EECS 120 Signals & Systems — University of California, Berkeley: Fall 2007

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Midterm Exam 1

Last name	First name	SID

Rules.

- You have two hours to complete this exam.
- There are 100 points for this exam.
- The exam is closed-book and closed-notes; calculators, computing and communication devices are *not* permitted.
- However, one handwritten and *not photocopied* double-sided sheet of notes is allowed.
- Moreover, you receive, together with the exam paper, copies of Tables 4.2 and 5.2 of the course textbook.
- No form of collaboration between the students is allowed. If you are caught cheating, you may fail the course and face disciplinary consequences.

Please read the following remarks carefully.

- Show all work to get any partial credit.
- Take into account the points that may be earned for each problem when splitting your time between the problems.

Problem	Points earned	Points possible	Problem	Points earned	Points possible
Problem 1		40	Problem 2		30
Problem 3		30			
Total					100

Problem 1 *(Short questions.)*

1. (a) 5 points

$$\text{Given} \quad x(t) = \begin{cases} t-2, & 2 \le t < 4, \\ 2, & 4 \le t < 6, \\ 0, & \text{otherwise.} \end{cases}$$

Plot $x\left(1 - \frac{t}{3}\right)$. Label your axes clearly and carefully!

1. (b) 5 points For the following system, with input $x[n]$ and output $y[n]$, circle whether the statements are true or false.

$$y[n] = \sum_{k=-\infty}^{-2n} 3x[k]$$

T F the system is linear

T F the system is time-invariant

T F the system is memoryless

T F the system is stable

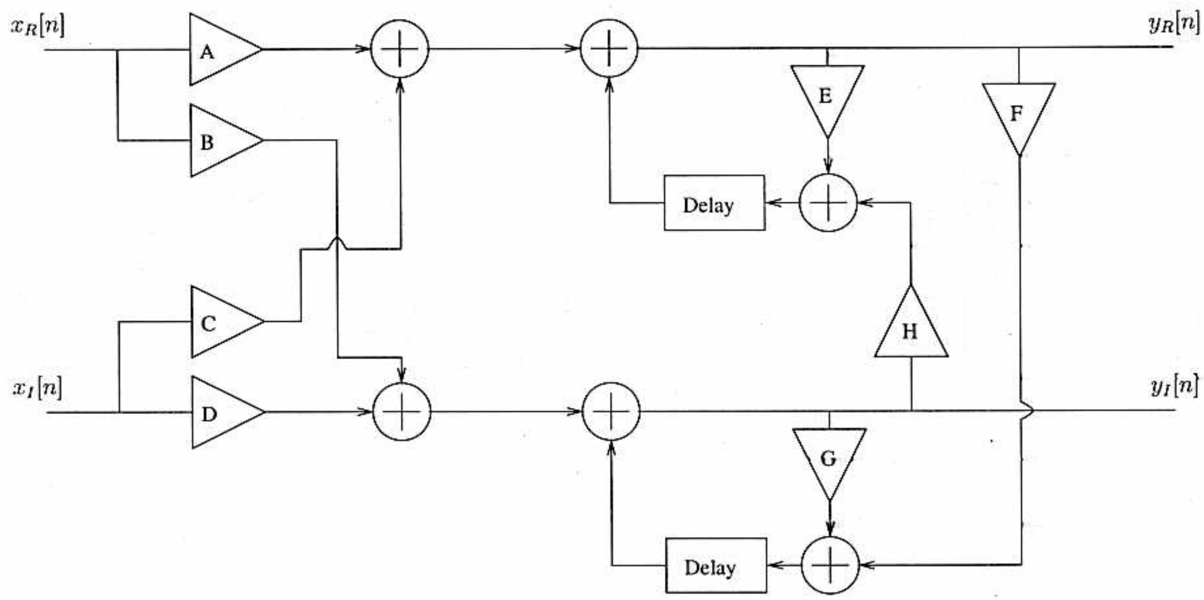
T F the system is causal

1. *(c) 7 points* An iron bar is heated to the temperature 300 degrees Celsius and placed in a room with ambient temperature S degrees Celsius, where it is allowed to cool.

Every minute, the temperature of the bar decreases by an amount equal to 2% of the difference between the current temperature (at the start of that minute) and the ambient temperature. In the box below, write a difference equation for $T[n]$, the temperature of the bar after it has been in the room for n minutes, and give any relevant initial conditions.

1. *(d) 4 points* Find the correct real gains in the block diagram below so that the input and output are related by the complex difference equation:

$$y[n] + (3 - 4j) \cdot y[n-1] = e^{-j\pi/2} x[n]$$



$A =$	$B =$	$C =$
$D =$	$E =$	$F =$
$G =$	$H =$	

1. (e) *6 points* A signal $x(t)$ is the input to an LTI system with impulse response $h(t) = \frac{\sin(500\pi t)}{\pi t}$. Which of the following signals could **not** be the output $y(t)$? (Circle your answer(s) and provide a brief explanation in the box below. No credit will be given for correct answers with incorrect reasoning.)

$y(t) = \cos(100\pi t)$

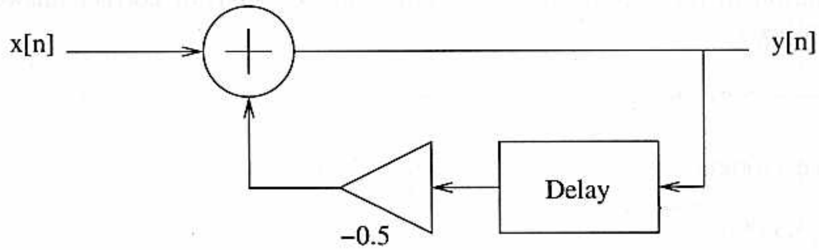
$y(t) = 12e^{j300\pi t}$

$y(t) = \sin(50\pi t) \cdot \cos(75\pi t)$

$y(t) = \sin(600\pi t)$

$y(t) = \sin(375\pi t)$

1. *(f) 6 points* Consider an LTI system with input $x[n]$ and output $y[n]$ that is implemented by the following block diagram



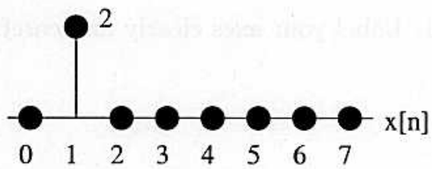
Find the frequency response $H(e^{j\Omega})$ of this system.

$H(e^{j\Omega}) =$

1. *(g) 7 points* A discrete-time LTI system, with input $x[n]$ and output $y[n]$, has frequency response

$$H(e^{j\Omega}) = \frac{1}{1 + 0.5e^{-j\Omega}}$$

The input signal $x[n]$ is periodic, with period $N = 8$. The following figure shows the value of $x[n]$ over the interval $0 \leq n \leq 7$.



Let b_k denote the discrete-time Fourier series coefficients of $y[n]$. Compute the coefficient b_4.

$b_4 =$

Problem 2 *(CTFT)*

Consider the signal

$$x(t) = x_1(t) + x_2(t)$$

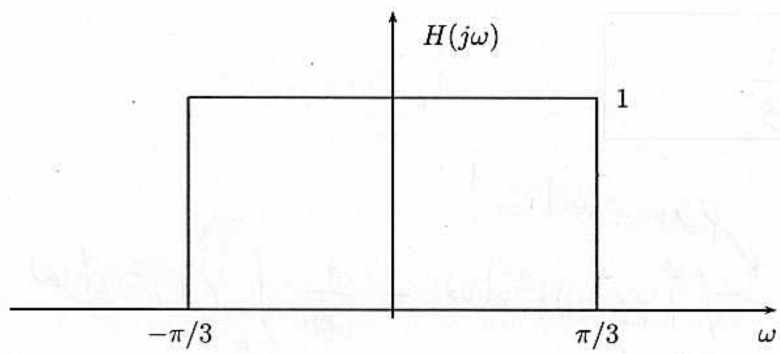
where

$$x_1(t) = \cos(20\pi t) \quad \text{and} \quad x_2(t) = \frac{\sin\left(\frac{\pi}{2}t\right)}{\pi t}$$

2. (a) 6 points Plot $x_2(t)$ from $-10 \le t \le 10$. Label your axes clearly and carefully!

2. (b) 8 points Plot the continuous-time Fourier transform of $x(t)$. Label your axes clearly and carefully!

2. *(c) 8 points* The signal $x(t)$ is now the input to an LTI system, whose frequency response $H(j\omega)$ is purely real and shown below.

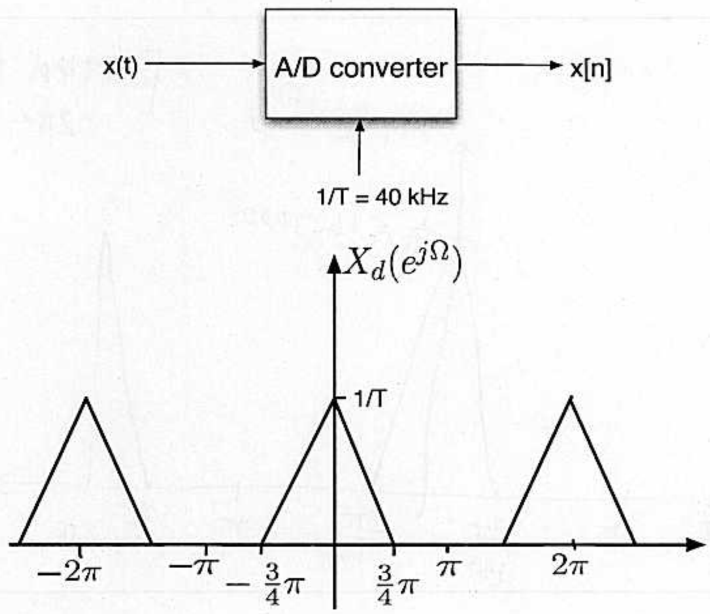


Write an expression for the output of the LTI system, $y(t)$.

2. *(d) 8 points* Compute the energy $E_Y = \int_{-\infty}^{\infty} |y(t)|^2 dt$ of $y(t)$ from part (c).

$E_Y =$

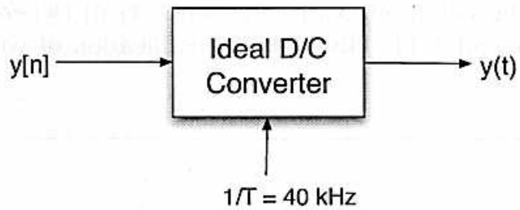
Problem 3 *(Sampling)*



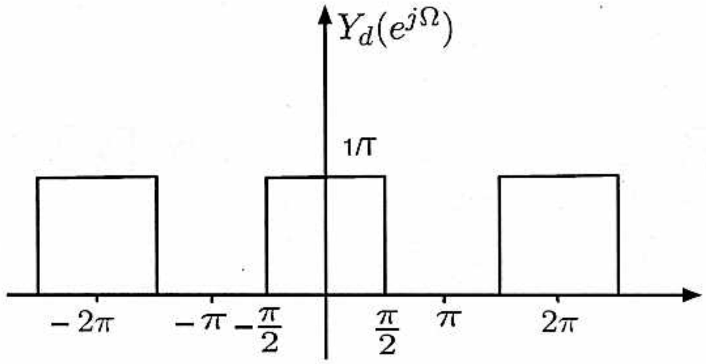
3. *(a) 5 points* x(t) is sampled above its Nyquist rate at $\frac{1}{T} = 40\ kHz$ to produce $x[n]$ whose spectrum, $X_d(e^{j\Omega})$, is shown in the figure above. Plot $X(j\omega)$, the spectrum of $x(t)$, clearly labeling your axes.

3. (b) 5 points For the same $x(t)$ as in part (a), suppose the A/D converter is now operated at $\frac{1}{T_1} = 160\ kHz$ to produce $x_1[n]$. Plot the spectrum of $x_1[n]$.

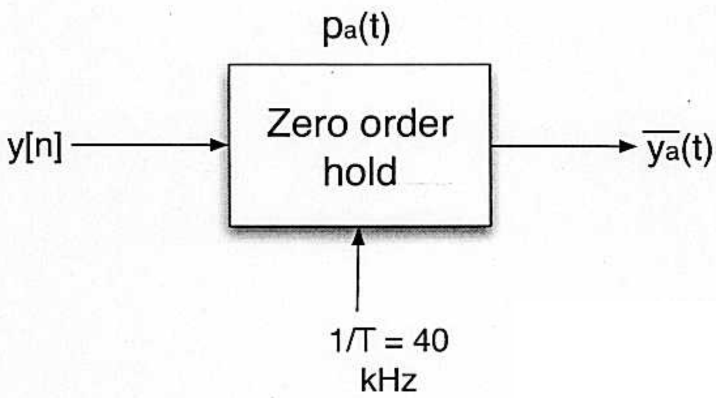
3. *(c) 6 points* Draw a discrete-time system with input $x[n]$ and output $x_1[n]$ (where $x[n]$ and $x_1[n]$ are the signals from parts (a) and (b)). Give a brief justification of your answer to receive full credit.

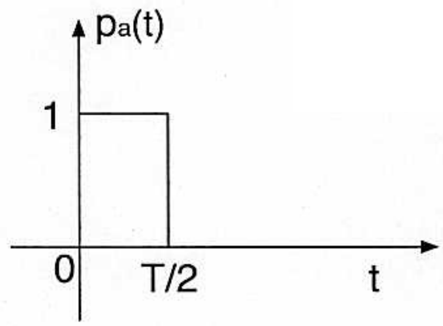


3. (d) 6 points



If $y[n]$ is the input to an ideal D/C running at $\frac{1}{T} = 40\ kHz$, and $y[n]$ has the spectrum shown in the figure above, plot $Y(j\omega)$, the spectrum of $y(t)$.





3. *(e) 8 points*

The signal $y[n]$ in part (d) is the input to a Zero-Order Hold circuit characterized by $\bar{y}_a(t) = \sum_{n=-\infty}^{\infty} y[n] p_a(t-nT)$, where $p_a(t)$ is shown above. Note that this ZOH is holding for $\frac{T}{2}$ seconds, rather than the classical T seconds. Plot the magnitude of the spectrum of $p(t)$ and the magnitude of the spectrum of $\bar{y}_a(t)$, both over the range $|\omega| < \frac{5\pi}{T}$.