1 Math H104 Midterm 2

Professor J. Sussan. Fall 2007.
Each problem is worth 20 points. You have 50 minutes to complete the exam. Please show your work; clarity of exposition counts!

1.1

Let α, β be Dedekind cuts. Prove that $\alpha\beta$ is also a cut.

1.2

Let $\bar{a_1} = (x_1, y_1), \bar{a_2} = (x_2, y_2) \in \mathbb{R}^2$. Prove that $d(\bar{a_1}, \bar{a_2}) = |x_1 - x_2| + |y_1 - y_2|$ is a metric of $\mathbb{R}^2$.

1.3

Prove that the finite union of compact sets is also compact.

1.4

Suppose $\{C_\alpha\}$ is a set of connected sets in a topological space such that $\cap_\alpha C_\alpha \neq \emptyset$. Prove that $\cup_\alpha C_\alpha$ is connected also.

1.5

Let X be a Euclidean space, let A, B be two compact subsets of X such that $A \cap B = \emptyset$. Prove that there exists open sets A', B' such that $A \subset A', B \subset B'$ and $A' \cap B' = \emptyset$.