Math 55: Midterm #2, 6 November 2003

Write your name, your student ID number, your section time and number, a three-problem grading grid, and your GSI's name on the cover of your blue book. Remain in your seat and hand in your exam book *to your GSI* at 3:30 pm. Books, notes, calculators, scratch paper and/or collaboration are not allowed. Justify your computations: correct answers with inadequate explanation may receive partial credit. You may leave your answers in terms of binomial coefficients or factorials where convenient.

Problem 1: Five pecans p_1, p_2, p_3, p_4 and p_5 are given to three squirrels s_1, s_2 and s_3, with each pecan given to one squirrel chosen independently with equal probabilities.
(a) Describe the sample space and compute its cardinality.
(b) Compute the probability that the first squirrel s_1 gets all the pecans.
(c) Compute the expected number of pecans the last squirrel s_3 gets.
(d) Compute the variance of the number of pecans the last squirrel gets.
(e) Compute the probability that each squirrel gets at least one pecan.

Problem 2: A discrete math midterm consists of 19 independent true–false questions on Martian literature. On each question, any non-Martian student has a 1/3 chance of guessing the correct answer. Two students (in the class of 28 students) are Martian, and therefore will get perfect scores.
(a) What is the probability that a student chosen at random from the non-Martian students gets a perfect score?
(b) What is the probability that a student chosen at random from the whole class gets a perfect score?
(c) Given that a student named Zřthjkpq got a perfect score on the midterm, compute the probability that Zřthjkpq is a Martian.

Problem 3: For any integer $k \geq 0$, let T_k be the set of all k–letter strings of Xs, Ys and Zs that **HAVE** two consecutive Xs. For example, $T_3 = \{XXY, XXZ, YXX, ZXX, XXX\}$. Let $t_k = |T_k|$.
(a) Evaluate t_0, t_1, t_2 and t_3 from the definition of T_k.
(b) Show that t_k satisfies the recurrence relation

$$t_{k+2} = 2t_{k+1} + 2t_k + 3^k$$

for $k \geq 0$.
(c) Find a closed form (like e^x/x or $1/(1-x)$) for the generating function

$$G(x) = \sum_{k=0}^{\infty} t_k x^k.$$