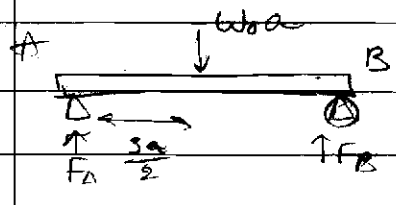
Problem 1 figure a

a) $q(x) = EIv'''' = -w_0 \langle x-a \rangle^0 + w_0 \langle x-2a \rangle^0$

$V(x) = EIv''' = -w_0 \langle x-a \rangle^1 + w_0 \langle x-2a \rangle^1 + C_1$

$M(x) = \int V(x)\,dx = -\frac{w_0}{2} \langle x-a \rangle^2 + \frac{w_0}{2} \langle x-2a \rangle^2 + C_1 x + C_2$



$\circlearrowleft \Sigma M_A = 0$

$F_B(3a) - w_0 a\left(\frac{3a}{2}\right) = 0$

$F_B = \frac{w_0(3a^2/2)}{3a} = \boxed{w_0(a/2)}$

$\Sigma F_y = 0 \; F_A = F_B - \boxed{w_0\left(\frac{a}{2}\right)}$

$V(0) = \frac{w_0 a}{2} \Rightarrow C_1 = \frac{w_0 a}{2}$

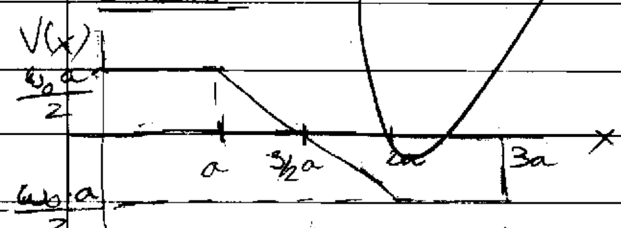
$M(0) = 0 \Rightarrow C_2 = 0$

$\Rightarrow V(x) = -w_0 \langle x-a \rangle^1 + w_0 \langle x-2a \rangle + \frac{w_0 a}{2}$

$M(x) = -\frac{w_0}{2} \langle x-a \rangle^2 + \frac{w_0}{2} \langle x-2a \rangle^2 + \frac{w_0 a}{2}(x)$

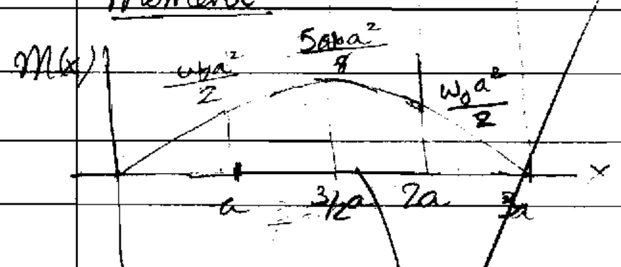
b) Diagrams

Shear



x	V(x)	x	V(x)
a	$\frac{w_0 a}{2}$	2a	$-\frac{w_0 a}{2}$
3/2 a	0	3a	$-\frac{w_0 a}{2}$

Moment



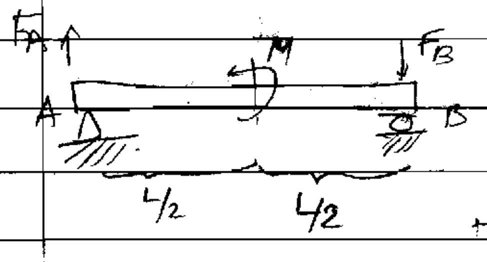
x	M(x)	x	M(x)
a	$w_0 a^2/2$	2a	$w_0 a^2/2$
3/2 a	$\frac{5}{8} w_0 a^2$	3a	0

Problem ① - Figure b

a) $q(x) = EIv'''' = -M\langle x - L/2\rangle^{-2}$

$V(x) = EIv''' = -M\langle x - L/2\rangle^{-1} + C_1$

$\mathcal{M}(x) = \int_0^x V(x)\,dx = -M\langle x - L/2\rangle^{0} + C_1 x + C_2$



$\circlearrowleft \Sigma M_A = 0$

$M - F_B(L) = 0 \Rightarrow \boxed{F_B = \frac{M}{L}}$

$+\Sigma F_y = 0$

$F_A - F_B = 0 \Rightarrow F_A = \boxed{\frac{M}{L}}$

$V(0) = \frac{M}{L} \Rightarrow C_1 = M/L$

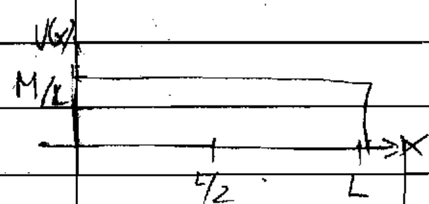
$M(0) = 0 \Rightarrow C_2 = 0$

$V(x) = -M\langle x - L/2\rangle^{-1} + \frac{M}{L}$

$M(x) = \int_0^x V(x)\,dx = -M\langle x - \frac{L}{2}\rangle^{0} + \left(\frac{M}{L}\right)x$

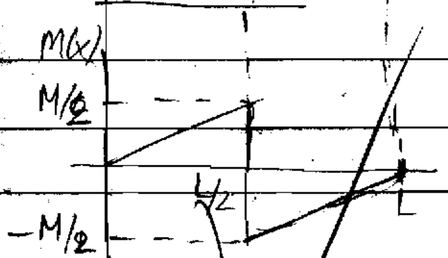
b) Diagrams

Shear



x	V(x)
0	M/L
L/2	M/L
L	M/L

Moment



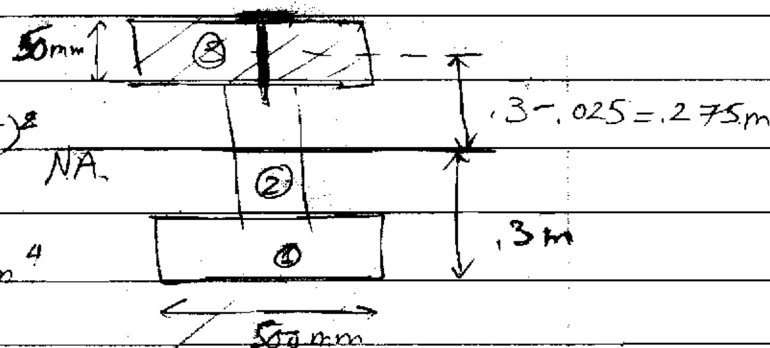
x	M(x)
0	0
L/2	±M/2
L	0

Problem ②

$I_{tot} = I_1 + I_2 + I_3$

$I_1 = \frac{1}{12}(.5)(.05)^3 + (.5)(.05)(.275)^2$

$= 1.893 \times 10^{-3}\ m^4$



$I_2 = \frac{1}{12}(0.05)(0.5)^3 = 5.208 \times 10^{-4}\ m^4$

$I_3 = I_1$

$I_{tot} = 2I_1 + I_2 = 2(1.893 \times 10^{-3}) + 5.208 \times 10^{-4} = 4.31 \times 10^{-3}\ m^4$

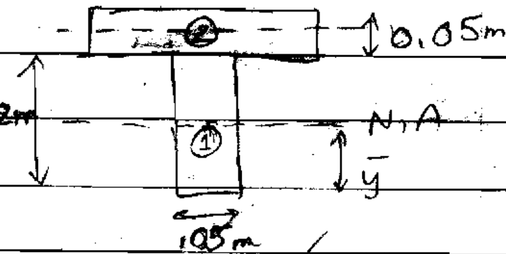
$Q = \bar{y}A = (0.275)(0.05)(0.5) = 6.875 \times 10^{-3}\ m^3$

$q = \frac{VQ}{I}$; $F_{allow} = q \cdot s \Rightarrow s = \frac{F_{allow}}{q} = \frac{F_{allow} \cdot I}{VQ}$

$s = \frac{(1000\,N)(4.31 \times 10^{-3}\,m^4)}{(10000\,N)(6.875 \times 10^{-3}\,m^3)} = \boxed{0.0627\,m}$

Problem ③

	A (m²)	$\bar{y}$ (m)	$A\bar{y}$ (m³)
①	(.2m)(.05) = 0.01	0.1	1×10^{-3}
②	(.2)(.05) = 0.01	.225	2.25×10^{-3}



$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{(1\times10^{-3}+2.25\times10^{-3})\,m^3}{(0.01+0.01)\,m^2} = 0.1625m$$

1) $\boxed{\bar{y} = 0.1625m}$

2) $I_{tot} = I_1 + I_2$

$I_1 = \frac{1}{12}(0.05)(0.2)^3 + (0.2)(0.05)(0.0625)^2 = 7.24\times10^{-5}\,m^4$

$I_2 = \frac{1}{12}(0.2)(0.05)^3 + (0.05)(0.2)(0.0625)^2 = 4.11\times10^{-5}\,m^4$

$I_{tot} = I_1 + I_2 = 7.24\times10^{-5}m^4 + 4.11\times10^{-5}m^4 = \boxed{1.135\times10^{-4}m^4}$

3) $\sigma_Y^T = 300MPa$, $\sigma_Y^C = -100MPa$, $\sigma_Y^T = 3\sigma_Y^C$

Assume compression occurs on top, and tension at bottom

Compression $\sigma_Y = -\frac{M_Y\cdot\bar{y}}{I} \Rightarrow M_Y = -\frac{\sigma_Y^C\cdot I}{\bar{y}} = \frac{(-100\times10^6)Pa(1.135\times10^{-4}m^4)}{0.0875m}$

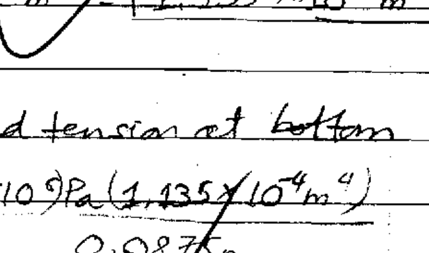
$M_Y = -129760\ N\cdot m$ or $-129.76kN\cdot m$

Tension $M_Y = -\frac{\sigma_Y^T\cdot I}{\bar{y}} = -\frac{(300\times10^6Pa)(1.135\times10^{-4}m^4)}{-0.1625m} = 209612.3\ N\cdot m$ or $209.61kN\cdot m$

Since M_Y for compression is less than M_Y for tension, the top surface will yield first. $M_Y = \underline{129.76kN\cdot m}$

4) Find neutral axis, $\sigma_Y^T = 3\sigma_Y^C$

$F_1 + F_2 = F_3$



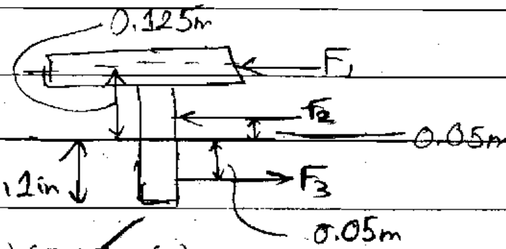
$\sigma_Y^C(bh) + \sigma_Y^C(h-y)b - \sigma_Y^T(y\cdot b)$

$\sigma_Y^C bh = \sigma_Y^T yb - \sigma_Y^C bh + \sigma_Y^C\cdot yb$

$2\sigma_Y^C bh = yb(\sigma_Y^T + \sigma_Y^C)$

$$y = \frac{2\sigma_Y^C\cdot h}{\sigma_Y^T+\sigma_Y^C} = \frac{2(100\times10^6Pa)(0.2m)}{(100\times10^6Pa+300\times10^6Pa)} = \boxed{0.1m}$$

5) Find ultimate bending moment, M_{ult}



$M_{ult} = \sigma_Y^T (0.05)(0.1)(0.05) +$

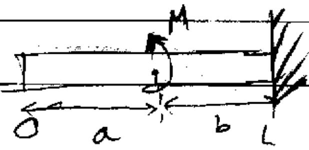
$\sigma_Y^C (0.05)(0.1)(0.05) + \sigma_c (.125)(.05)(.2)$

$= (300\times10^6)(.05)(.1)(0.05) +$

$(100\times10^6)(.05)(.1)(.05) + (100\times10^6)(.125)(.05)(.2)$

$M_{ult} = 225\,000 \text{ N}\cdot\text{m}$ or $\boxed{225 \text{ kN}\cdot\text{m}}$

Problem (4) — Cantilever beam



$EI v'''' = -w(x)$

1) Choice (C) $w(x) = M\langle x-a\rangle^{-2}$

2) State 4 conditions: $V(0) = 0$ $\quad \theta(L) = \theta(a+b) = 0$

$M(0) = 0 \quad v(L) = v(a+b) = 0$

3) Find $v(x)$

$EI v'''' = -M\langle x-a\rangle^{-2}$

$V(x) = -M\langle x-a\rangle^{-1} + C_1$

$M(x) = -M\langle x-a\rangle^{0} + C_1 x + C_2$

$EI\theta(x) = -M\langle x-a\rangle^{1} + \frac{C_1 \cdot x^2}{2} + C_2 x + C_3$

$EI v(x) = -\frac{M}{2}\langle x-a\rangle^{2} + \frac{C_1 \cdot x^3}{6} + \frac{C_2 x^2}{2} + C_3 \cdot x + C_4$

$V(0) = 0 \Rightarrow C_1 = 0$

$M(0) = 0 \Rightarrow C_2 = 0$

$\theta(L) = 0 \Rightarrow 0 = -Mb + C_3 \Rightarrow C_3 = Mb$

$v(L) = 0 \Rightarrow 0 = -\frac{Mb^2}{2} + Mba + Mb^2 + C_4 \Rightarrow C_4 = -\frac{Mb^2}{2} - Mba = -Mb\left(\frac{b}{2} + a\right)$

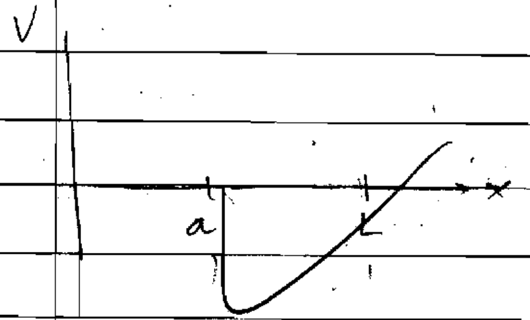
$$\boxed{v(x) = \frac{1}{EI}\left[-\frac{M}{2}\langle x-a\rangle^{2} + Mbx - \frac{Mb^2}{2} - Mab\right]}$$

4) Find $v(x=0)$

$v(x=0) = \frac{1}{EI}\left[-\frac{M}{2}(0) + Mb(0) - \frac{Mb^2}{2} - Mab\right] = \boxed{\frac{1}{EI}\left[-Mab - \frac{Mb^2}{2}\right]}$

5) Draw moment and shear diagrams

SHEAR



MOMENT