CE152: Civil and Environmental Engineering Systems Analysis

Final

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Problem 1 (5 points)

CalAirways has a San Francisco hub. It has three crews based in San Francisco. Table 1 shows CalAirways flights and twelve sequences of flights originating and ending in San Francisco. The abbreviations used in Table 1 are summarized in the title of the table. Each crew needs to be assigned a sequence to fly, with the objective of minimizing costs. Each flight needs to have at least one crew. The last row of table 1 gives the cost in thousands of dollars per sequence. The longer sequences are more expensive.

Formulate the optimization problem of assigning the crews. You are not to solve this problem.

		Sequence											
		1	2	3	4	5	6	7	8	9	10	11	12
	SFO-LAX	1			1			1			1		
	SFO-DEN		1			1			1			1	
F	SFO-SEA			1			1			1			1
l	LAX-ORD				2			2		3	2		3
i	LAX-SFO	2					3				5	5	
g	ORD-DEN				3	3		3		4			
h	ORD-SEA								3		3	3	4
t	DEN-SFO		2		4	4				5			
s	DEN-ORD					2			2			2	
	SEA-SFO			2				4	4				5
	SEA-LAX						2			2	4	4	2
Cost ($1000s)		2	3	4	6	7	5	7	8	9	9	8	9

Table 1: Flight sequences. The numbers on the rows of the flights indicate the order of the flight in a particular sequence. For example, for sequence 3, the cell on the SFO-LAX row is empty, indicating that this flight does not belong to sequence 3. For sequence 3, the row SEA-SFO contains 2, indicating that this flight is the second flight in sequence 3. For sequence 3, the first flight is the one whose row contains 1, i.e. SFO-SEA. The abbreviations used are the following: SFO (San Francisco), LAX (Los Angeles), DEN (Denver), SEA (Seattle), ORD (Chicago).

Problem 2 (7 points)

A machine is either running or broken down. If it runs throughout one week, it makes a gross profit of \$100. If it fails during the week, gross profit is zero. If it is running at the start of the week and we perform preventive maintenance, the probability of failure is 0.4. If we do not perform such maintenance, the probability of failure is 0.7. However, maintenance will cost \$20. When the machine is broken down at the start of the week, it may either be repaired at a cost of \$40, in which case it will fail during the week with a probability of 0.4, or it may be replaced at a cost of \$150 by a new machine that is guaranteed to run through its first week of operation.

(a) Find the optimal maintenance, repair and replacement policy that maximizes total profit over four weeks, assuming that we have a brand new machine at the start of the first week.

Assume that the machine can only be repaired or replaced at the beginning of a week, i.e. if it fails during a week, it cannot be repaired or replaced until the beginning of the following week.

(b) This problem was given to a class of 5 students, with different numerical values. The problem also asked them to find maximum total profit over 4 weeks for two cases: starting with a machine in working condition, and starting with a broken machine. The results were the following:

student	starting with working machine	starting with broken machine
1	100	120
2	140	120
3	140	120
4	100	120
5	150	140

Whose answers can you be sure are incorrect? Why?

Problem 3 (10 points)

(a) `intprog` is a computer function that solves linear integer programs. `intprog(f,A,b)` provides the solution of the following linear integer program:[1]

$$\max_x f'.x \qquad (f' \text{ denotes the transpose of } f)$$

subject to

$$A.x \leq b$$

$$x_i \text{ integer}, \ i = 1, 2, \ldots, n \qquad (n \text{ denotes the dimension of } x)$$

Consider the following optimization problem. Notation: for any vectors p and q of the same dimension, $< p, q >$ denotes the inner product or dot product of p and q.

$$\max x_3 + x_4$$

subject to

$$(1) \quad \left\langle \begin{bmatrix} -1 \\ 10 \\ 2 \\ 3 \end{bmatrix}, x \right\rangle \leq 1 \text{ if } x_1 \geq 0$$

$$(2) \quad \left\langle \begin{bmatrix} 1 \\ 10 \\ 2 \\ 3 \end{bmatrix}, x \right\rangle \leq 1 \text{ if } x_1 < 0$$

$$(3) \quad x_1 \geq -\sqrt{2}$$

$$(4) \quad x_1 \leq \pi$$

$$(5) \quad x_i \text{ integer}, \ i = 1, 2, 3, 4$$

You are to provide the matrix A, and the vectors f and b, so that `intprog(f,A,b)` returns the solution to the optimization problem above.

[1] `intprog` could be seen as a version of the Matlab function `linprog` for integer programs. However, you do not need to know anything about `linprog` to solve this problem.

(b) Consider the following optimization problem. Notation: for any vectors a and b of the same dimension, $< a, b >$ denotes the inner product or dot product of a and b.

$$\max x_3$$
subject to

$$(1) \quad \left\langle \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}, x \right\rangle \leq 1 \text{ if } x_1 \geq 0$$

$$(2) \quad \left\langle \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, x \right\rangle \leq 1 \text{ if } x_1 < 0$$

$$(3) \quad x_1 \geq -\sqrt{2}$$

$$(4) \quad x_1 \leq \pi$$

$$(5) \quad x_i \text{ integer, } i = 1, 2, 3$$

Solve this problem using branch and bound. You will receive no credit if you use another algorithm, even if you obtain the correct solution. **This question is completely independent from the previous one: you are not to use the `intprog` function, and you are not to write the arguments with which you would call this function.**

Problem 4 (8 points)

Solve the following optimization problem by the Lagrangian method for every two dimensional vectors c. You are to find the optimal value(s) of the objective function and of the decision variable(s). A correct solution for every c will get 3 points (even without explanation). Please note that c is not a decision variable, but a fixed parameter.

$$\max \langle c, z \rangle$$

subject to

$$(1) \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} cos(\theta) & sin(\theta) \\ -sin(\theta) & cos(\theta) \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

$$(2) \quad x_1^2 + x_2^2 \leq 1$$

$$(3) \quad z_1 + z_2 \leq 1$$

$$(4) \quad 0 \leq \theta < 2\pi$$

Notation: for any two vectors a and b of the same dimension, $< a, b >$ denotes the inner product or dot product of a and b.