Mathematics 54. Spring Semester 2008 **Professor: Daniel Tataru**

Midterm 2

April 8, 2008, 11:10-12:30

Your Name: ______________________________________

TA's Name: ______________________________________

Section time: ______________________________________

Directions: This is a *closed* book exam. No calculators, cell phones, pagers, mp3 players and other electronic devices are allowed.
Remember: Answers without explanations will not count. You should **show your work**. Solve each problem on its own page. If you need extra space you can use backs of the pages and the extra page attached to your exam paper, but make a note you did so.

Problem	Score
1	
2	
3	
4	
5	
6	
Total	
Grade	

(30) 1. **Problem 1**. Consider the symmetric matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

a) Calculate its characteristic polynomial and its eigenvalues.

b) Find an orthonormal base in $\mathbb{R}^3$ consisting of eigenvectors of A.

c) Orthogonally diagonalize the matrix A.

(30) 2. **Problem 2**. a) Define an inner product in a vector space V.

b) For $p, q \in \mathbb{P}_2$ define $\langle p, q \rangle = \displaystyle\int_0^1 xp(x)q(x)dx$. Show that this is an inner product.

c) Find an orthogonal basis in $\mathbb{P}_2$ with respect to the above inner product.

(20) 3. Find a least squares solution of $Ax = b$ where

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 1 & 2 \end{bmatrix}, \qquad b = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}$$

(20) 4. Mark each statement True or False. Justify your answers.

a)Each eigenvalue of a square matrix A is also an eigenvalue of A^2.

b) Let A, B be invertible $n \times n$ matrices. If AB is diagonalizable then BA is diagonalizable.

(20) 5. Let $A = \begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$. Find an invertible matrix P and a matrix C of the form $C = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ so that $A = PCP^{-1}$.

(20) 6. Mark each statement True or False. Justify your answers.

a) Let $\{v_1, \cdots, v_p\}$ be an orthonormal set in $\mathbb{R}^n$. If $x = c_1v_1 + \cdots + c_nv_n$ then $\|x\|^2 = c_1^2 + \cdots + c_n^2$.

b) Any orthogonal matrix is invertible.