UNIVERSITY OF CALIFORNIA, BERKELEY
Spring SEMESTER 2002

Sample

FINAL EXAMINATION

(CE130-1 Mechanics of Materials)

Problem 1: (10 points)

A pin-jointed 3-bar structure is shown in the Figure 1. There is an external force, P, acting on the point C. (1) find internal axial forces for bar AC, BC, and CD; (2) find the vertical displacement as well as horizontal displacement at nodal point C.

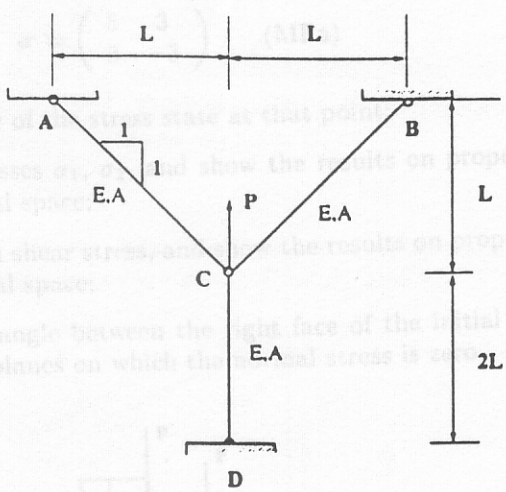


Figure 1: Schematic illustration of problem 1

(Hint: (1) use Castigliano's second theorem, the energy for axially deformed bar is, $U = \dfrac{P^2 L}{2EA}$, where P is the axial force, L is the length of the bar, E is the Young's modulus, and A is the cross section of the bar; (2) $\Delta = \dfrac{LP}{AE}$.)

Problem 2 (10 points)

A simply supported beam subjected two concentrated forces that have the same magnitude, P, shown in Figure 2. Draw shear and moment diagrams. (10 points)

Problem 3: (20 points)

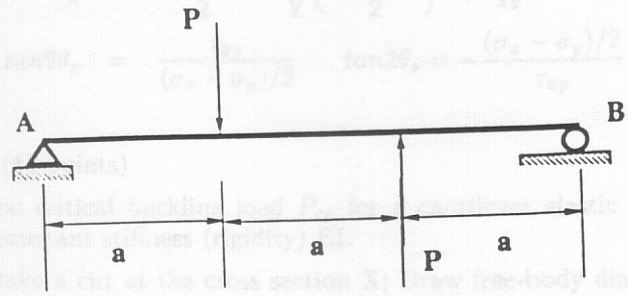


Figure 2: Simply supported beam with concentrated forces.

Consider a plane stress state as follows

$$\sigma = \begin{pmatrix} 5 & 3 \\ 3 & -3 \end{pmatrix} \quad \text{(MPa)}$$

A. draw Mohr's circle of the stress state at that point;

B. find principal stresses σ_1, σ_2, and show the results on properly oriented element in physical space;

C. find the maximum shear stress, and show the results on properly oriented element in physical space;

D. find at least one angle between the right face of the initial infinitesimal element and the planes on which the normal stress is zero.

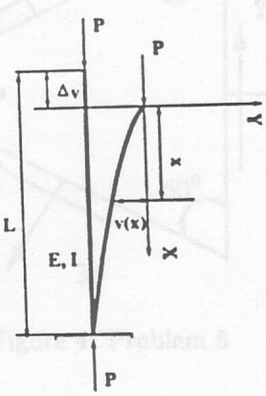


Figure 3: Problem 4

(Hint:

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} \qquad tan 2\theta_s = \frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

)

Problem 4 (10 points)

Determine the critical buckling load P_{cr} for a cantilever elastic column with span L and constant stiffness (rigidity) EI.

(Hint: (1) Make a cut at the cross section X; Draw free-body diagram for the isolated part, and derive the second order differential equation that governs the stability of a cantilever beam, and find the critical load, or

(2) Use the fourth order differential equation

$$\frac{d^4 v}{dx^4} + \lambda^2 \frac{d^2 v}{dx^2} = 0 \qquad (1)$$

where $\lambda^2 = \dfrac{P}{EI}$. Write down the boundary conditions, solve the differential equation, and find the critical load. The general form of homogeneous solution of Eq. (1) is $v(x) = C_1 \cos \lambda x + C_2 \sin \lambda x + C_3 x + C_4$.)

Problem 5 (10 points)

Consider an infinitesimal element shown in Figure 4. The normal stresses and shear stresses on two oblique planes are given. Find σ_x and τ_{xy}.

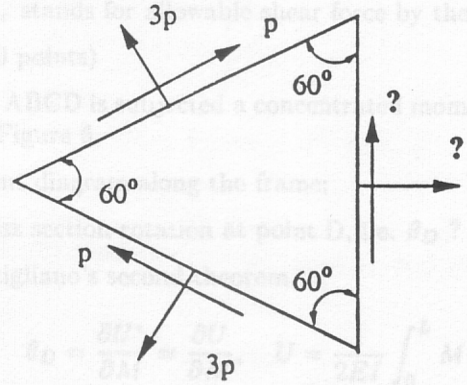


Figure 4: Problem 5

Problem 6 (20 points)

A box beam is made by nailing together four boards in the configurations shown in Figure 5 and labeled as *Config.1* and *Config.2*. The beam supports a concentrated load of 1000 N at its midspan, and it rests on simple supports as shown in Figure 5. Assume each nail can withstand an allowable shear force of 200 N.

a . draw shear diagram;

b . what is the maximum spacing (Δ_s) for configuration 1;

c . what is the maximum spacing (Δ_s) for configuration 2;

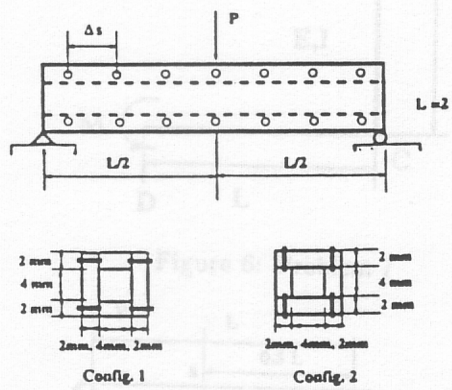


Figure 5: Problem 6

(Hint:

$$q = \frac{VQ}{I_z}, \quad Q = \int_A y dA = \bar{y}A, \quad q = \frac{N_{allowable}}{\Delta_s} \tag{2}$$

where $N_{allowable}$ stands for allowable shear force by the nails.)

Problem 7 (10 points)

A planar frame ABCD is subjected a concentrated moment, M, at the end point D as shown in Figure 6.

(a) draw moment diagram along the frame;

(b) find the cross section rotation at point D, i.e. θ_D ?

(Hint: use Castigliano's second theorem,

$$\theta_D = \frac{\partial U^*}{\partial M} = \frac{\partial U}{\partial M}, \quad U = \frac{1}{2EI}\int_0^L M(s)^2 ds$$

)

Problem 8 (10 points)

A L-shaped beam is made of a rectangular section and a solid cylinder section with radius $R = 0.1m$. The span of the both section is $L = 2.0\ m$. There is a concentrated load, $P = 300N$, acting on the free-end of the rectangular (as shown in Figure 7.). (1) Draw the moment diagram, shear diagram, and

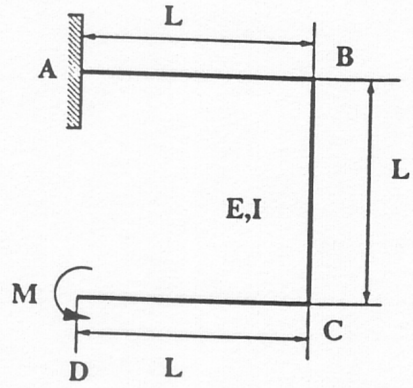


Figure 6: Problem 7

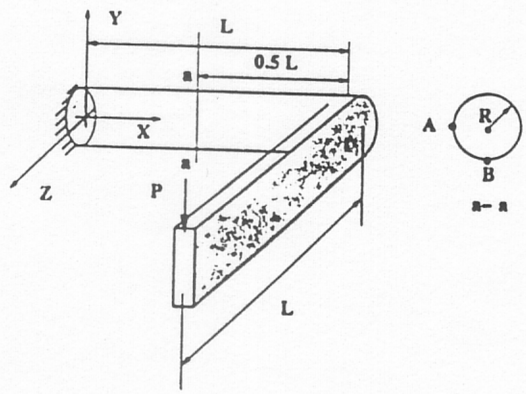


Figure 7: Problem 8

internal torque diagram; (2) Find the normal stress σ_x, shear stresses τ_{xy} and τ_{xz} at point A; (3) Find the normal stress σ_x and shear stress τ_{xy} and τ_{xz} at point B;

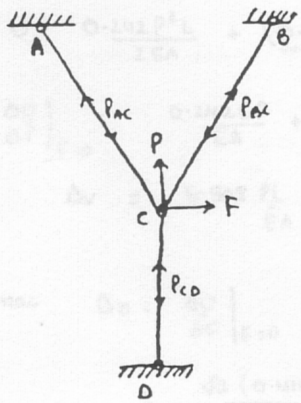
Hint:

$$\sigma_x = -\frac{M_z y}{I_z}, \quad \tau = \frac{VQ(y)}{I_z t}, \quad \tau = \frac{T\rho}{I_p}, \quad I_z = \frac{1}{2}I_p = \frac{R^4\pi}{4}$$

$$\text{For semi-circle,} \quad : \; Q(y)\Big|_{y=0} = \frac{2}{3}R^3$$

Practice Final Solutions
CE-130-1

(1) Problem 1



writing Equilibrium Equations at node C

$$\frac{P_{AC}}{\sqrt{2}} + \frac{P_{BC}}{\sqrt{2}} + P - P_{CD} = 0$$

$$P_{AC} = P_{BC} + \sqrt{2}F$$

if $P_{AC} = x$

$P_{BC} = x + \sqrt{2}F$

$$P_{CD} = P + \frac{1}{\sqrt{2}}(2x + \sqrt{2}F)$$

$$= P + F + \sqrt{2}x$$

F is a Fictious force introduced at C, ∵ we wish to determine horizontal displacement at C.

we can write strain energy of all three rods as:

$$U = \frac{x^2\sqrt{2}L}{2EA} + \frac{(x+\sqrt{2}F)^2\sqrt{2}L}{2EA} + \frac{(P+F+\sqrt{2}x)^2 2L}{2EA}$$

now ∵ A is a fixed support ∴ $\Delta_A = 0$

$$\Rightarrow \left.\frac{\partial U}{\partial x} = 0\right|_{F=0}$$

$$\Rightarrow \frac{x\sqrt{2}L}{EA} + \frac{x\,[\text{illegible}]\,L}{EA} + \sqrt{2}\,\frac{(P+\sqrt{2}x)2L}{EA} = 0$$

$$\Rightarrow 2x + 2P + 2\sqrt{2}x = 0$$

$$x = \frac{P}{1+\sqrt{2}} = \underline{0.414P}$$

internal axial force in $AC = BC = 0.414P$

in $DC = P + \frac{\sqrt{2}P}{1+\sqrt{2}} = \underline{1.585P}$

Now for vertical displacement at C

$$\Delta v = \left.\frac{\partial U}{\partial P}\right|_{F=0}$$

$$U = \frac{0.242P^2L}{2EA} + \frac{(0.414P+\sqrt{2}F)^2\sqrt{2}L}{2EA} + \frac{(P+F+0.585P)^2 2L}{2EA}$$

$$\left.\frac{\partial U}{\partial P}\right|_{F=0} = \frac{0.242LP}{EA} + \frac{0.242PL}{EA} + \frac{5.02445PL}{EA}$$

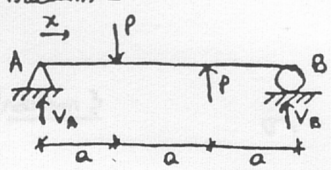
$$\Delta v = 5.508\frac{PL}{EA}$$

now $\Delta_H = \left.\frac{\partial U}{\partial F}\right|_{F=0}$

$$= \frac{\sqrt{2}(0.414P)\sqrt{2}L}{EA} + \frac{(1.585P)2L}{EA}$$

$$= 3.998\frac{PL}{EA}.$$

Problem 2



$V_A + V_B = 0$ -(1)

$\Sigma M_A = 0$

$\Rightarrow 3aV_B + 2aP = Pa$

$V_B = -\frac{P}{3}$

$\Rightarrow V_A = \frac{P}{3}$

$\therefore$ for $0 < x < a$

$V = P/3$

$M = \frac{P}{3}x$

for $a < x < 2a$

$V = -2P/3$

$M = \frac{Px}{3} - P(x-a) = Pa - \frac{2Px}{3}$

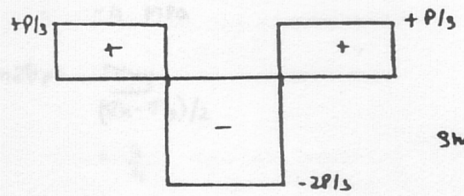
for $2a < x < 3a$

$$V = P/3$$

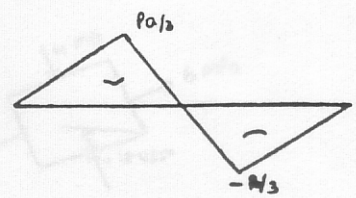
$$M = Pa - \frac{2}{3}Px + P(x-2a)$$

$$= \frac{Px}{3} - Pa$$

we now draw shear force and moment diagrams



Shear force diagram



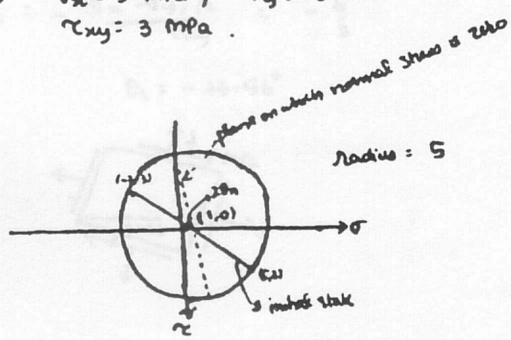
Moment diagram

Problem 3

$$\sigma = \begin{pmatrix} 5 & 3 \\ 3 & -3 \end{pmatrix} \text{ MPa}$$

$\Rightarrow$ $\sigma_x = 5$ MPa , $\sigma_y = -3$ MPa
$\tau_{xy} = 3$ MPa .

A) Mohr Circle



radius = 5

B) principal stresses

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 6 \text{ MPa}$$

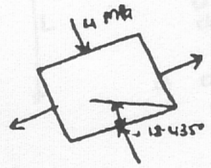
$$\sigma_2 = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= -4 \text{ MPa}$$

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x-\sigma_y)/2}$$

$$= \frac{3}{4}$$

$$\Rightarrow \theta_p = 18.435°$$



(c) maximum shear stress

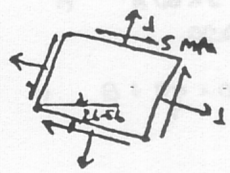
$$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma' = \frac{5-3}{2} = 1 \text{ MPa}$$

$$= 5 \text{ MPa}$$

$$\tan 2\theta_s = -\frac{(\sigma_x-\sigma_y)/2}{\tau_{xy}} = -\frac{4}{3}$$

$$\theta_s = -26.56°$$



(D) One of the plane on which normal stress is zero is shown with a dotted line on mohr circle (see part A)

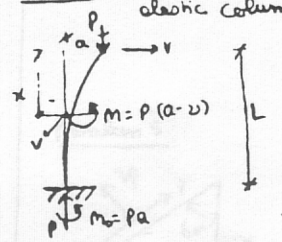
$$\cos 2\theta_n = \frac{1}{5}$$

$$\Rightarrow \theta_n = 39.23$$

$\therefore$ angle between initial element and hur face

$$= \theta_p + 45 + (45 - \theta_n)$$
$$= 18.435 + 45 + 5.77$$
$$= 69.205 .$$

Problem 4 To determine critical buckling load P_{cr} for a cantilever elastic column with span L and constant stiffness EI.



$$\therefore \quad \frac{d^2v}{dx^2} = \frac{M}{EI} = \frac{P(a-v)}{EI}$$

$$\Rightarrow \frac{d^2v}{dx^2} + \lambda^2 v = \frac{Pa}{EI} \qquad \text{where } \lambda^2 = \frac{P}{EI}$$

$$a = M_0/P$$

$$\frac{d^2v}{dx^2} + \lambda^2 v = \lambda^2 \frac{M_0}{P}$$

Complete solution of this equation is given as

$$v = A\sin\lambda x + B\cos\lambda x + \frac{M_0}{P}$$

now $v(L) = v'(L) = 0$

$$\Rightarrow A\sin\lambda L + B\cos\lambda L + \frac{M_0}{P} = 0$$

$$A\lambda\cos\lambda L - B\lambda\sin\lambda L = 0$$

$$\Rightarrow A\cos\lambda L = B\sin\lambda L$$

also $v(0) = a \Rightarrow B + \frac{M_0}{P} = a \qquad \because \frac{M_0}{P} = a$

$$\Rightarrow B = 0$$

$\Rightarrow \quad A \cos \lambda L = 0$

$\Rightarrow \quad \cos \lambda L = 0$

first root of this equation is

$$\lambda L = \frac{\pi}{2}$$

$$\Rightarrow \quad \lambda = \frac{\pi}{2L}$$

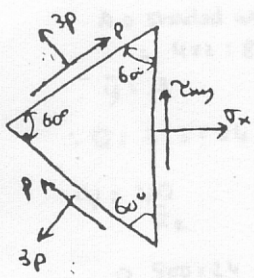
$$\lambda^2 = \frac{P}{EI}$$

$$\Rightarrow \quad \frac{P_{cr}}{EI} = \frac{\pi^2}{4L^2}$$

This Load is critical

$$\therefore \quad P_{cr} = \frac{\pi^2 EI}{4L^2}$$

Problem 5



assume area of one side of the element = 1

all the sides have equal area

we can write equilibrium equations for this element as:-

$$\sigma_x \times 1 + P \cos 30 = 2 \times 3P \cos 60 + P \cos 30$$

$$\Rightarrow \quad \sigma_x = 3P$$

$$\tau_{xy} + P \sin 30 + P \sin 30 + 3P \sin 60 = 3P \sin 60$$

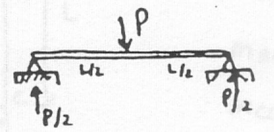
$$\Rightarrow \quad \tau_{xy} = -P$$

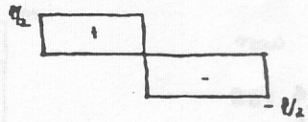
$\Sigma F_x = \sigma_x \times 1 - 3P \cos 60°$

$\sigma_x = 2 \cdot 3P \cos 6$

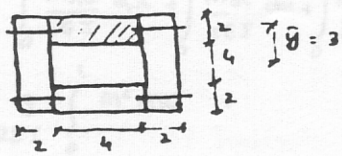
Problem 6

a) Shear diagram





b) Config 1



$I_z = \frac{1}{12}(8)^4 - \frac{1}{12}(4)^4 = 320 \text{ mm}^4$

$Q = \bar{y}A$

$A \Rightarrow$ shaded area

$= 4 \times 2 = 8 \text{ mm}^2$

$\bar{y} = 3$

$\therefore Q = 8 \times 3 = 24 \text{ mm}^3$

$\therefore q = \frac{VQ}{I_z}$ $\quad V = \frac{P}{2} = 500 \text{ N}$

$\Rightarrow \frac{500 \times 24}{320} = 37.5$

$N_{allowable} = 200$

There are two nails $\therefore$

$q = N_{allowable} / \Delta s \Rightarrow \Delta s = N_{allowable} / q/2 = 2 \times 200 / 37.5$

$= 10.66$

≈ 11 mm spacing

c) Config 2. $\therefore$ $I_z = 320 \text{ mm}^2$

$Q \Rightarrow A = 8 \times 2 = 16 \text{ mm}^2$, $\bar{y} = 3$ $\Rightarrow$ $Q = A\bar{y} = 48 \text{ mm}^3$

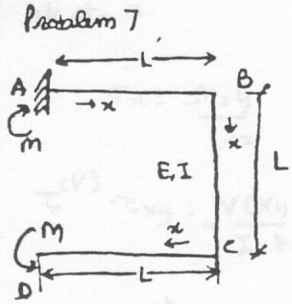
$\therefore q = \frac{500 \times 48}{320} = 75$

$\therefore \Delta s = \frac{2 \times 200}{75} = 5.33$ mm

≈ 6 mm spacing

$\therefore$ Config 1 is better.

Problem 7

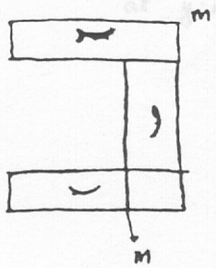


for AB

$M_x = M$

$M_{BC} = -M$

$M_{CD} = M$



now to find rotation at pt. D θ_D

$$\theta_D = \frac{\partial U}{\partial M}$$

where $$U = \int_0^L \frac{M_{AB}^2}{2EI}\,dx + \int_0^L \frac{M_{BC}^2}{2EI}\,dx + \int_0^L \frac{M_{CD}^2}{2EI}\,dx$$

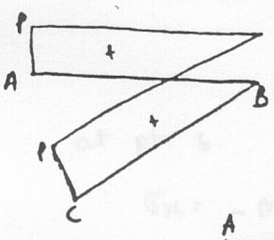
$$= \frac{3}{2EI}\int_0^L M^2\,dx$$

$$U = \frac{3M^2L}{2EI}$$

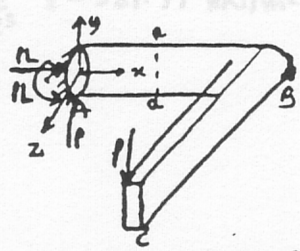
$$\frac{\partial U}{\partial M} = \frac{3ML}{EI} = \theta_D .$$

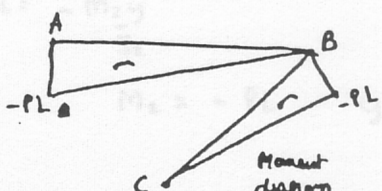
Problem 8

(1) Force, Moment and Torque diagrams

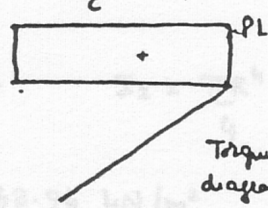


Shear force diagram



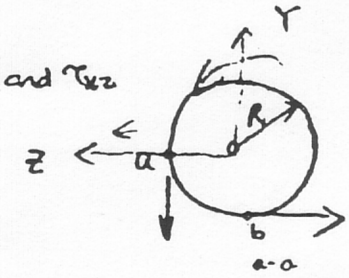


Moment diagram



Torque diagram

(2) Find normal stress σ_x, shear stress τ_{xy} and τ_{xz} at pt. a



$y = 0$

$$\sigma_x = -\frac{M_z y}{I_z} = 0 \quad \because y = 0$$

$$\tau^{(V)} \quad \tau_{xy} = -\frac{V Q(y)}{I_z t} \quad \text{(for semicircle)}$$

at $y = 0$ $\quad Q(y) = \frac{2}{3} R^3$

$V = P$

$I_z = \frac{\pi R^4}{4}$

$t = 2R$

$P = 300\,N$; $R = 0.1\,m$ $\quad L = 2.0\,m$

$$\tau^{(T)} \quad \tau_{xy} = \frac{P \cdot \frac{2}{3} R^3}{\frac{\pi}{4} R^4 \cdot 2R} = \frac{4P}{3\pi R^2} = -12.732\ kN/m^2$$

$$\tau_{xy} = -\frac{Tc}{I_p}$$

$c = R$, $T = PL$, $I_p = \frac{\pi R^4}{2}$

$$\Rightarrow \tau_{xy} = \frac{2PLR}{\pi R^4} = \frac{2PL}{\pi R^3} = -381.97\ kN/m^2$$

(3) at pt. b

$$\sigma_x = -\frac{M_z y}{I_z}$$

$M_z = -PL \qquad y = -R \qquad I_z = \frac{\pi R^4}{4}$

$$\sigma_x = -\frac{4PLR}{\pi R^4} = -\frac{4PL}{\pi R^3} = -763.94\ kN/m^2$$

$$\tau_{xy} = \frac{V Q(y)}{I_z t} = 0 \quad \because Q(y) = 0 \text{ at b} \quad \because \bar{y} = 0$$

$$\tau_{zx} = \frac{Tc}{I_p} = \frac{PLR}{\pi R^4/2} = \frac{2PL}{\pi R^3} = -381.97\ kN/m^2$$