MATH 185 — MIDTERM 1

e^{u+iv}
$e^{u}e^{iv}$

Problem #1. Prove that

$$\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$$

for all complex numbers z_1, z_2.

Problem #2. Write down the precise definition of the complex logarithm

$$log\, z.$$

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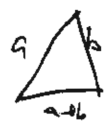
Problem #3. Find all the solutions z of the equation

$$z^6 = 2 + 2i.$$



Problem #4. Use the Cauchy-Riemann equations in polar coordinates to show that the function

$$f(z) = z^a = r^a e^{ia\theta}$$

is analytic, where a is real and $z = re^{i\theta}$ for $r > 0, -\pi < \theta < \pi$.

$|a| + |b| \geq |a+b|$

$|a+b| + |b| \geq |a|$

$|a+b| \geq |a| - |b|$

Problem #5. Show that if $|z| = 3$, then

$$\left| \frac{1}{z^2 + 3z + 2} \right| \leq \frac{1}{2}.$$

~~$|a+b| \geq |a|-|b|$~~ $|a+b| + |a| \geq |b|$

$|a+b| \geq ||a|-|b||$ $|a+b| \geq |b| - |a|$

(Hint: factor the denominator.)

$|a+b| \geq \max(|a|-|b|, |b|-|a|)$

$\geq ||a|-|b||$

Problem #6. Assume $f : \mathbb{C} \to \mathbb{C}$ is differentiable at a point z_0. State and prove the Cauchy–Riemann equations.

Problem #7. Let u and v be harmonic functions, and suppose that v is the conjugate of u. Show that

$z = u + iv$

$z^2 = u^2 - v^2 + 2iuv$

$$e^{u^2-v^2}\cos(2uv) \quad \text{and} \quad e^{u^2-v^2}\sin(2uv)$$

~~$(\cos + i\sin)^2$~~

are harmonic.

(Hint: e^{z^2} is analytic.) $e^{z^2} = e^{u^2-v^2}(\cos$ $\cos^2 - \sin^2$ ~~$+ 2i\cos\sin$~~

Problem #8. Show directly from the definition of the derivative that the function

$$f(z) = \bar{z}^2$$

is differentiable only at $z = 0$.

(Do not use the Cauchy-Riemann equations.)