Mathematics 54
Midterm #1 Spring 2009
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Instructions Write your name, section number, and GSI's name on your Blue Book and Answer Sheet RIGHT NOW. Show your work as indicated on Problems 11,12.1,12.3, 13.1,13.2 . Best wishes on the exam !

Problem # 1 The matrix $\begin{bmatrix} 1 & -1 & 1 & 0 & 6 \\ 0 & 0 & -1 & -9 & 2 \\ 0 & 0 & 0 & 1 & 10 \end{bmatrix}$ is in reduced row echelon form

(A) True **(B)** False

Problem #2 Let α and β be fixed constants. The function $T(x, y) = \alpha^3 x - \beta y$ from R^2 to R is a linear transformation.

(A) True **(B)** False

Problem #3 Subset H of R^3 consisting of vectors of the form $\begin{bmatrix} t \\ -t \\ t^3 \end{bmatrix}$ where t is any number is a subspace of R^3 of dimension equal to 1.

(A) True **(B)** False

Problem #4 Let P_3 denote the vector space of all polynomials $p(t) = p_0 + p_1 t + p_2 t^2 + p_3 t^3$ of degree at most 3. Consider the subset H of P_3 consisting of those $p(t)$ satisfying $p''(0) = 0$ where $p''(t)$ is the second derivative of $p(t)$. Which of the following is correct ?

(A) H is not a subspace of P_3.
(B) H is a subspace of P_3 of dimension 3.
(C) H is a subspace of P_3 of dimension 2 because the condition $p''(0) = 0$ involves the second derivative .

Problem #5 For any 3×4 matrix A we have

$$4 = dim(Col(A)) + dim(Nul(A))$$

(A) True **(B)** False

Problem #6 Let A and B be two 3×4 matrices such that $Col(A) = R^3$ and $Col(B) = R^3$. Then $Col(A + B) = R^3$.

(A) True **(B)** False

Problem #7 Four non-zero vectors in R^4 are linearly independent iff they span R^4.

(A) True **(B)** False

Problem # 8 Let A be a 5×3 matrix with 3 pivot columns, and let B be the reduced row echelon form of A. We know B is unique, and there is 5×5 invertible matrix M such that $MA = B$. Question: Is there only one such matrix M ?

(A) Yes **(B)** No

Problem # 9 Consider the matrix $A = \begin{bmatrix} 1 & -7 \\ 3 & 0 \end{bmatrix}$
The entry in the first row and second column of the classical adjoint of A is

(A) -7 **(B)** 7 **(C)** -3 **(D)** -2

Problem # 10 Consider the system of equations

$$(*) \qquad \begin{aligned} x - 3sy &= p \\ 2sx + (1 - 6s^2)y &= q \end{aligned}$$

where s is a real parameter. Suppose these equations are written in matrix form $Mv = b$ where $v = \begin{bmatrix} x \\ y \end{bmatrix}$ and $b = \begin{bmatrix} p \\ q \end{bmatrix}$.

(10.1) The 2×2 matrix M is

(A) $\begin{bmatrix} 1 & -3s \\ 2s & 1 - 6s^2 \end{bmatrix}$ **(B)** $\begin{bmatrix} 1 - 6s^2 & 3s \\ -2s & 1 \end{bmatrix}$

(10.2) The inverse M^{-1} of M is

(A) $\begin{bmatrix} 1/(1-6s^2) & 3s/(1-6s^2) \\ -2s/(1-6s^2) & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1-6s^2 & 3s \\ -2s & 1 \end{bmatrix}$

(10.3) The values of s for which there is a solution to the system (*) for every choice of values p and q are

(A) $s \neq \pm 1/\sqrt{6}$ **(B)** $s = \pm 1/\sqrt{6}$ **(C)** All s

Problem # 11 Let $A = \begin{bmatrix} t & 1 & -1 \\ 1 & t+1 & 0 \\ 1 & -1 & t \end{bmatrix}$

Compute the determinant of A using expansion along the third column.
SHOW WORK

(A) $1-t+t^2+t^3$ **(B)** $1+2t^2-t^3$ **(C)** $2+t^2+t^3$

Problem # 12 Consider the matrix and the vector

$$A = \begin{bmatrix} 3 & 6 & 2 & 1 \\ -5 & -10 & -3 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

(12.1) The reduced row echelon form of A is

(A) $\begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ **(B)** $\begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ **(C)** $\begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

SHOW WORK in transforming A to reduced row echelon for by row operations.

(12.2) The equation(s) expressing the basic (i.e., dependent) variables in terms of the free variables are

(A) $\begin{array}{l} x_1 = -2x_2 + x_4 \\ x_3 = -2x_4 \end{array}$ **(B)** $\begin{array}{l} x_1 = 2x_3 - x_4 \\ x_2 = 2x_4 \end{array}$ **(C)** $x_1 = -x_2 - 2x_3 + x_4$

(12.3) $Nul(A)$ consists of linear combinations of the form

(A) $s \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \end{bmatrix}$ **(B)** $s \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ -2 \\ -1 \end{bmatrix}$

where s and t are arbitrary parameters. **SHOW WORK**

(12.4) The dimension of $Col(A)$ is

(A) 1 **(B)** 2 **(A)** 3 **(B)** 4

(12.5) The dimension of $Nul(A)$ is

(A) 1 **(B)** 2 **(A)** 3 **(B)** 4

Problem #13 Let $A = \begin{bmatrix} 3 & 2 \\ -5 & -3 \end{bmatrix}$.

(13.1) We have $A = E_{21}(-1)E_{21}(-1)E_{12}(1)E_{12}(1)E_{21}(1)$. Can A be written as a product of three $E_{ij}(\lambda)$'s where λ is an integer ? **SHOW WORK**

(A) Yes **(B)** No

(13.2) What is the smallest number of $E_{ij}(\lambda)$ that can be used to write A as a product of $E_{ij}(\lambda)$'s in some way ? Here we allow λ to be any number.

(A) 2 **(B)** 3 **(C)** 4

SHOW WORK For example, can we have $A = E_{12}(a)E_{21}(b)$ for numbers a and b ?