

Friday, November 3rd, 2006.

Math 113, Section 1, Fall 2006 (Caviglia) SECOND MIDTERM

Open book, open notes. In your proofs you may use any results from the lectures, from the parts of the textbook we covered, or from the homework exercises.

- 6 $\overline{) 990}$
160
6000
393
360
36
- (1) [10 Points] No proof required, just make an educated guess.
 - (a) Find a subgroup of \mathbb{Z}_{1101} of order 3.
 - (b) Give an example of two permutations σ and τ such that $(\sigma\tau)^2 \neq \sigma^2\tau^2$.
 - (c) Find an infinite group containing an element of order 11.
 - (2) [10 points] Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 7 & 5 & 4 & 6 & 3 & 2 & 1 \end{pmatrix}$ be an element of S_8 . Compute the order of σ , its parity, and calculate σ^{1000} .
 - (3) [10 Points] Determine whether the given relation is an equivalence relation.
 - (a) $a \sim b$ in $\mathbb{N} \setminus \{0\}$ if $\gcd(a, b) = \min\{a, b\}$.
 - (b) $a \sim b$ in $S \subset \mathbb{R}$, where S is a given subset of \mathbb{R} , if $(2a - 2b) \in \mathbb{Z}$.
 - (c) $a \sim b$ in \mathbb{Z} if $a + b$ is even.
 - (4) [10 Points] Let A be a set, let B be a subset of A , and let b be one particular element of B . Determine whether the given set
$$X = \{\sigma \in \text{Sym}(A) \mid \sigma(b) \in B\}$$
is sure to be a subgroup of $\text{Sym}(A)$.
 - (5) [10 Points] Let G be a group with the following property: whenever a, b and c belong to G and $ab = ca$, then $b=c$. Prove that G is Abelian. (i.e. "Cross" cancelation implies commutativity.)
 - (6) [10 Points] Suppose that G is a group with the property: whenever $a, b, c, d, x \in G$ and $axb = cxd$, then $ab = cd$. Prove that G is Abelian.
 - (7) [10 Points] Let $x, y \in G$, where G is a group. Assume that $y^2 = e, x \neq e$ and $yxxy^{-1} = x^2$. Find the order of $\langle x \rangle$.
 - (8) [10 Points] Let H_1 and H_2 be two subgroups of an Abelian group G . Prove that the set of products $H_1H_2 = \{g \in G \mid g = h_1h_2 \text{ for some } h_1 \in H_1, h_2 \in H_2\}$ is a subgroup of G .
 - (9) [10 Points] Let $p, q \in \mathbb{Z}^+$ be two prime numbers. Prove that all the subgroups of \mathbb{Z}_{pq} are cyclic.
 - (10) [10 Points] Let G be a finite group such that $|G| = 2n, n \in \mathbb{Z}$. Let H be a subgroup of G such that $|H| = n$. Prove that for all $a \in G$ and $h \in H$ we have $aha^{-1} \in H$.

GOOD LUCK!