UNIVERSITY OF CALIFORNIA, BERKELEY
SPRING SEMESTER 2003

FINAL EXAMINATION

(CE130-1 Mechanics of Materials)

Problem 1: (10 points)

In the three-bar truss shown Fig. 1, all members are two-force member. There is an external force applied at the point C. All bars have the same Young's modulus, E, and they have the uniform cross-sectional area as indicated.

(1) Determine the strain energy of the system;

(2) Find the vertical displacement at point C.

(1) Ans: $U = \frac{19P^2L}{48EA}$

(2) $\Delta_v = \frac{19PL}{24EA}$

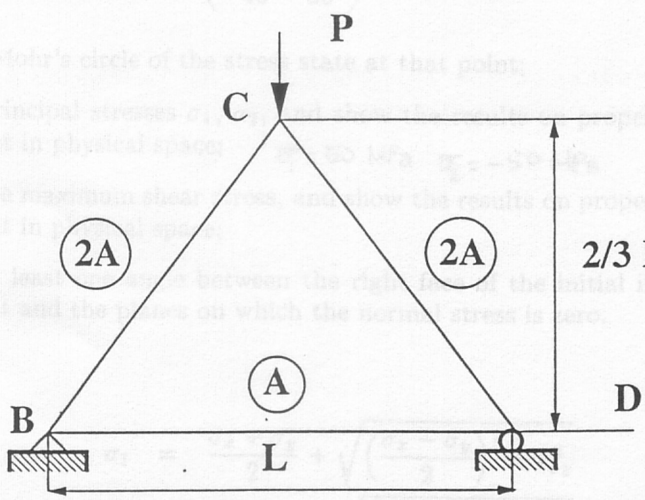


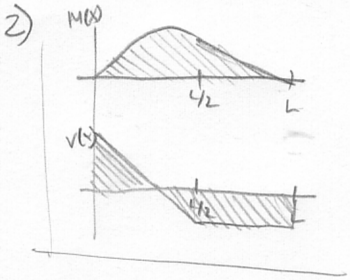
Figure 1: Schematic illustration of problem 1

(Hint: use Castigliano's second theorem, the energy for axially deformed column is, $U = \dfrac{P^2 L}{2EA}$, where P is the internal axial force, L is the length of the column, E is the Young's modulus, and A is the cross section of the bar.)

Problem 2 (10 points) 7

A simply supported beam subjected piece-wise constant distributed load as shown in Fig. 2. Draw shear and moment diagrams. (10 points)

Problem 3: (20 points) 16



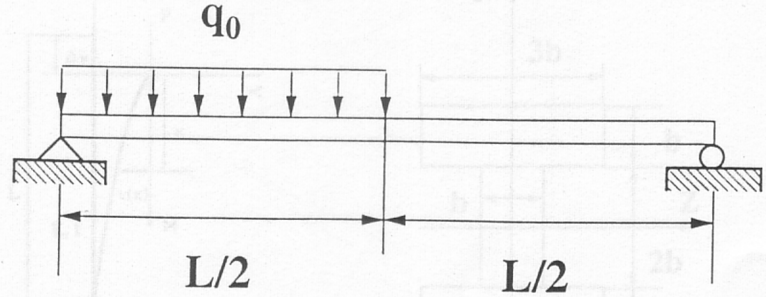
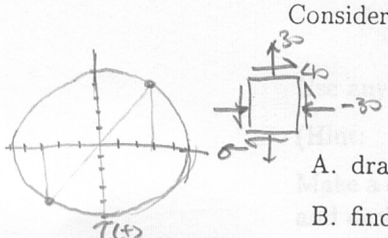


Figure 2: Simply supported beam distributed laod.

Consider a plane stress state as follows



$$\sigma = \begin{pmatrix} -30 & 40 \\ 40 & 30 \end{pmatrix} \text{ (MPa)}$$

A. draw Mohr's circle of the stress state at that point;

B. find principal stresses σ_1, σ_2, and show the results on properly oriented element in physical space;

$\sigma_1 = 50$ MPa $\sigma_2 = -50$ MPa

C. find the maximum shear stress, and show the results on properly oriented element in physical space;

D. find at least one angle between the right face of the initial infinitesimal element and the planes on which the normal stress is zero.

(Hint:

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$tan2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

$$tan2\theta_s = \frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

)

Problem 4 (10 points)

Determine the critical buckling load P_{cr} for a cantilever elastic column with span L and constant stiffness (rigidity) EI. The cross section of the beam is shown in Fig. 4(b)

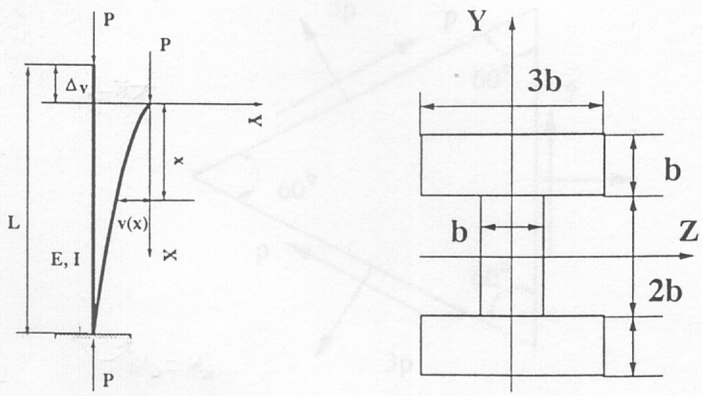


Figure 3: Problem 4

Use any methods that you feel comfortable with.

(Hint:

Make a cut at the cross section X; Draw free-body diagram for the isolated part, and derive the second order differential equation that governs the stability of a cantilever beam, and find the critical load;

$I = \frac{44}{3} b^4$

Boundary conditions:

$$v(0) = 0, \quad \text{and} \quad v'(L) = 0 \; .$$

$\cancel{P_{cr} = \frac{11 E \pi^2 b^4}{3L^2}}$

)

Problem 5 (~~10~~ points) 15

Consider an infinitesimal element shown in Figure 4. The normal stresses and shear stresses on two oblique planes are given. Find σ_x and τ_{xy}.

$\sigma_x = 3p$
$\tau_{xy} = -p$

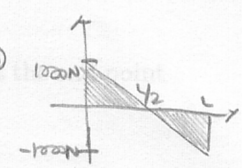
Problem 6 (~~10~~ points) 9

A box beam is made by nailing together four boards in the configurations shown in Figure 5. The beam is subjected a distributed load of $q_0 = 1000N/m$, and it rests on simple supports as shown in Figure 5. Assume each nail can withstand an allowable shear force of 200 N.

a . draw shear diagram and find maximum shear force;

b . find maximum nail spacing (Δ_s);

a)



b) $\Delta_s = 2.667$ mm

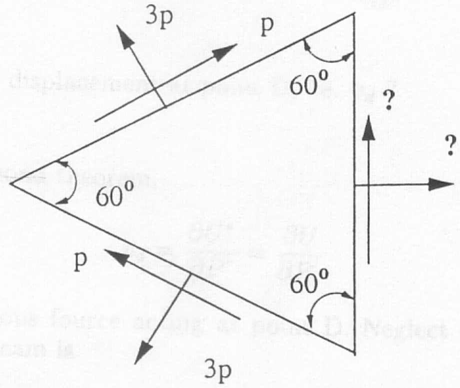


Figure 4: Problem 5

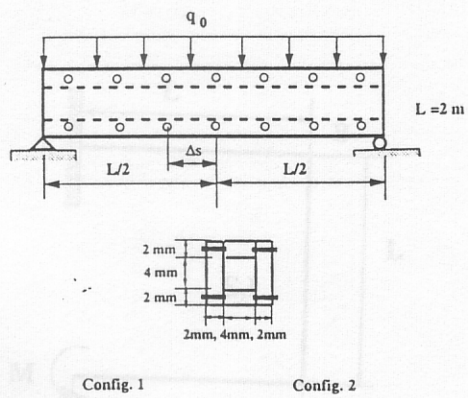


Figure 5: Problem 6

(Hint:

$$q = \frac{VQ}{I_z};$$

$$Q = \int_A y dA = \bar{y} A$$

$$q = \frac{N_{allowable}}{\Delta_s}$$

where $N_{allowable}$ stands for allowable shear force by the nails.)

Problem 7 (10 points) S

A planar frame ABCD is subjected a concentrated moment, M, at the end point D as shown in Figure 6.

(a) draw moment diagram along the frame;

(b) find the vertical displacement at point D, i.e. v_d ?

(Hint:

use Castigliano's second theorem,

$$v_d = \frac{\partial U^*}{\partial P'} = \frac{\partial U}{\partial P'}$$

where P' is a fictitious fource acting at point D. Neglect shear deformation, strain energy for a beam is

$$U = \frac{1}{2EI}\int_0^L M(s)^2 ds$$

)

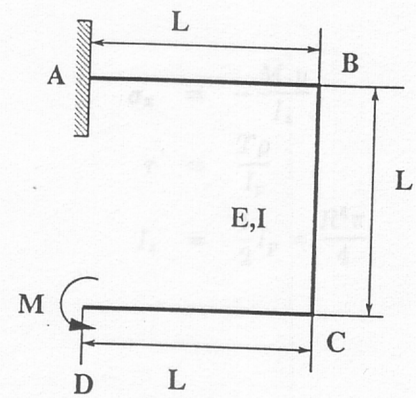


Figure 6: Problem 7

$\frac{2ML^2}{EI} = \Delta v$

~~Problem 8~~ ~~(20~~ points) 17

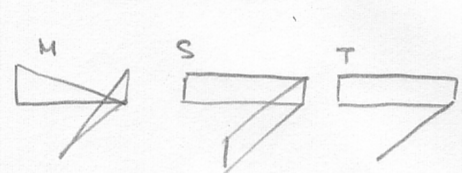
A L-shaped beam is made of a rectangular section and a solid cylinder section. The span of the both section is $L = 2.0\ m$. Suppose the allowable normal stress, $\sigma_{allow} = 120MP_a$ and allowable shear stress $\tau_{allow} = 60MP_a$.

There is a concentrated load, $P = 2kN$ acting on the free-end of of rectangular handel (as shown in Figure 7.).

(1) Draw the moment diagram, shear diagram, and internal torque diagram; (2) Find the maximum normal stress σ_x inside the beam of circular cross section; (3) Find the maximum shear stress inside the beam of circular cross section due to torsion (Neglect the shear stress due to transverse shear force). (4) Determine the smallest radius that can carry a $2 = kN$ load on tip of rectangular handel.

Hints:

(1)



(2) $\frac{4PL}{R^3\pi} = \sigma_x$ (3) $\frac{2PL}{R^3\pi} = \tau$ (4) $R = 34.88$ cm

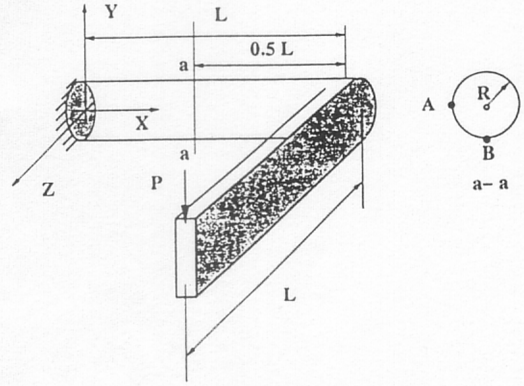


Figure 7: Problem 8

$$
\begin{aligned}
\sigma_x &= -\frac{M_z y}{I_z} \\
\tau &= \frac{T\rho}{I_p} \\
I_z &= \frac{1}{2} I_p = \frac{R^4 \pi}{4}
\end{aligned} \tag{1}
$$