UNIVERSITY OF CALIFORNIA
College of Engineering
Department of Electrical Engineering and Computer Sciences

EECS 130
Spring 2008

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Midterm I - Solutions

Name: ________________________________

SID: ________________________________

Grad/Undergrad: ________________________

Closed book. One sheet of notes is allowed.
There are 12 pages of this exam including this page.

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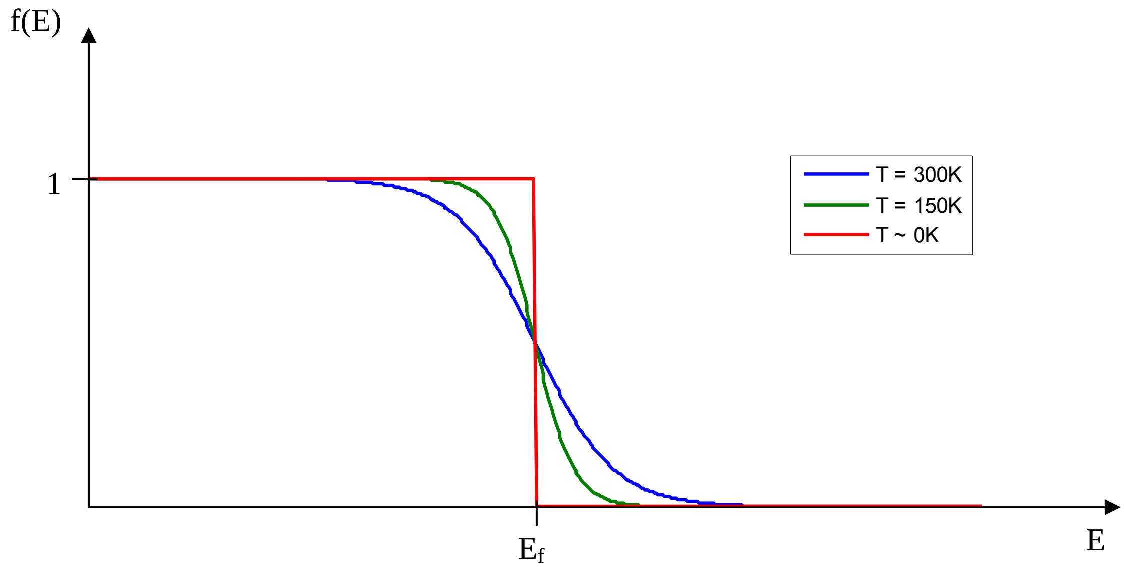
Total		100

Physical Constants

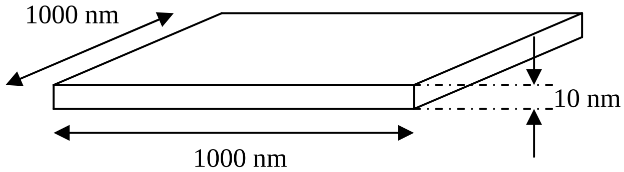
Electronic charge	q	1.602×10^{-19} C
Permittivity of vacuum	ε_0	8.845×10^{-14} F cm^{-1}
Relative permittivity of silicon	$\varepsilon_{Si}/\varepsilon_0$	11.8
Boltzmann's constant	k	8.617×10^{-5} eV/ K or
		1.38×10^{-23} J K^{-1}
Thermal voltage at T = 300K	kT/q	0.026 V
Effective density of states	N_c	2.8×10^{19} cm^{-3}
Effective density of states	N_v	1.04×10^{19} cm^{-3}

<u>Problem 1</u> [15pts]

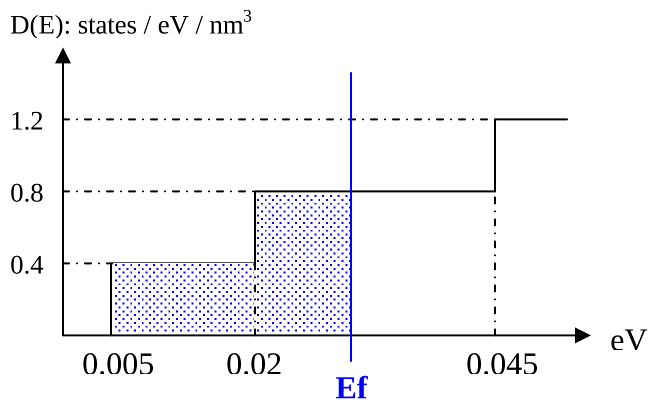
[5pts] **(a)** Draw qualitatively the Fermi-Dirac distribution at T=300K, T=150K, and T≈0K.



[10pts] **(b)** Quantum wells are often used in applications such as semiconductor lasers. In a quantum well, electrons are confined in a thin slab of material, as shown below.



The density-of-states, *D(E)* in an ideal quantum-well is step-like:



At T≈0K, there are 1.4×10^5 states in this system, mark the location of Fermi Level, E_f? (In this problem, there is no need to consider spin explicitly since it is already taken into account in *D(E)*.)

At 0K, all the states below E_f are filled with electrons; all the states above E_f are empty. Therefore,

$$N = V \bullet n = V \bullet \int_{Ec}^{\infty} f(E)D(E)dE = V \bullet \int_{Ec}^{Ef} D(E)dE$$

This means the number of electrons in the system, N, equals the area under the D(E) curve up to E_f, times the volume of the semiconductor, V.

$$V = 10nm \times 1000nm \times 1000nm = 10^7\,nm^3.$$

To determine the location of E_f, we divide the system in two regions and calculates the number of states in these regions.

(1) 0.005eV < E < 0.02eV $\quad 0.4 \times (0.02 - 0.005) \times 10^7 = 60000(states)$

(2) 0.02eV < E < 0.045eV $\quad 0.8 \times (0.045 - 0.02) \times 10^7 = 200000(states)$

60000 electrons will completely fill region (1); 80000 are left for region (2). Region (2) will only be partially filled. The precise location of E_f:

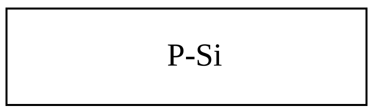
$$(E_f - 0.02eV) \times 0.8 \times 10^7 = 80000$$

$$E_f = 0.03eV$$

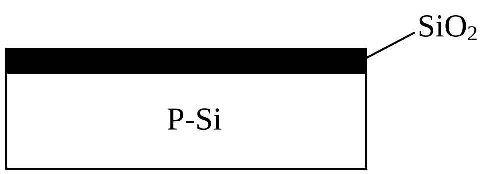
Problem 2 [30pts]

[10pts] **(a)** A P-N junction is fabricated by the process sequence shown below. The cartoons show cross sectional views of the wafer after each processing step. Fill in the names of each process steps shown below:

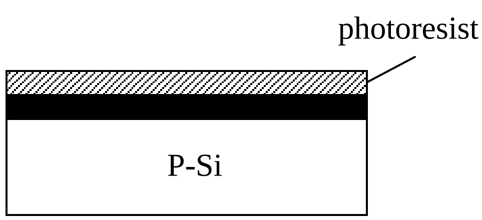
1. (Starting Wafer)



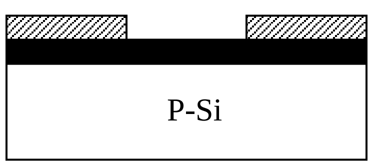
2. *thermal oxidation* .



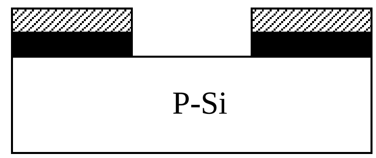
3. photoresist spin-coating



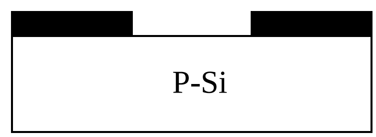
4. *lithography* .



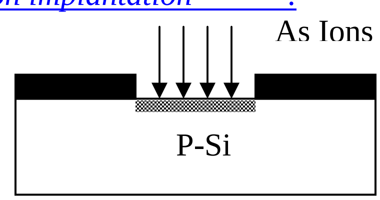
5. *etching* .



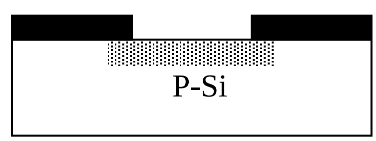
6. *resist strip* .



7. *ion implantation* .



8. Diffusion at 1100C for 60min



Process Available: ion implantation, sputtering, resist strip, etching, lithography, chemical mechanical polishing, epitaxy, thermal oxidation

[5pts] **(b)** The starting wafer is P-type and doped with $N_A=10^{15}cm^{-3}$. Find the depletion region width. (You may assume $N_d >> N_a$ and Φ_{bi}=0.85 V)

The implanted N-type region is usually much more heavily doped than the background P-doping ($10^{15}cm^{-3}$), this is why we can assume $N_d >> N_a$.

We may use the lightly-doped side doping concentration to calculate W_{dep}.

$$W_{dep} = \sqrt{\frac{2\varepsilon_{si}\phi_{bi}}{qN_a}} = \sqrt{\frac{2\times 11.8\times 8.854\times 10^{14} F/cm \times 0.85V}{1.6\times 10^{-19} C \times 10^{15} cm^{-3}}} = 1.05\times 10^{-4} cm = 1.05\mu m$$

[5pts] **(c)** Find the depletion region width when a reverse bias of V_R = 4.15V is applied. (Also assume $N_d >> N_a$)

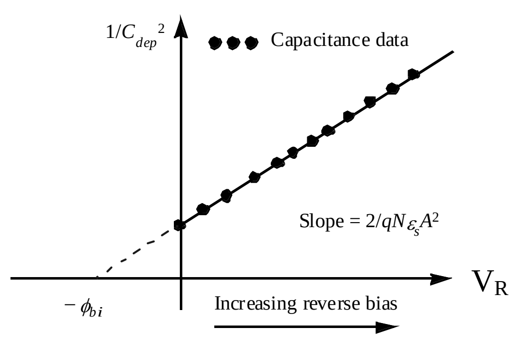
The depletion region is widened under reverse bias.

$$W_{dep} = \sqrt{\frac{2\varepsilon_{si}\left(\phi_{bi}+V_R\right)}{qN_a}} = \sqrt{\frac{2\times 11.8\times 8.854\times 10^{14} F/cm \times 5V}{1.6\times 10^{-19} C \times 10^{15} cm^{-3}}} = 2.56\times 10^{-4} cm = 2.56\mu m$$

[10pts] **(d)** If the diode area is 0.01cm^2, plot $\frac{1}{C_{dep}{}^2}$ versus V_R. What are the x-intercept and the slope of the curve, respectively?

The depletion capacitance can be calculated using the parallel-plate capacitor formula with plate separation W_{dep}.

$$C_{dep} = \frac{\varepsilon_{si}A}{W_{dep}} = \sqrt{\frac{qN_a\varepsilon_{si}A^2}{2\left(\phi_{bi}+V_R\right)}}$$ *(Here we also assume $N_d >> N_a$)*



Arrange terms:

$$\frac{1}{C_{dep}{}^2} = \frac{2\left(\phi_{bi}+V_R\right)}{qN_a\varepsilon_{si}A^2}$$

x-intercept = $-\Phi_{bi}$ = -0.85

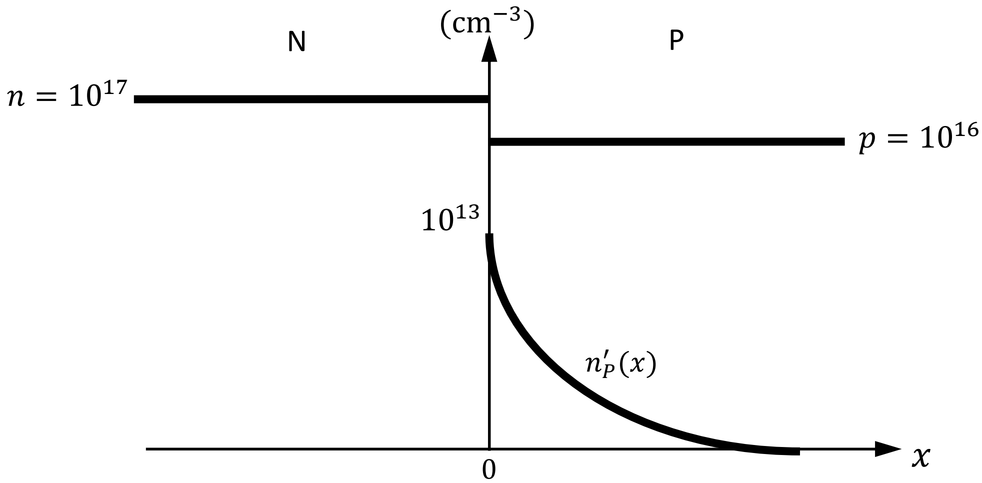
slope:

$$\frac{2}{qN_a\varepsilon_{si}A^2} = \frac{2}{1.6\times 10^{-19} C\times 10^{15} cm^{-3}\times 11.8\times 8.854\times 10^{-14} F/cm \times \left(0.01cm^2\right)^2}$$

$$= 1.20\times 10^{20}\frac{1}{F\bullet C} = 1.20\times 10^{20}\frac{1}{F^2\bullet V}$$

<u>Problem 3.</u> [35pts]

The carrier profiles in a forward biased PN diode are shown below. The majority carrier densities for both P and N-side are given. Also shown is the minority carrier density on the P-side.



[5pts] **(a)** Calculate the bias voltage.

From the above graph you can find the doping density.

$$N_d = n = 10^{17} cm^{-3}, \qquad N_a = p = 10^{16} cm^{-3}$$

From the minority carrier profile,

$$n_P'(0) = \left(\frac{n_i^2}{N_a}\right)\left[exp\left(\frac{qV}{kT}\right) - 1\right] = 10^{13} cm^{-3}$$

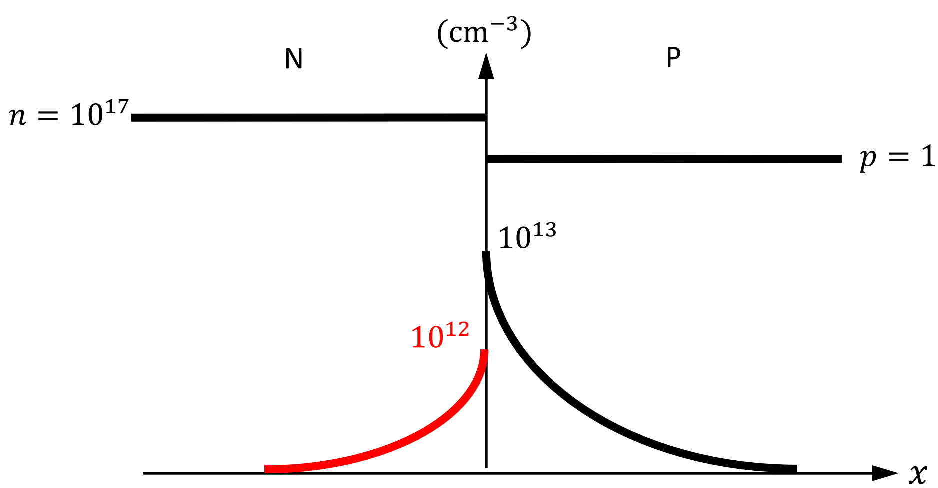
$$\therefore V = \left(\frac{kT}{q}\right) ln\left(\frac{n'(0)}{n_i^2/N_a} + 1\right) = (0.026V) ln\left(\frac{10^{13}}{(10^{10})^2/(10^{16})} + 1\right) = 0.539\ V$$

[5pts] **(b)** Find the minority electron and hole current density $J_{nP}(0)$ and $J_{pN}(0)$. Use $D_p = 12cm^2/s$, $D_n = 35cm^2/s$, $\tau_p = 0.01\mu s$ and $\tau_n = 0.02\mu s$.

$$J_{nP}(0) = q\frac{D_n}{L_n}\left(\frac{n_i^2}{N_a}\right)\left[exp\left(\frac{qV}{kT}\right) - 1\right] = q\frac{D_n}{\sqrt{D_n\tau_n}}n'(0) = 6.69 \times 10^{-2}\ A/cm^2$$

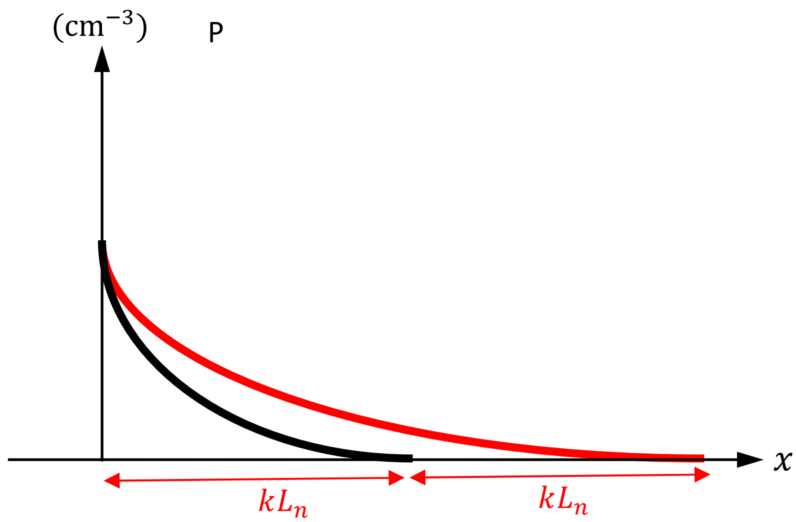
$$J_{pN}(0) = q\frac{D_p}{\sqrt{D_p\tau_p}}\left(\frac{n_i^2}{N_d}\right)\left[exp\left(\frac{qV}{kT}\right) - 1\right] = 5.54 \times 10^{-3}\ A/cm^2$$

[5pts] **(c)** Add the minority hole concentration profile on the N-side and show $p_N'(0)$.



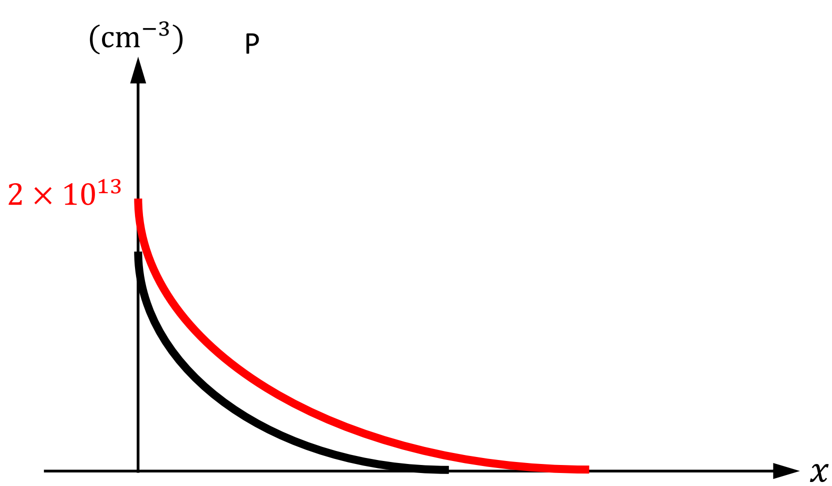
$$p_N'(0) = \left(\frac{n_i^2}{N_d}\right)\left[exp\left(\frac{qV}{kT}\right) - 1\right] = 10^{12}cm^{-3}$$

[5pts] **(d)** Redraw $n'_P(x)$ if the electron diffusion constant D_n is 4 times larger. Assume all other conditions are the same.



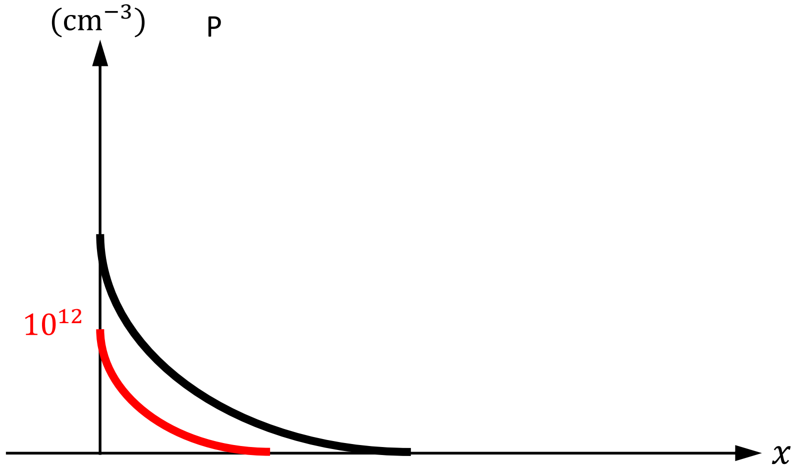
If the diffusion constant D_n is increased by a factor of 4, the diffusion length is doubled. So the minority carrier decay length will double.

[5pts] **(e)** Redraw $n'_P(x)$ if $5 \times 10^{15}\text{cm}^{-3}$ donors are added to the P-side and show $n'_P(0)$. Assume all other conditions are the same.



$$n'_P(0) = \left(\frac{n_i^2}{N_a - N_d}\right)\left[exp\left(\frac{qV}{kT}\right) - 1\right] = 2 \times 10^{13} cm^{-3}$$

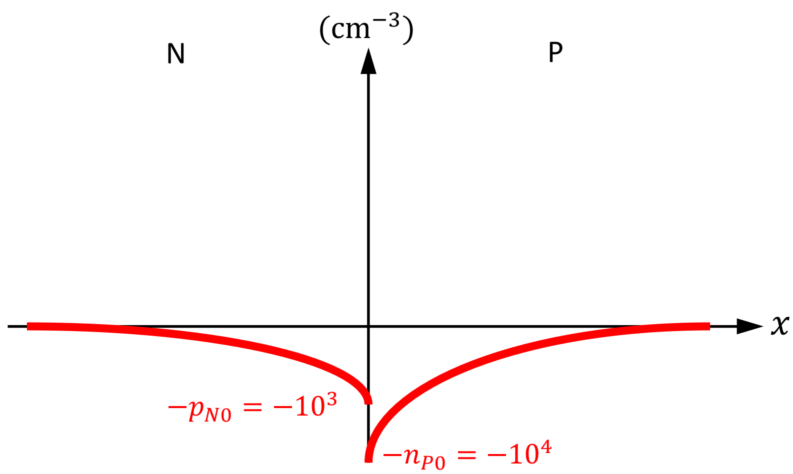
[5pts] **(f)** Redraw $n'_P(x)$ the applied bias voltage is reduced by $60mV (= 0.06V)$.
Assume all other conditions are the same.



With a 60mV reduction the minority carrier concentration is reduced by a factor of 10.

$$n'_P(0) = \left(\frac{n_i^2}{N_a}\right)\left[exp\left(\frac{q(V - 60mV)}{kT}\right) - 1\right] = 10^{12}cm^{-3}$$

[5pts] **(h)** Draw the excess minority carrier distribution if the diode is reverse biased at $\sim 1V$.



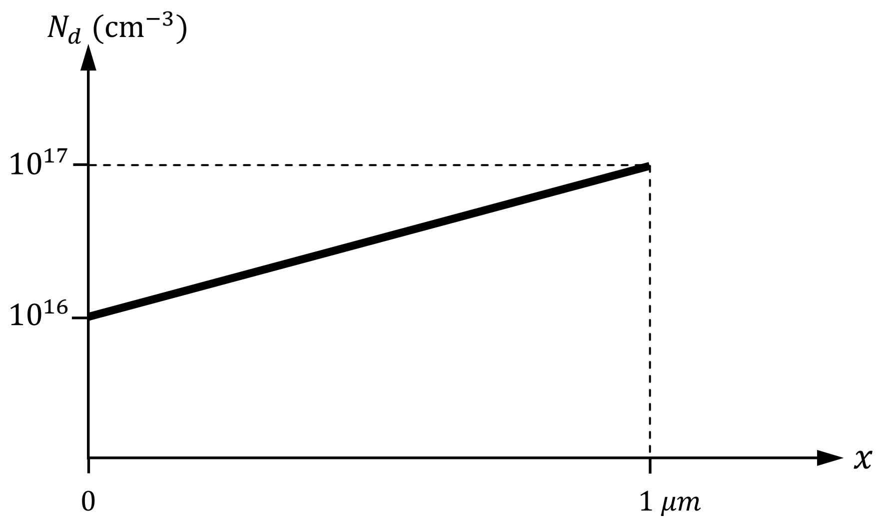
With a reverse bias the excess minority carrier concentration at the edge of the deletion region will be

$$n'_P(0) = \left(\frac{n_i^2}{N_a}\right)\left[exp\left(\frac{q(-1V)}{kT}\right) - 1\right] \approx -\left(\frac{n_i^2}{N_a}\right) = -n_{P0} = -10^4cm^{-3}$$

$$p'_N(0) = \left(\frac{n_i^2}{N_d}\right)\left[exp\left(\frac{q(-1V)}{kT}\right) - 1\right] \approx -\left(\frac{n_i^2}{N_d}\right) = -p_{N0} = -10^3cm^{-3}$$

<u>Problem 4.</u> [20pts]

You are given an N-type piece of silicon with $N_d(x) = (10^{16}\ cm^{-3}) + (9 \times 10^{16} cm^{-3}/\mu m) \cdot x$. Assume $T = 300K$ and there are no acceptor dopants, i.e. $N_a = 0$.



[5pts] **(a)** Given that E_f is constant, what is the electron current density $J_n(x)$?

When the Fermi level is constant, the system is in equilibrium. So there will be no current flowing. The diffusion current and drift current will cancel each other out.

$$J_n(x) = J_{n,drift}(x) + J_{n,diff}(x) = 0$$

[5pts] **(b)** Find the electron diffusion current density $J_{n,diff}(x)$. Use $D_n = 30\ cm^2/s$.

$$J_{n,drift}(x) = qD_n \frac{dn}{dx}$$

$$= qD_n \frac{d}{dx}[(10^{16}\ cm^{-3}) + (9 \times 10^{16} cm^{-3}/\mu m) \cdot x]$$

$$= qD_n \times (9 \times 10^{16} cm^{-3}/\mu m)$$

$$= (1.6 \times 10^{-19} C) \times (30 cm^2/s) \times (9 \times 10^{16} cm^{-3}/\mu m) \times (10^4 \mu m/cm)$$

$$= 4.32 \times 10^3\ A/cm^2 \quad (or \quad 0.432\ A/cm \cdot \mu m)$$

[5pts] **(c)** Derive an expression for the electron drift current density $J_{n,drift}(x)$ in terms of μ_n.

$$J_{n,drift}(x) = q\mu_n n(x)\mathcal{E}(x)$$

Using Poisson's equation and $n(x) = N_c exp\left(-\frac{(E_c-E_f)}{kT}\right)$

$$\mathcal{E}(x) = -\frac{d\phi}{dx} = \frac{1}{q}\frac{dE_c(x)}{dx} = \frac{1}{q}\frac{d}{dx}\left[-kT\cdot ln\left(\frac{n(x)}{N_c}\right)\right] = -\frac{kT}{q}\cdot\frac{N_c}{n(x)}\cdot\frac{1}{N_c}\cdot\frac{dn(x)}{dx}$$

$$= -\frac{kT}{q}\cdot\frac{1}{n(x)}\cdot\frac{dn(x)}{dx} = -\frac{kT}{q}\cdot\frac{9\times 10^{16}cm^{-3}/\mu m}{(10^{16}\ cm^{-3}) + (9\times 10^{16}cm^{-3}/\mu m)\cdot x}$$

Therefore,

$$J_{n,drift}(x) = q\mu_n\cdot n(x)\cdot\left[-\frac{kT}{q}\cdot\frac{9\times 10^{16}cm^{-3}/\mu m}{n(x)}\right]$$

$$= -kT\cdot(9\times 10^{16}cm^{-3}/\mu m)\cdot\mu_n$$

$$= -(0.026\ eV)\cdot(1.6\times 10^{-19}\,J/eV)\ \cdot(9\times 10^{16}cm^{-3}/\mu m)\cdot(10^4\mu m/cm)\cdot\mu_n$$

$$= -(3.744\ \ C\cdot V/cm^4)\cdot(\mu_n\ \ cm^2/V\cdot s)\ \ [A/cm^2]$$

[2pts] **(d)** Using the total electron current density, find the value of μ_n.

From $J_n(x) = J_{n,drift}(x) + J_{n,diff}(x) = (4.32\times 10^3) + (-3.744\mu_n) = 0,$

$$\mu_n = \frac{4.32\times 10^3\ A/cm^2}{3.744\ C\cdot V/cm^4} = 1.154\times 10^3\ cm^2/V\cdot s$$

[3pts] **(e)** Numerically verify Einstein's relationship with the values you've calculated above.

$$\frac{D_n}{\mu_n} = \frac{30\ cm^2/s}{1.154\times 10^3\ cm^2/V\cdot s} = 0.026\ V = \frac{kT}{q}\bigg|_{T=300K}$$