column strong and weak axis, as shown. The unbraced length of this column is 14 feet about both axes. To compute amplification factor B_2 use the total factored axial load above this story ($\sum P_u$) of 6000 kips and the sum of Euler loads for the story ($\sum P_{e2}$) of 36,000 kips.



Problem 1 Select lightest A992 W-Section for beam AB.



normal wt concrete 6"-thick slab
self-wt of beam
LL = 100 psf

Assume slab provides continuous lateral bracing

$\Delta_{allow} = \frac{L}{240}$

DL 1) concrete slab $\frac{6"}{12"} \cdot 150 \text{ lb/ft}^3 = 75$ psf

2) self-wt assume 50 lb/ft

$w_D = 75 \cdot 8' + 50 = 650$ lb/ft

LL 100 psf

$w_L = 110 \cdot 8 = 880$ lb/ft

$w_u = 1.2w_D + 1.6w_L = 1.2(650) + 1.6(880) = 2188$ lb/ft

$w = w_D + w_L = 1530$ kip/ft

$\Delta_{allow} = \frac{L}{240} = \frac{38' \cdot 12"}{240} = 1.9"$

$\Delta_{allow} = 1.9"$

req'd $I = \frac{5wL^4}{384E\Delta_{allow}} = \frac{5(0.65+0.88)(38)^4 \cdot 12^3}{384(29000)(1.9)} = 1302.7$ in^4

$I_{req'd} = 1302.7$ in^4

Try 24x55 ($I_x = 1350$ in^4) (Table 3-3)

$w_D = 75 \cdot 8 + 55 = 655$ lb/ft

check Δ $\Delta = \frac{5wL^4}{384EI} = \frac{5(0.655+0.88)(38)^4 \cdot 12^3}{384(29000)(1350)} = 1.83"$ OK

CHECK MOMENT: $\frac{b_f}{2t_f} = 6.94 < \lambda_p = 9.15$; $\frac{h}{t_w} = 53.6 < 90.55 = \lambda_p$

SECTION IS COMPACT FOR $F_y = 50$ KSI

$L_b = 0'$ SLAB

$\Rightarrow \phi_b M_n = \phi_b M_p = 503$ KIP-FT (TABLE 3-2)

$w_u = 1.2 \cdot w_D + 1.6 \cdot w_L = 1.2 \cdot 0.655 + 1.6 \cdot 0.88 = 2.20$ KIP/FT

$M_u = \frac{1}{8} w_u L^2 = \frac{1}{8} \cdot 2.2 \cdot 38^2 = 396$ KIP-FT $< \phi_b M_p$ (OK)

CHECK SHEAR

$\frac{h}{t_w} = 53.6 > 2.24\sqrt{\frac{E}{F_y}} = 53.9 \Rightarrow$ G.2(b) APPLIES

$\Rightarrow k_v = 5$

$\Rightarrow \frac{h}{t_w} = 53.6 \leq 1.10\sqrt{\frac{k_v \cdot E}{F_y}} = 59.24 \Rightarrow C_v = 1.0$

$\Rightarrow \phi_v V_n = (1.0) \cdot 0.6 F_y \cdot A_w \cdot C_v =$

$= (1.0)\ 0.6 \cdot 50 \cdot (0.395 \cdot 23.6) \cdot 1.0 = 279.66$ KIPS

OR TABLE 3-2 $\phi_v V_n = 251$ KIPS

$V_u = \frac{1}{2} w_u L = \frac{1}{2} \cdot 2.2 \cdot 38 = 41.8$ KIPS $< \phi_v V_n$ (OK)

TOPS 35500

Problem 2 Determine max service load P that can be carried by this beam.

Consider 1) bending
2) Shear
3) deflection limit $\Delta_{allow} = \frac{L}{360}$

$P = 0.25D + 0.75L$

Disregard self-wt

Use steel A992 $F_y = 50$ ksi $F_u = 65$ ksi

Verify if shape is compact

flange $\lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 9.15$

$\lambda = \frac{b_f}{2t_f} = \frac{14}{2(1)} = 7'' < \lambda_p$ compact

web $\lambda_p = 3.76\sqrt{\frac{E}{F_y}} = 90.55$

$\lambda = \frac{h}{t_w} = \frac{30}{1} = 30 < \lambda_p$ compact

For this built-up shape,

$A = 2(14)(1) + 30(1) = 58 \text{ in}^2$

$d = 32''$ $t_f = 1''$

component	$\bar{I}_x$	A	dx	$(\bar{I} + Ad_x^2)_x$	$\bar{I}_y$	d_y	Ad_y^2
Flange	1.167	14	15.5	3364.7	228.67	∅	∅
Flange	1.167	14	15.5	3364.7	228.67	∅	∅
web	2250	—	—	2250	2.5	∅	∅
sum				8979.4 in⁴	459.84 in⁴		

$I_x = 8979.4 \text{ in}^4$ $I_y = 459.84 \text{ in}^4$ $r_x = \sqrt{\frac{I_x}{A}} = 12.4$ in

$S_x = \frac{I}{c} = 561.2 \text{ in}^3$ $r_y = \sqrt{\frac{I_y}{A}} = 2.82$ in

component	A	y	Ay
flange	14	15.5	217
web/2	15	7.5	112.5
sum	29		329.5

$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{329.5}{29}$

$\bar{y} = 11.36''$

$Z_x = \left(\frac{A}{2} \cdot \bar{y} \cdot 2\right) = 659 \text{ in}^3$

From user note in section F2:

$$r_{ts} = \frac{b_f}{\sqrt{12\left(1 + \frac{1}{6}\frac{h t_w}{b_f t_f}\right)}} = \frac{14}{\sqrt{12\left(1 + \frac{1}{6}\frac{(30)(1)}{(14)(1)}\right)}}$$

$r_{ts} = 3.47''$

$$L_r = \pi r_{ts} \sqrt{\frac{E}{0.7 F_y}} = \pi(3.47)\sqrt{\frac{29000}{0.7(50)}} = 313.8 \text{ in}$$

$L_r = 26.15'$

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} = 1.76(2.82)\sqrt{\frac{29000}{50}} = 119.5 \text{ in}$$

$L_p = 9.96' \sim 10'$

1) BENDING: $L_p < L_b = 14' < L_r$

$$M_n = C_b\left[M_p - (M_p - 0.7F_y S_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] < M_p$$

$M_p = F_y Z_x = 32950$ kip-in $\quad C_b = 1.67$ (TABLE 3-1)

$0.7F_y S_x = 19672$ kip-in

$\Rightarrow M_n = 49480$ kip-in $> M_p \gg M_n = M_p = 2745.8$ kip-ft

Let $M_u = \frac{P_u L}{4} = \phi_b M_p = 2471.25$ kip-ft $\longrightarrow P_u = 353.0$ kips

$P_u = 1.2(0.25 \cdot P) + 1.6(0.75 \cdot P) = 353.0$ kips

$P = 235.33$ kips

2) Shear

$V_n = 0.6 F_y A_w C_v = 0.6(50)(30 \cdot 1)(1.0) = 900$ kips

$\phi_v V_n = 900$ kips

Let $V_u = \phi_v V_n$

$\frac{P_u}{2} = 900$ kips $\longrightarrow P_u = 1800$ kips

$1800 = 1.2(0.25 \cdot P) + 1.6(0.75 \cdot P) = 1.5P$

$P = 1200$ kips

3) deflection limit

$$\Delta_{allow} = \frac{L}{360} = \frac{28 \cdot 12}{360} = 0.93''$$

$$\Delta = \frac{P_s L^3}{48EI} \qquad (P_s \text{ - unfactored})$$

$$P = \frac{\Delta\, 48EI}{L^3} = \frac{(0.93) \cdot 48 \cdot (29000) \cdot (6979)}{(28 \cdot 12)^3}$$

$P = 306.4$ kips (unfactored)

max value of service load P

$$\boxed{P = 235.33 \text{ kips}}$$

Problem 3 W24x94 A992

Bearing Plate : A36

Check for
1) Bending
2) shear
3) web yielding & web crippling under column
4) web yielding & web crippling at support.

P_D = 110 kips $P_{self wt}$ = 94 · 6 = 564 kips = 0.564 kips

P_L = 280 kips

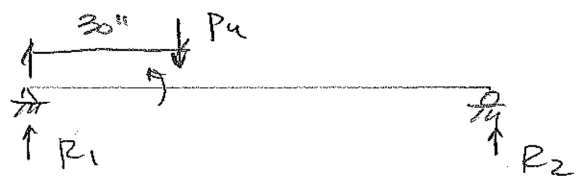
$P_u = 1.2(P_D + P_{self wt}) + 1.6 P_L$ = 580 kips

1) For a W24x94 $L_b = 6'$

From the Zx table : $L_p = 6.99'$ $L_r = 21.2'$

$L_b < L_p$ → no LTB

$\phi_b M_n = \phi_b M_{px}$ = 953 k · ft



$R_2(72) - P_u(30) = 0$ → $R_2 = 0.42 P_u$ = 243.6 kips

$R_1(72) - P_u(42) = 0$ → $R_1 = 0.58 P_u$ = 336.4 kips

$M_u = 0.58 P_u (30")$ = 841 k · ft < $\phi_b M_p$ = 953 k · ft
<u>Ok</u>

2) Shear

$\phi_v V_n = \phi_v (0.6 F_y A_w C_v)$

$\frac{h}{t_w} = 41.9 < 2.24\sqrt{\frac{E}{F_y}} = 54$ → $C_v = 1.0$

$\phi_v V_n = 1.0(0.6(50))(24.3 \cdot 0.515)(1.0) = 375.4$ kips

R_2 = 0.42 (580) = 243.6 kips < $\phi_v V_n$ <u>Ok</u>

R_1 = 0.58 (580) = 336 kips > $\phi_v V_n$ <u>OK</u>

3) web yielding and web crippling under column.

A36 PL 12 x 10 x 1

web yielding

$R_n = (5k + N) F_y t_w$

$N = 12''$

$k = 1.38''$

$t_w = 0.515''$

$F_y = 50ksi$

$t_f = 0.875''$

$d = 24.3''$

$R_n = 486.7$ kips

$\phi R_n = 486.7$ kips

web crippling (DISTANCE $> \frac{d}{2} \times 12''$)

$$R_n = 0.8 t_w^2 \left[1 + 3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{E F_y t_f}{t_w}}$$

$$= 0.8(0.515)^2\left[1 + 3\left(\frac{12}{24.3}\right)\left(\frac{0.515}{0.875}\right)^{1.5}\right]\sqrt{\frac{(29000)(50)(0.875)}{0.515}}$$

(J10-4)

$R_n = 555$ kips $\xrightarrow{\phi = 0.75}$ $\phi R_n = 416.25$ kips

web yielding $\phi R_n < P_u$ (N.G.)

web crippling $\phi R_n < P_u$ (N.G.)

4) web yielding and web crippling @ support

A36 PL 8 x 10 x 1 $N = 8''$

web yielding $R_n = (2.5k + N) F_{yw} t_w$

$R_n = 295$ kips $\longrightarrow$ $\phi R_n = 295$ kips (J10.5b)

web crippling $\frac{N}{d} = \frac{8}{24} = 0.33 > 0.2$ $R_n = 0.40 t_w^2 \left[1 + \left(\frac{4N}{d} - 0.2\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{E F_{yw} t_f}{t_w}}$

$R_n = 250.5$ kips $\longrightarrow$ $\phi R_n = 187.88$ kips

web yielding $\phi R_n < R_1$ (NG)

$\phi R_n > R_2$ OK

web crippling $\phi R_n < R_1$ (NG)

$\phi R_n < R_2$ (NG)

Beam is inadequate

Problem 4 Is A992 W14x145 used as a 14' column adequate.

frame not braced in plane $k_x = 1.7$
braced out of plane → $k_y = 1.0$

$L_b = 14'$

for B_2 : $\Sigma P_u = 6000$ kips $\Sigma P_{e2} = 36,000$ kips

W14x145 $\phi_b M_{px} = 975$ k·ft $L_p = 14.1'$ $L_r = 61.7'$

1) Column action

$k_x L_x = 1.7(14) = 23.8'$ $k_y L_y = 14'$

$r_x = 6.33''$ $r_y = 3.98''$

$\frac{(kL)_x}{r_x} = 45.1$ $\frac{(kL)_y}{r_y} = 42.2$

→ governs.

check compactness

$\lambda_r = 0.56\sqrt{\frac{E}{F_y}} = 13.48 > \frac{b_f}{2t_f} = 7.11$ ok

$\lambda_r = 1.49\sqrt{\frac{E}{F_y}} = 35.88 > \frac{h}{t_w} = 16.88$ ok

$\frac{(kL)_x}{r_x} < 4.71\sqrt{\frac{E}{F_y}} = 113.4 \rightarrow F_e = \frac{\pi^2 E}{(kL/r)^2} = 140.7$ ksi

$F_{cr} = (0.658^{F_y/F_e}) \cdot F_y = 43.1$ ksi

$\phi P_n = \phi A_g F_{cr} = 0.9(42.7)(43.1) = 1656.3$ kips

2) combination action $P_u / \phi P_n = 600 \text{ kips} / 1656.3 = 0.36 > 0.2$

Use H1-1a

3) Beam Action.

FLB $\lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 9.15 > \frac{b_f}{2t_f}$ → compact

WLB $\lambda_p = 3.76\sqrt{\frac{E}{F_y}} = 90.6 > \frac{h}{t_w}$ → compact

LTB $L_b = 14' < L_p \rightarrow$ no LTB

x-axis $M_{n,x} = F_y Z_x = 50(260) = 13000 \text{ k-in} = 1083 \text{ k}\cdot\text{ft}$

y-axis $M_{n,y} = F_y Z_y = 50(133) = 6650 \text{ k}\cdot\text{in} = 554 \text{ k}\cdot\text{ft}$

i) Case 1 : nt-only

$$B_{1,x} = \frac{C_{mx}}{1-(\alpha P_u/P_{e,x})} \qquad C_{m,x} = 0.6-0.4\left(\frac{M_{1,x}}{M_{2x}}\right)$$

$$= 0.6 - 0.4\left(\frac{-100}{120}\right) = 0.93$$

$$P_{e1,x} = \frac{\pi^2 E I_x}{(K_1 L)^2} = \frac{\pi^2(29000)(1710)}{(23.8\cdot 12)^2} = 6000 \text{ kips}$$

$$B_{1,x} = \frac{0.93}{1-(600/6000)} = 1.03$$

$$B_{1,y} = \frac{C_{my}}{1-(\alpha P_u/P_{e1,y})} \qquad C_{m,y} = 0.6-0.4\left(\frac{-10}{12}\right) = 0.93$$

$$P_{e1,y} = \frac{\pi^2 E I_y}{(K_1 L)^2} = \frac{\pi^2(29000)(677)}{(14\cdot 12)^2} = 6865 \text{ kips}$$

$$= \frac{0.93}{1-(600/6865)} = 1.02$$

$M_{u,x} = B_{1x} M_{ntx} = 1.03(120) = 124 \text{ k}\cdot\text{ft}$

$M_{u,y} = B_{1y} M_{nty} = 1.02(12) = 12.2 \text{ k}\cdot\text{ft}$

$$\frac{P_u}{\phi P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi M_{nx}} + \frac{M_{uy}}{\phi M_{ny}}\right) = 0.36 + \frac{8}{9}\left(\frac{124}{0.9\,1083} + \frac{12.2}{0.9\,554}\right) = 0.49 < 1 \quad \underline{ok}$$

ii) Case 2 : nt + lt

$B_{1,x} = 1.03 \qquad B_{1,y} = 1.02$

$$B_{2x} = B_{2,y} = \frac{1}{1-\frac{\alpha \Sigma P_{nt}}{\Sigma P_{e2}}} = \frac{1}{1-\frac{6000}{36000}} = 1.2$$

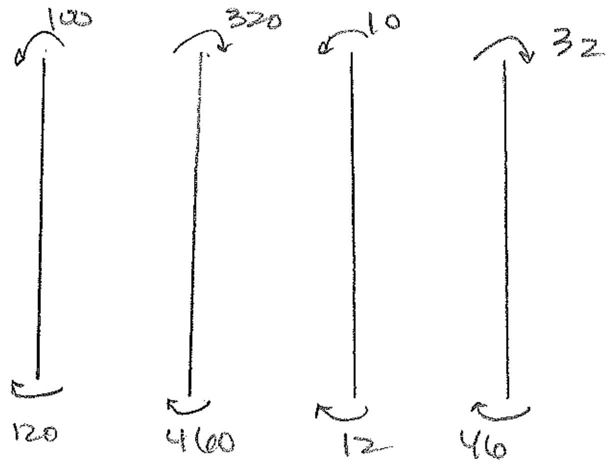
$M_{u,x} = B_{1x} M_{nt,x} + B_{2x} M_{lt,x} = 1.03(100) + 1.2(320) = 487 \text{ k}\cdot\text{ft}$

$M_{u,y} = 1.02(10) + 1.2(32) = 48.6 \text{ k}\cdot\text{ft}$

$$\frac{P_u}{\phi P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi M_{nx}} + \frac{M_{uy}}{\phi M_{ny}}\right) = 0.36 + \frac{8}{9}\left(\frac{487}{0.9\,1083} + \frac{48.6}{0.9\,554}\right) = 0.893 < 1 \quad \underline{ok}$$

adequate

* in a reel design case, "lt" moments occur in both directions.



$B_{1,x} = 1.03$

$B_{1,y} = 1.02$

$M_{u,x} = 1.03(120) + 1.2(460) = 676 \text{ k}\cdot\text{ft}$

$M_{u,y} = 1.02(12) + 1.2(46) = 67 \text{ k}\cdot\text{ft}$

$$\frac{P_u}{\phi P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi M_{nx}} + \frac{M_{uy}}{\phi M_{ny}}\right) = 0.36 + \frac{8}{9}\left(\frac{676}{0.9\ 1083} + \frac{67}{0.9\ 554}\right) = 1.10 > 1$$

since this is only 2% greater say ok.

<u>Column is adequate</u>