September 27, 2006.

Math 113, Section 1, fall 2006 (Caviglia) **FIRST MIDTERM**
Open book, open notes. In your proofs you may use any results from the lectures, from the parts of the textbook we covered, or from the homework exercises.

1) **[10 points]** Let A and B be two subsets of a given set S. Prove the following equality between sets:
$$S \setminus (A \cap B) = (S \setminus A) \cup (S \setminus B).$$

2) **[10 points]** Find integers m, n such that $m16 + n21 = 20$.

3) **[10 points]** Use the euclidian algorithm to find the G.C.D. of 89 and 144.

4) **[10 points]** Let $n \in \mathbb{Z}$ be an integer. Prove that $(5n + 4, 6n + 1)|19$.

5) **[10 points]** Let $a, n \in \mathbb{Z}$ and $n > 0$. Show that $a = qn + r$ with $-n/2 \leq r < n/2$ for some $q, r \in \mathbb{Z}$.

6) **[10 points]** Prove that the following set A contains no integer square.
$$A = \{x \in \mathbb{Z} | x = 3q + 2 \text{ for some } q \in \mathbb{Z}\}.$$

7) **[10 points]** Find the smallest integer in the given set:
   i) $\{x \in \mathbb{Z} | x > 0 \text{ and } x = 34s + 15t \text{ for some } s, t \in \mathbb{Z}\}$.
   ii) $\{x \in \mathbb{Z} | x > 100 \text{ and } x = 80s + 25t \text{ for some } s, t \in \mathbb{Z}\}$.

8) **[10 points]** Find all the incongruent solutions of $35X \equiv 10 \pmod{20}$.

9) **[10 points]** Solve the following system of congruences:
$$X \equiv 14 \pmod{16} \quad X \equiv 9 \pmod{15}\ .$$