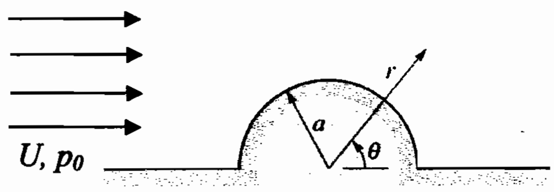
Problem 1 (40 points): Potential flow past a cylindrical dome

A practice facility for the Cal football team is to be covered by a semi-circular cylindrical dome, as shown in the sketch. The dome with radius a is placed normal to the cross flow of velocity U. Assume that the dome is very long (into the page) so you can ignore end effects.



a) The stream function and velocity potential for this flow $(0 \le \theta \le \pi)$ are given by

not needed

$$\psi = Ur\left(1-\frac{a^2}{r^2}\right)\sin\theta, \qquad \phi = Ur\left(1+\frac{a^2}{r^2}\right)\cos\theta$$

Use potential flow theory to find the tangential velocity along the surface of the dome as a function of the angle θ. (15 points)

$$u_\Theta = \frac{1}{r}\frac{\partial\phi}{\partial\Theta} = \frac{1}{r}\left[\frac{\partial}{\partial\Theta}\left(Ur\left(1+a^2/r^2\right)\cos\Theta\right)\right]$$

$$u_\Theta = \frac{1}{r}\left[-Ur\left(1+a^2/r^2\right)\sin\Theta\right]$$

$$u_\Theta\Big|_{r=a} = -2U\sin\Theta$$

b) Choose a reference condition to be $p=p_0$ at $z=0$ (on the ground) far from the cylindrical dome. Use the Bernoulli equation to find the pressure field on the surface of the dome as a function of θ. Do *not* neglect hydrostatic variations. (15 points)

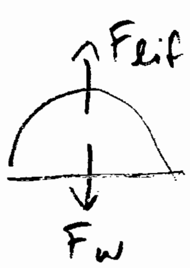
Irrotational, so Bernoulli applies between any 2 points

$$P_o + \tfrac{1}{2}\rho V_o^2 + \gamma z_o = P_s + \tfrac{1}{2}\rho V_s^2 + \gamma z_s \qquad (s \to \text{on surface})$$

$$P_o + \tfrac{1}{2}\rho U^2 + 0 = P_s + \tfrac{1}{2}\rho(-2U\sin\theta)^2 + \gamma a \sin\theta$$

$$\underline{P_s = P_o + \tfrac{1}{2}\rho U^2 - 2\rho U^2 \sin^2\theta - \gamma a \sin\theta}$$

c) If the net vertical force on the dome per unit length (upwards) is given by $F_{lift} = \frac{5}{3}\rho U^2 a$, find the mass per unit length (m_c) required to keep the cylindrical dome on the ground when the wind blows at 30 m/s, with air density ρ=1.2 kg/m^3, and a = 50 m. (10 points)



$$\Sigma F_z = 0$$

$$F_{lift} - F_w = 0$$

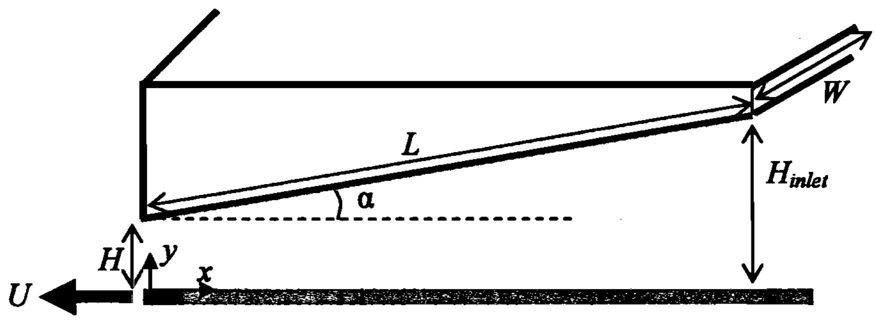
$$\tfrac{5}{3}\rho U^2 a = m_c g$$

$$m_c = \tfrac{5}{3}(1.2\ \text{kg/m}^3)(30\ \text{m/s})^2(50\ \text{m})\left(\frac{1}{9.81\ \text{m/s}^2}\right)$$

$$\underline{m_c = 9170\ \text{kg/m}}$$

Problem 2 (60 points): Slider bearing

The slider bearing (or "slider block") consists of a smooth plate, which is inclined at a small angle α to the horizontal (typically $\alpha < 10°$) and separated by a small distance from a smooth lower surface which moves steadily at a velocity U as shown below. Because of the no-slip condition at the lower and upper plates, a fluid such as air (of density ρ and viscosity μ) is dragged between the plates and must accelerate through the gap. This is found to generate a vertical lift force F_y on the upper plate.



a) Use the Buckingham Pi theorem to find the dimensionless groups important for determining the vertical force F_y generated by the slider bearing. Explain why you should not include H_{inlet} if you include α in your analysis. Choose ρ, U, and H as your repeating variables. Hint: you should be able to identify a few of the dimensionless groups by inspection (but show your work to verify they are indeed dimensionless). (30 points)

$$F_y = \phi(\rho, U, H, L, \alpha, W, \mu)$$

$$[F] \quad [FL^{-4}T^2]\,[LT^{-1}]\,[L]\,[L]\,[-]\,[L]\,[FL^{-2}T]$$

$H_{inlet} = H + L\sin\alpha$ so it is not independent and does not need to be included because it can be derived from the other terms.

$n - r = 8 - 3 = 5$ Π terms needed

$$\Pi_1 = F_y \rho^a U^b H^c$$

$$[-] = [F][FL^{-4}T^2]^a [LT^{-1}]^b [L]^c$$

$$\left.\begin{aligned} F)\ 0 &= 1 + a && \rightarrow a = -1 \\ L)\ 0 &= -4a + b + c && \rightarrow c = -2 \\ T)\ 0 &= 2a - b && \rightarrow b = -2 \end{aligned}\right\} \quad \Pi_1 = \frac{F_y}{\rho U^2 H^2}$$

$\Pi_2 = L/H \qquad \frac{[L]}{[L]} \doteq [-]$ by inspection

$\Pi_3 = \alpha \qquad [-]$ already nondimensional

$\Pi_4 = W/H \qquad \frac{[L]}{[L]} \doteq [-]$ by inspection

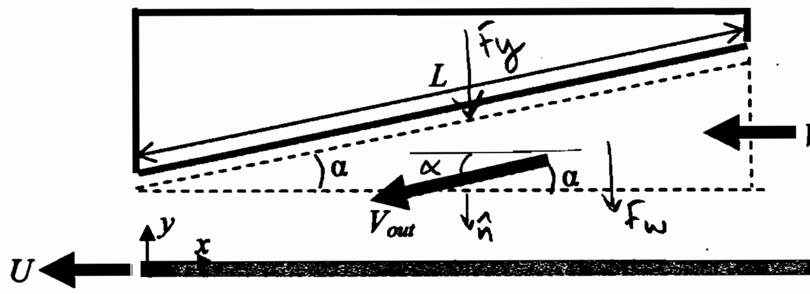
$\Pi_5 = \mu \rho^d U^e H^f$

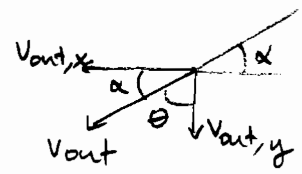
$\Pi_5 = \frac{\mu}{\rho U H} = \frac{1}{Re}$ by inspection

$$\frac{\mu}{\rho U H} \doteq \frac{[F L^{-2} T]}{[F L^{-4} T^2][L T^{-1}][L]} \doteq [-]$$ ✓ checks

$$\boxed{\frac{F_y}{\rho U^2 H^2} = f\left(\frac{L}{H}, \alpha, \frac{W}{H}, \frac{1}{Re}\right)}$$

b) Use the triangular control volume shown below to find an expression for the vertical force on the plate due to the flow in terms of L, W, α, and the average inlet (V_{in}) and outlet (V_{out}) velocities over this region. You can ignore viscous stresses and approximate the flow as uniform over each cross section. Is the force on the upper plate up or down? (20 points)





$$\sum F_y = \underbrace{\frac{\partial}{\partial t}\int_{cv} \rho V_y \, d\forall}_{=0 \text{ steady state}} + \int_{cs} \rho V_y (\underline{v}\cdot\underline{n})\, dA$$

outflow surface: $\underline{v}\cdot\underline{n} = V_{out}\sin\alpha$

$V_{out,y} = -V_{out}\sin\alpha$

let $P = P_o = 0$

$$P_{out}(L\cos\alpha)W - F_y - F_W = \rho V_{out,y} V_{out}\sin\alpha\,[L\cos\alpha \cdot W]$$

$$-F_y - \underbrace{\tfrac{1}{2}\rho(L\sin\alpha)(L\cos\alpha)Wg}_{\text{weight of fluid is often small so we can neglect this (e.g. for air)}} = -\rho V_{out}^2 \sin^2\alpha\,[L\cos\alpha\cdot W]$$

$F_y = \rho V_{out}^2 \sin^2\alpha\cos\alpha\, LW - \frac{1}{2}\rho L^2 \sin\alpha\cos\alpha\, W$ downward reaction force

Lift force from air flow on plate:

$$F_y = \rho V_{out}^2 \sin^2\alpha\cos\alpha\, LW - \underbrace{\tfrac{1}{2}\rho L^2\sin\alpha\cos\alpha\, W}_{\text{again, can usually neglect}} \quad \text{upward}$$

c) Given $H = 0.5$ mm, $\rho = 1.2$ kg/m^3, $\mu = 1.79\text{x}10^{-5}$ N·s/m^2, $\alpha = 5°$, $U = 2$ cm/s, L=1 cm, and W =2 cm, do you expect inertial or viscous forces to be more important? Which famous nondimensional number can help you decide? What do you think of your analysis in part (b) given this information? (10 points)

$$Re = \frac{\rho U H}{\mu} = \frac{1.2\ \text{kg/m}^3 \cdot 0.02\ \text{m/s} \cdot 0.0005\ \text{m}}{1.79 \times 10^{-5}\ \text{N}\cdot\text{s/m}^2} = 0.67 < 1$$

$$Re = \frac{\text{inertial forces}}{\text{viscous forces}}$$ so viscous forces are important

The analysis in part (b) neglected viscous stresses so it is not valid for the given values.