Math 53 Midterm #1, 10/8/07, 3:10 PM – 4:00 PM
(please do not leave the exam between 3:50 and 4:00)

No calculators or notes are permitted. Each of the 5 questions is worth 10 points. Please write your solution to each of the 5 questions on a separate sheet of paper with your name, SID number, and GSI's name on it. For each question, to get full credit, you must put a box around your final answer and show correct work or justification. Good luck!

1. Find the area of the region enclosed by the polar curves $r = 5\sec\theta$ and $\theta = -\pi/4$ and $\theta = \pi/4$.

2. Find the tangent plane to the surface

$$z = \frac{9}{x+y}$$

at the point $(1, 2, 3)$. Write your answer as an equation of the form $ax + by + cz = d$.

3. Does the following limit exist? If so, what is it? Justify your answer.

$$\lim_{(x,y)\to(0,0)} \frac{\sqrt{x^2+y^2+xy^2}}{\sqrt{x^2+y^2}}$$

4. The surfaces $x^2 + y^2 = 2$ and $y = z$ intersect in a curve C. Find a unit tangent vector to the curve C at the point $(1, 1, 1)$.

5. Let $\mathbf{r}(t)$ be a vector-valued function of t. Suppose that $\mathbf{r}(0) = \langle 2, 2, 1\rangle$ and $\mathbf{r}'(0) = \langle 1, 1, 2\rangle$. Compute the derivative

$$\frac{d}{dt}\|\mathbf{r}(t)\|\Big|_{t=0}.$$