Student name:___________________________

CE93 -- Engineering Data Analysis
Final Examination
Thursday, December 18, 2003

Work on all five problems. Write clearly and state any assumptions you make. The exam is closed books and notes, except for three sheets of paper and table of distributions.

The problems have the following weights:

Problem 1 (25 points) _________________________
Problem 2 (20 points) _________________________
Problem 3 (20 points) _________________________
Problem 4 (15 points) _________________________
Problem 5 (20 points) _________________________

Exam grade (100 points) ______________________

Student name:________________________________

Problem 1. (10+10+5 = 25 points)

Suppose during the holidays you are planning a ski trip to Lake Tahoe. As the map below shows, past Sacramento there are two main roads to Lake Tahoe: Interstate 80 (I80) and Highway 50 (H50). Suppose from past experience we know that, if there is a storm, then there is probability 0.3 that I80 will be closed and probability 0.2 that H50 will be closed. Furthermore, if H50 is closed due to a storm, then the probability that I80 is also closed is 0.6. In case of no storm, each of these roads could be closed due to major accidents, repair or other causes. Assume such closure events for the two roads are statistically independent with probabilities 0.01 for I80 and 0.02 for H50 on a given day.

a) Suppose on your planned day of travel there is a 30% chance that a storm will occur. What is the probability that you will be able to get to Lake Tahoe through I80?

b) Under the condition in (a), what is the probability that you will not be able to get to Lake Tahoe through either road?

c) Suppose on a stormy day you hear your friend has arrived in Lake Tahoe. What is the probability that I80 is open?

Solution:

Let I80 = Interstate 80 is open; H50 = Highway 50 is open

a) $P(I80) = P(I80 \mid storm)P(storm) + P(I80 \mid no\ storm)P(no\ storm)$

$= (1-0.3)(0.3)+(1-0.01)(0.7)$

$= 0.903$ *Ans.*

b) $P(\overline{I80}\,\overline{H50}) = P(\overline{I80}\,\overline{H50} \mid storm)\,P(storm) + P(\overline{I80}\,\overline{H50} \mid no\ storm)P(no\ storm)$

$= P(\overline{I80} \mid storm, \overline{H50})\,P(\overline{H50} \mid storm)\,P(storm)$

$+ P(\overline{I80} \mid no\ storm, \overline{H50})P(\overline{H50} \mid no\ storm)\,P(no\ storm)$

$= (0.6)(0.2)(0.3) + (0.01)(0.02)(0.7)$

$= 0.0361$ *Ans.*

c) $P(I80 \mid storm, I80 \cup H50) = \dfrac{P(I80 \mid storm)}{P(I80 \cup H50 \mid storm)}$

$= \dfrac{1 - P(\overline{I80} \mid storm)}{1 - P(\overline{I80}\,\overline{H50} \mid storm)} = \dfrac{1 - P(\overline{I80} \mid storm)}{1 - P(\overline{I80} \mid storm, \overline{H50})P(\overline{H50} \mid storm)}$

$= \dfrac{1-0.3}{1-(0.6)(0.2)}$ *Ans.*

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Problem 2. (10+10 = 20 points)

The force on a fixed body submerged in moving water is given by the formula

$$F = DAV^2$$

where A is the projected area of the body on the plane perpendicular to the direction of motion, V is the water velocity and D is the drag coefficient. Suppose A, V and D are random variables with the following second moment properties:

variable	mean	stdev	Correlation coefficient		
			A	V	D
A, m^2	1	0.25	1	0	0
V, m/sec	2	0.4	0	1	–0.3
D, $Nsec^2/m^4$	20	6	0	–0.3	1

a) Using first order approximations, determine the mean and standard deviation of F.

b) Determine the order of importance of the random variables A, V and D in contributing to the uncertainty in F.

a) $\mu_F \cong (20)(1)(2)^2 = 80\,\text{N}$ *Ans.*

$$\left(\frac{\partial F}{\partial A}\right)_{\mathbf{x=M}} = \left(DV^2\right)_{\mathbf{x=M}} = (20)(2)^2 = 80$$

$$\left(\frac{\partial F}{\partial V}\right)_{\mathbf{x=M}} = \left(2DAV\right)_{\mathbf{x=M}} = 2(20)(1)(2) = 80$$

$$\left(\frac{\partial F}{\partial D}\right)_{\mathbf{x=M}} = \left(AV^2\right)_{\mathbf{x=M}} = (1)(2)^2 = 4$$

$$\sigma_F^2 \cong (80)^2(0.25)^2 + (80)^2(0.4)^2 + (4)^2(6)^2 - 2(0.3)(80)(4)(0.4)(6)$$
$$= 400 + 1024 + 576 - 461$$
$$= 1{,}539$$

$\sigma_F \cong= 39.2$ *Ans.*

c) Imp(A) = (80)(0.25) = 20

Imp(V) = (80)(0.4) = 32

Imp(D) = (4)(6) = 24

Imp(A) < Imp(D) < Imp(V) *Ans.*

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Problem 3. (10+10 = 20 points)

Wave heights in open sea are known to have a probability density function of the form

$$f_X(x) = ax\exp(-bx^2) \qquad 0 < x$$

where a and b are positive-valued parameters.

a) Determine parameter a in terms of parameter b.

b) For $b = 0.1\,\mathrm{m}^{-2}$, determine the probability that the height of a wave will exceed 4m.

Solution:

a) Area underneath the PDF must equal 1:

$$\int_0^\infty axe^{-bx^2}dx = a\left(\frac{-1}{2b}\right)e^{-bx^2}\Big|_0^\infty = 0 - a\left(\frac{-1}{2b}\right) = 1$$

$a = 2b$ *Ans.*

b) $b = 0.1 \quad \rightarrow \quad a = 0.2$

$$f_X(x) = 0.2xe^{-0.1x^2}$$

$$P(X > 4) = \int_4^\infty 0.2xe^{-0.1x^2}dx$$

$$= -e^{-0.1x^2}\Big|_4^\infty = 0 + e^{-0.1(4)^2}$$

$= 0.202$ *Ans.*

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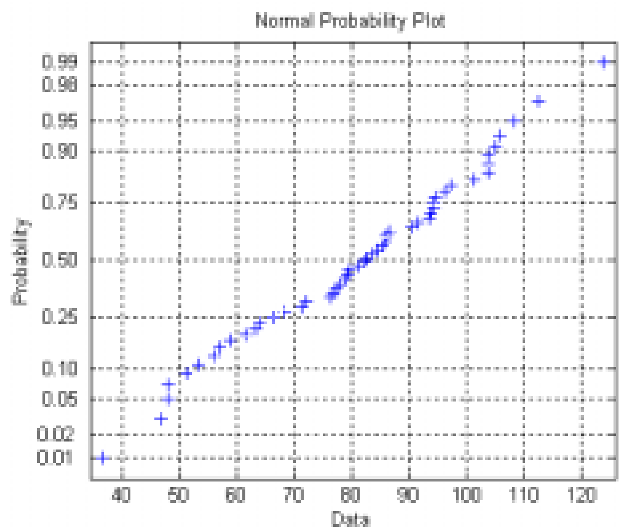
Problem 4. (5+10 = 15 points)

Shown below is data plotted on normal probability paper by use of the Matlab® command `normplot`.

a) In your opinion, does the plot suggest the normal distribution for this data? Explain your reasoning.

b) Approximately estimate the mean and standard deviation of a normal distribution fitted to the data. You may make use of any of the following Matlab® calculations:

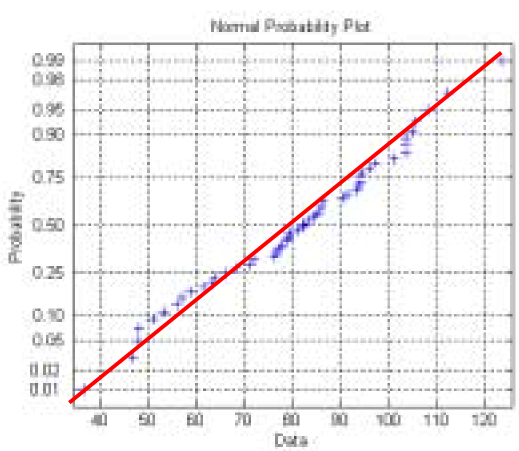
```
>> normcdf(2,0,1)
ans =
    0.9772

>> norminv(0.95,0,1)
ans =
    1.6449
```



Solution:

a) Yes. The data closely fall on a straight line on the normal probability plot, as shown below. This indicates that the normal distribution is a good fit to this data.

b) The mean is the 50% value; the mean plus 2 standard deviations is approximately the 98% value. Thus, mean $\cong$ 80, mean+2sigma $\cong$ 117, sigma $\cong$ (117–80)/2 = 18.5



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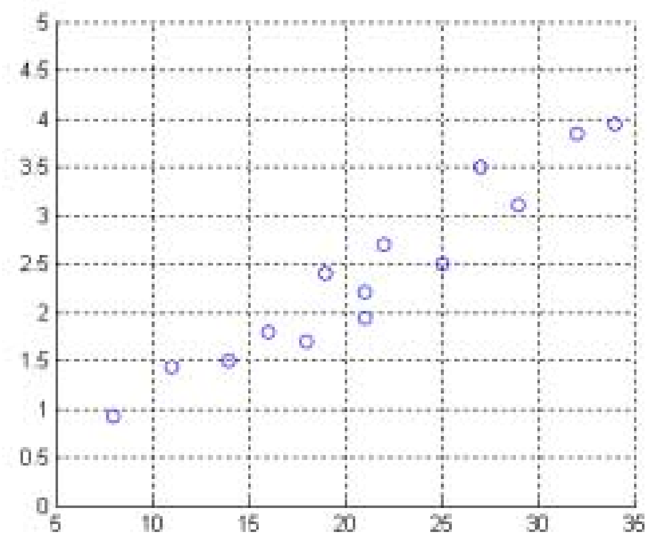
Problem 5. (10+10 = 20 points)

Shown below is a Matlab® session with data on blow counts (first column) and unconfined compressive strength of stiff clay soils (second column, in tons per square foot). Using the information provided, (a) compute and plot the mean regression line as well as the mean plus/minus one standard deviation regression lines on the scatter diagram shown. Clearly indicate coordinates at two points on each line. (b) Determine the R^2 value of the regression. Knowing the blow count, what reduction in the standard deviation of the soil strength is achieved?

```
x =
    8.0000    0.9200
   14.0000    1.5000
   16.0000    1.8000
   11.0000    1.4300
   19.0000    2.4000
   34.0000    3.9500
   21.0000    1.9500
   32.0000    3.8500
   25.0000    2.5000
   21.0000    2.2000
   18.0000    1.6900
   27.0000    3.5000
   29.0000    3.1000
   22.0000    2.7000
```




```
>> scatter(x(:,1),x(:,2))
>> axis([5 35 0 5]);
>> [b bint r rint stats] = regress(x(:,2),[ones(size(x(:,1))) x(:,1)]);
>> b

b =
   -0.0954
    0.1173

>> r'*r
ans =
    0.8698

>> stats
stats =

    0.9234  144.6166    0.0000
```

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Solution:

a) $E[Y \mid X = x] = -0.0954 + 0.1173x$

Two points on the line are (10,1.078) and (30,3.424)

The mean line is shown in solid red.

$$S_{Y|X} = \sqrt{\frac{0.8698}{14-2}} = 0.269$$

Pair of points on the lines are

(10,1.347) and (30,3.693) for the mean plus one standard deviation

(10,0.809) and (30,3.155) for the mean minus one standard deviation

The two lines are plotted in dashed red.

b) $R^2 = 0.9234$

$$1 - \frac{s_{Y|X}^2}{s_Y^2} = 0.9234$$

$$\frac{s_{Y|X}}{s_Y} = \sqrt{1 - 0.9234} = 0.277$$

The standard deviation is reduced by 100 – 27.7 = 72.3%.