University of California at Berkeley
College of Engineering
Dept. of Electrical Engineering and Computer Sciences

EE 105 Midterm II

Fall 2005 Prof. Borivoje Nikolić November 17, 2005

Your Name: ______________________________

Student ID Number: ________________________

Guidelines

Closed book and notes; there are some useful formulas in the end of the exam.
You may use a calculator.
You can unstaple the pages with formulas, but do not unstaple the exam.
Show all your work and reasoning on the exam in order to receive full or partial credit.
Time: 80 minutes = 1 hour, 20 minutes.

Score

Problem	*Points* Possible	Score
1	20	
2	14	
3	18	
Total	52	

1. MOS current source [20 points]

For the current source shown in Figure 1, dimensions of MOS transistors M1 and M2 are $(W/L)_1 = 2$, $(W/L)_2 = 10$. $V_{DD} = 2.5$V. $\mu_n C_{ox} = 100\ \mu A/V^2$, $V_{Tn} = 0.5$ V, $\lambda_n = 0.05\ V^{-1}$, $\gamma = 0$.

a) [4 points] Find the value of R_{REF} such that $I_{REF} = 100\mu A$. You can ignore channel length modulation in this part.

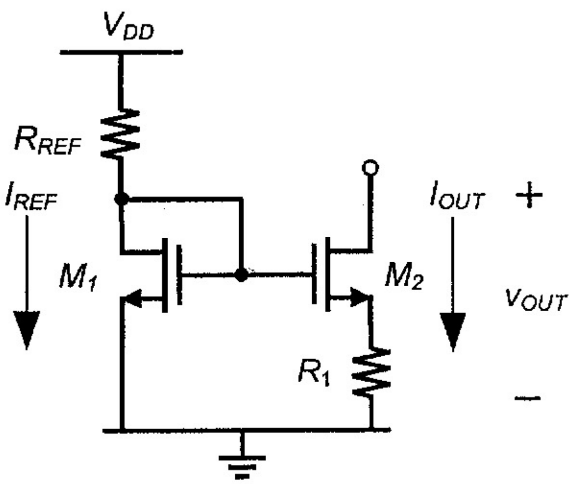


Figure 1.

$$\frac{V_{DD} - V_{GS_1}}{R_{ref}} = 100\mu A.$$

$$100\mu A = \left(\frac{W}{L}\right)_1 \frac{\mu_n C_{ox}}{2} (V_{GS1} - Vt)^2$$

$$\Rightarrow (V_{GS1} - Vt)^2 = \frac{100\mu \times 2}{2} \times \frac{1}{100\mu} = 1.$$

$$V_{GS} = 1 + 0.5 = 1.5\ V$$

So $$R_{ref} = \frac{2.5 - 1.5}{100\mu} = 10\,k\Omega$$

$R_{REF} = 10\,k\Omega$

(b) [4 points] Find the value of R_1 that gives $I_{OUT} = 40\mu A$? Assume that M_2 is in saturation. You can ignore channel length modulation in this calculation.

$\frac{V_{S2}}{R_1} = 40\mu A$. and $40\mu A = \left(\frac{W}{L}\right)_2 \frac{\mu_n C_{ox}}{2} (V_{GS2} - V_t)^2$

$\Rightarrow (V_{G2} - V_{S2} - V_t)^2 = \frac{40}{10} \times \frac{2}{100}$

$\Rightarrow (1.5 - V_{S2} - 0.5)^2 = 0.08$

$\Rightarrow (1 - V_{S2})^2 = 0.08$

$\Rightarrow V_{S2} = 0.717\,V$

So $R_1 = \frac{V_{S2}}{40\mu} = 17.9\,k\Omega$

$\boxed{R_1 = 17.9\,k\Omega}$

(c) [4 points] Find the lowest output voltage v_{OUT} for which the circuit in Figure 1 still acts as a current source. You can ignore channel length modulation.

Condition for saturation

$V_{DS} > V_{GS} - V_{tn}$

$\Rightarrow V_D > V_G - V_{tn}$

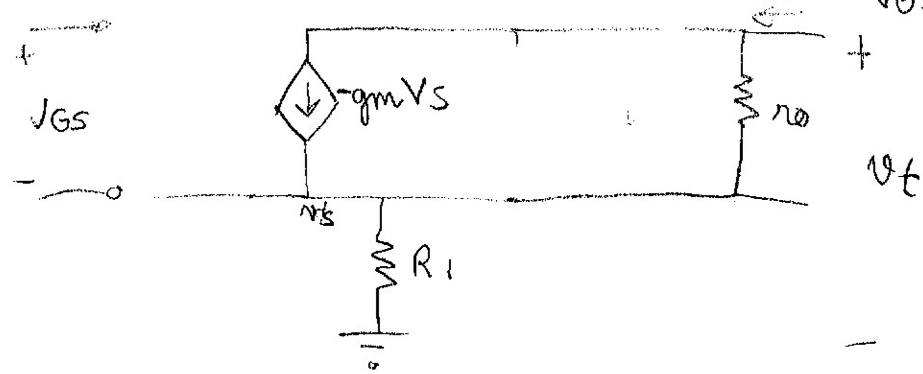
$V_D = V_{out}$

$V_{out\,min} = V_G - V_{tn} = 1.5 - 0.5 = \boxed{1\,V}$

$\boxed{v_{OUT,\,min} = 1V}$

(d) [6 points] Find the small-signal output resistance of the current source in Figure 1.

$V_{GS} = -V_S \quad (V_G = 0V)$



$i_t = \frac{v_s}{R_1}$ (1) $\quad i_t = -v_s g_m + \frac{v_t - v_s}{r_o}$ (2)

Replacing the value of $v_s = i_t R_1$ in (2), we get

$$i_t\left(1 + g_m R_1 + \frac{R_1}{r_o}\right) = \frac{v_t}{r_o}$$

$$R_{out} = \frac{v_t}{i_t} = r_o\left(1 + g_m R_1 + \frac{R_1}{r_o}\right)$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.05 \times 40\mu} = 500\,k\Omega$$

$$g_m = \left(\frac{W}{L}\right)_2 \mu_n C_{ox}(V_{GS} - V_t) = 10 \times 100\mu\,(1.5 - 0.717 - 0.5) = 0.283\,mS$$

$R_1 = 17.9\,k\Omega$

So $R_{out} = 500 \times 10^3\left(1 + 0.283 \times 10^{-3} \times 17.9 \times 10^3 + \frac{17.9 \times 10^3}{500 \times 10^3}\right) = 3.05\,M\Omega$

$\boxed{R_{out} = 3.05\,M\Omega}$

(e) [2 points] If body effect parameter, $\gamma > 0$, would it increase or decrease the value of output resistance from part (d)? Explain your answer.

Increase

Reason 1

$V_{tn} = V_{tno} + \gamma\left(\sqrt{V_{SB} - 2\phi_p} - \sqrt{-2\phi_p}\right)$

$V_{tn} \uparrow \Rightarrow I_D \downarrow$. $r_o = \frac{1}{\lambda I_D}$ so $r_o \uparrow$.

R_{out} $g_m \propto \sqrt{I_d}$ so $g_m \downarrow$ but slower than r_o increases.

so $R_{out} \propto g_m r_o \uparrow$.

Reason 2

$g_m \rightarrow g_m + g_{mb}$.

$R_{out} = r_o[(g_m + g_{mb})R_1 + 1] + R_1$.

so $R_{out} \uparrow$.

R_{out} increases / decreases (circle one)

2. MOS amplifiers [14 pts]

For the MOS amplifier in Figure 2, $(W/L)_1 = 10$, $(W/L)_2 = 20$, $(W/L)_3 = 10$, $I_{BIAS} = 50\mu A$. $V_{DD} = 2.5V$. $\mu_n C_{ox} = 100\ \mu A/V^2$, $\mu_p C_{ox} = 30\ \mu A/V^2$, $V_{Tn} = V_{Tp} = 0.5$ V, $\lambda_n = \lambda_{np} = 0.05V^{-1}$, $\gamma = 0$. $C_{GS2} = 2C_{GS1} = 2C_{GS3} = 100fF$. $C_{GD2} = 2C_{GD1} = 2C_{GD3} = 10fF$.

Input voltage v_S has negligible input resistance and contains a DC and an AC component.

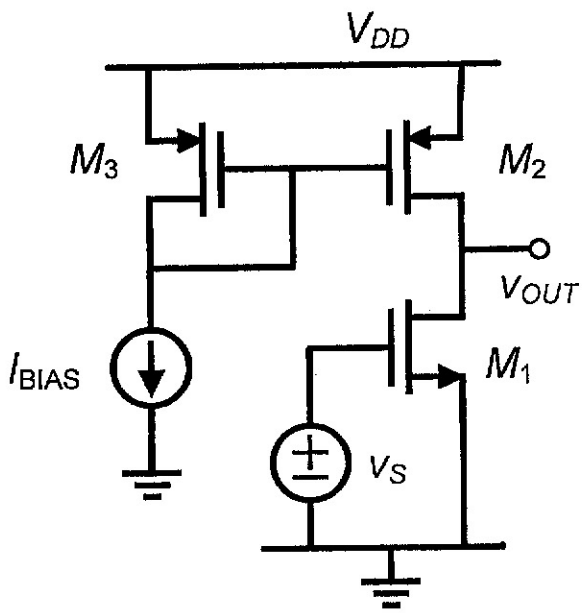


Figure 2.

(a) [2 points] Find the bias current of transistor M_1.

$$I_{M1} = I_{bias} \frac{(W/L)_2}{(W/L)_1} = I_{bias} \times 2 = 100\mu A$$

$I_{M1} = 100\mu A$

(b) [4 points] Find the small-signal voltage gain $A_v = v_{out}/v_s$.

$$A_v = \frac{v_{out}}{v_s} = -g_{m1}(r_{o1} \parallel r_{o2})$$

$$r_{o1} = \frac{1}{\lambda_n I_{M1}} = \frac{1}{0.05 \times 100\mu} = 200\,k\Omega$$

$$r_{o2} = \frac{1}{\lambda_p I_{M1}} = \frac{1}{0.05 \times 100\mu} = 200\,k\Omega$$

$$R_{out} = r_{o1} \parallel r_{o2} = 100\,k\Omega$$

$$g_{m1} = \sqrt{2\left(\frac{W}{L}\right)_1 I_{M1} \mu_n C_{ox}} = \sqrt{2 \times 10 \times 100\mu \times 100\mu} = 0.447\,mS$$

80 $\boxed{A_v = -44.7\ V/V}$

$A_v =$ -44.7 V/V

(c) [4 points] Find the maximum and the minimum voltage at the output of this amplifier.

For M1: $I_{M1} = \left(\frac{W}{L}\right)_1 \frac{\mu_n C_{ox}}{2} (V_{GS_1} - V_{tn})^2$

$\Rightarrow 100\mu A = \frac{10}{2} \times 100\mu\, (V_{GS_1} - 0.5)^2 \Rightarrow V_{G1} = 0.947$ $V_{GS_1} = V_{G1}$

$V_{out\,min} = V_{G1} - V_{tn} = 0.447V.$

For M2: $I_{M2} = \left(\frac{W}{L}\right)_2 \frac{\mu_p C_{ox}}{2} (V_{SG2} - |V_{tp}|)^2$

$\Rightarrow 100\mu A = \frac{20}{2} \times 30\mu\, (V_{SG2} - 0.5)^2$

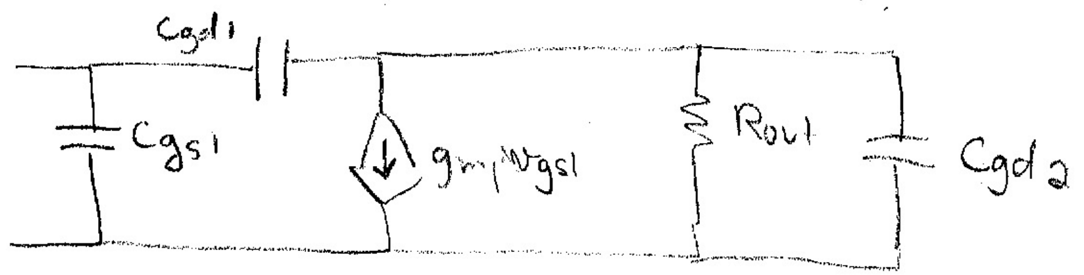
$\Rightarrow V_{SG2} = 1.077V \Rightarrow V_{G_2} = 2.5 - 1.077 = 1.42V.$

so $V_{out\,max} = V_{G_2} + |V_{tp}| = 1.42 + 0.5 = 1.92V$

$V_{out,\,max}$ = 1.92 V; $V_{out,\,min}$ = 0.447V

(d) [4 points] Find the frequency of the dominant pole of this amplifier.

$\tau = R_{out} \times C_{out} = R_{out} \times \left(C_{gd2} + C_{gd1}\left(1 - \frac{1}{A_v}\right)\right)$



$P1 = \frac{1}{\tau} = \frac{1}{100k \times (20f)} = 5 \times 10^8$ rad/s.

$\omega = 5 \times 10^8$ rad/s

3. Amplifier frequency response [18 points]

An amplifier has all of its poles and zeros in the left-hand frequency plane (it is a stable, minimum-phase system) and an amplitude frequency response, as shown in Figure 3.

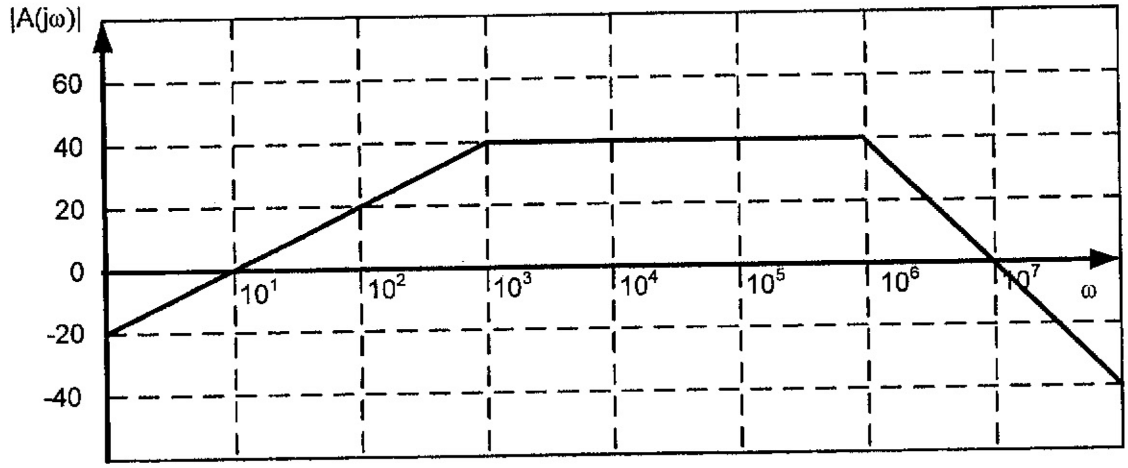
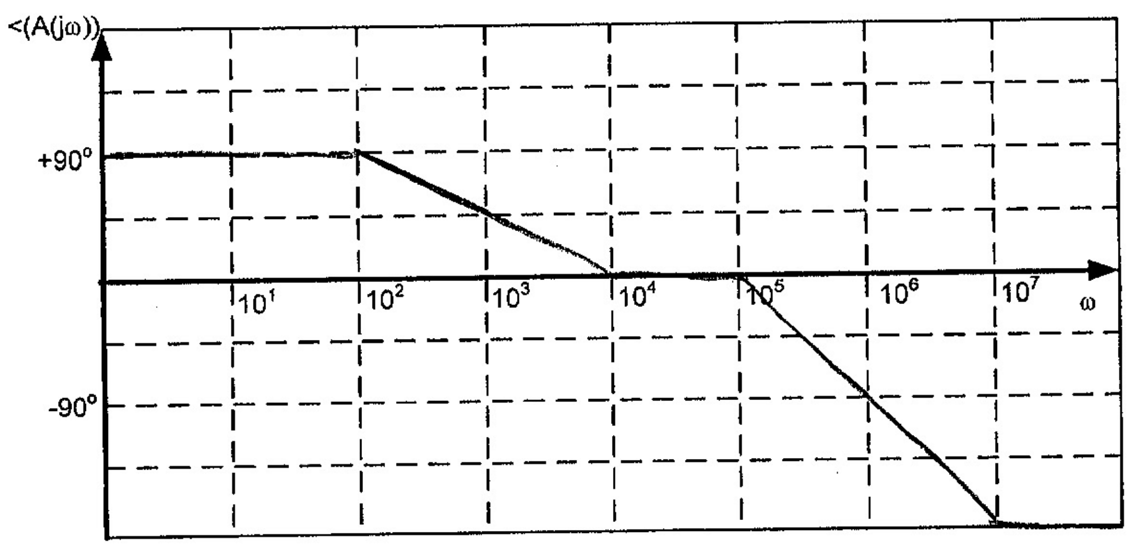


Figure 3

a) [6 points] Write the transfer function that produces this response.

$$H(j\omega) = \frac{j\frac{\omega}{10}}{\left(1+j\frac{\omega}{10^3}\right)\left(1+j\frac{\omega}{10^6}\right)^2}$$

b) [6 points] Draw the phase response that corresponds to this amplitude response.



(c) [6 points] Write the transfer function that produces the response from Figure 4.

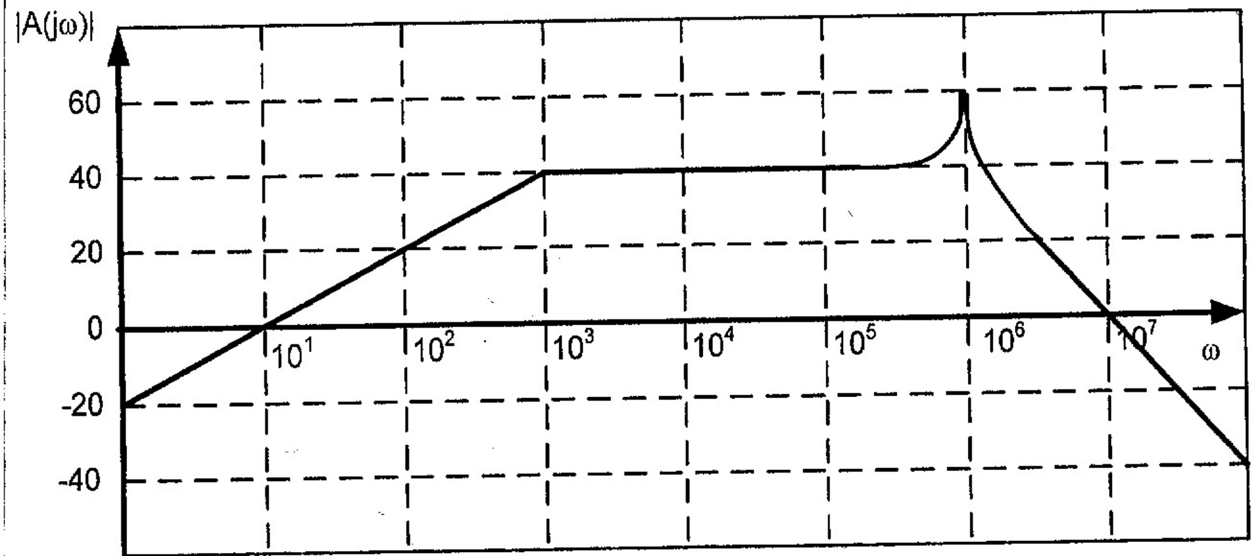


Figure 4.

$$H(j\omega) = \frac{j\frac{\omega}{10} \times 10^{12} \leftarrow \omega_0^2}{\left(1+j\frac{\omega}{10^3}\right)\left((j\omega)^2 + j\omega\frac{10^6}{Q} + 10^{12}\right)}$$

$20 \log|Q| = 20 \Rightarrow Q = 10.$

$$H(j\omega) = \frac{j\omega 10^{11}}{\left(1+j\frac{\omega}{10^3}\right)\left((j\omega)^2 + j\omega 10^5 + 10^{12}\right)}$$

Note : denominator

$1 + \left(\frac{j\omega}{\omega_0}\right)^2 + \left(\frac{j\omega\omega_0}{Q}\right)\frac{1}{\omega_0^2} \Rightarrow$ multiply by ω_0^2 on numerator.