

Midterm II, November 16,

Work each problem on a separate sheet of paper. Be sure to put your name, your section number, and your GSI's name on each sheet of paper. Also, at the top of the page, in the center, write the problem number, and be sure to put the pages in order. Write clearly: explanations (with complete sentences when appropriate) will help us understand what you are doing. Note that there are problems on the back of this sheet, for a total of four problems.

1. Let $A := \begin{pmatrix} 1 & -1 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$.

- (a) (10 pts) Find the eigenvalues of A .
- (b) (10 pts) For each eigenvalue, give a basis for the corresponding eigenspace.
- (c) (5 pts) Is A diagonalizable?

Notes: Explain your work step by step. Partial credit will only be given if your writing is very clear and easy to follow, with little or no partial credit for answers which would have been correct based on a mistake in an early step. If you copy the matrix wrong at the beginning, you will get no credit.

2. Answer the following questions using complete sentences.

- (a) (5 pts) What does it mean for two matrices to be *similar*?
- (b) (5 pts) Suppose that A and B are similar matrices. Is it true that every eigenvalue of A is an eigenvalue of B ? Give a proof or a counterexample.
- (c) (5 pts) Suppose that A and B are similar matrices. Is it true that every eigenvector of A is an eigenvector of B ? Give a proof or a counterexample.
- (d) (5 pts) Suppose that A and B are similar matrices and $A^2 + 2A = 3I$. Is it true that $B^2 + 2B = 3I$? Give a proof or a counterexample.

3. Let V be the vector space consisting of polynomials of degree less than or equal to 2, and let $T: V \rightarrow V$ be the map sending f to $xf' + f''$.

- (a) (5 pts) Find the eigenvalues of T .
- (b) (10 pts) If possible, find a basis for V consisting of eigenvectors for T .

4. (a) (5 pts) Use the Gram-Schmidt procedure to find an orthogonal basis for the column space of the matrix $\begin{pmatrix} 1 & 2 \\ 1 & 0 \\ 1 & 1 \end{pmatrix}$

- (b) You are told to solve the system of equations

$$\begin{aligned}x_1 + 2x_2 &= 2 \\x_1 &= 4 \\x_1 + x_2 &= 0\end{aligned}$$

After hearing your explanation that this is impossible, your boss (that's me) tells you to find x_1 and x_2 so that the vector

$$\mathbf{y}^* := \begin{pmatrix} x_1 + 2x_2 \\ x_1 \\ x_1 + x_2 \end{pmatrix}$$

is as close as possible to the vector $\mathbf{y} := \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix}$.

- i. (10 pts) What is your answer for $\mathbf{x}^* := \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$?
- ii. (5 pts) What is the your answer for \mathbf{y}^* ?
- iii. (5 pts) What is the distance between \mathbf{y}^* and \mathbf{y} ?