**UNIVERSITY OF CALIFORNIA**
**College of Engineering**
**Department of Electrical Engineering and Computer Sciences**

EECS 130
Fall 2006

Professor Ali Javey

# Midterm I

**Name:** Solutions

1. Closed book. One sheet of notes is allowed.
2. Show all your work in order to receive partial credit.
3. Include correct units when appropriate.
4. Make sure everything is on the exam papers. Work on additional papers will *NOT* be accepted.
5. There are a total of 10 pages of this exam including this page. Make sure you have them all.

| | |
|---|---|
| **Problem 1** | 30 |
| **Problem 2** | 15 & 5 extra credit |
| **Problem 3** | 28 |
| **Problem 4** | 27 |
| **Total** | 100 |

# Physical Constants

| | | |
|---|---|---|
| Electronic charge | $q$ | $1.602\times10^{-19}$ C |
| Permittivity of vacuum | $\varepsilon_0$ | $8.845\times10^{-14}$ F cm$^{-1}$ |
| Relative permittivity of silicon | $\varepsilon_{Si}/\varepsilon_0$ | 11.8 |
| Boltzmann's constant | $k$ | 8.617 x $10^{-5}$ eV/ K or<br>$1.38\times10^{-23}$ J K$^{-1}$ |
| Thermal voltage at $T$ = 300K | $kT/q$ | 0.026 V |
| Effective density of states | $N_c$ | 2.8 x $10^{19}$ cm$^{-3}$ |
| Effective density of states | $N_v$ | 1.04 x $10^{19}$ cm$^{-3}$ |
| Silicon Band Gap | $E_G$ | 1.12 eV |
| Intrinsic Carrier Concentration in Si at 300K | $n_i$ | $10^{10}$ cm$^{-3}$ |

1. Carriers Concentrations [30 pts]

This problem concerns a specimen of gallium arsenide, GaAs, which has $2\text{x}10^{17}$ $\text{cm}^{-3}$ donors and an unknown number of acceptors. A measurement is made on the specimen and it is found that it is p-type with an equilibrium hole concentration, $p_o$, of $5 \text{ x } 10^{17}\ \text{cm}^{-3}$.

At room temperature in GaAs, the intrinsic carrier concentration, $n_i$, is $10^{7}\ \text{cm}^{-3}$, the hole mobility, $\mu_h$, is 300 $\text{cm}^2$/V-s, and the electron mobility, $\mu_e$, is 4000 $\text{cm}^2$/V-s. The minority carrier lifetime, $t_{min}$, is $10^{-9}$ s.

a) [6 pts] What is the net acceptor concentration, $N_A$ ($= N_a - N_d$), in this sample, and what is the total acceptor concentration, $N_a$?

$$p_o = \frac{N_a - N_d}{2} + \left( \left(\frac{N_a - N_d}{2}\right)^2 + n_i^2 \right)^{1/2}$$

$N_A =$ $5 \times 10^{17}$ $\text{cm}^{-3}$

$$\left(\frac{N_a - N_d}{2}\right)^2 \gg n_i^2 \Rightarrow p_o = \frac{N_a - N_d}{2} + \frac{N_a - N_d}{2} = N_a - N_d \Rightarrow N_A = p_o$$

$$N_a = N_A + N_d = 7 \times 10^{17}\ \text{cm}^{-3}$$

$N_a =$ $7 \times 10^{17}$ $\text{cm}^{-3}$

b) [6 pts] What is the equilibrium electron concentration, $n_o$, in this sample at room temperature?

$$n_o \cdot p_o = n_i^2$$

$$n_o = \frac{n_i^2}{p_o} = \frac{(10^7\ \text{cm}^{-3})^2}{5 \times 10^{17}\ \text{cm}^{-3}} = 2 \times 10^{-4}\ \text{cm}^{-3}$$

$n_0 =$ $2 \times 10^{-4}$ $\text{cm}^{-3}$

c) [6 pts] Calculate $E_F - E_i$ in this sample at room temperature.

$$p_o = n_i e^{E_i - E_F / KT}$$

$$\frac{p_o}{n_i} = e^{-(E_F - E_i)/KT}$$

$$E_F - E_i = -KT \ln \frac{p_o}{n_i} = -26\ \text{meV} \ln \frac{p_o}{n_i}$$

$E_F - E_i =$ $-0.64$ eV

$$= -60\ \text{meV} \log \frac{5 \times 10^{17}\ \text{cm}^{-3}}{10^7\ \cancel{\text{cm}^{-3}}} = -60\ \text{meV} (10 + \log 5) = -642\ \text{meV}$$

d) [6 pts] What is the electrical conductivity, $\sigma_0$, of this sample in thermal equilibrium at room temperature?

$$\sigma_0 = (\overset{\approx 0}{\cancel{q \mu_n n_0}} + q \mu_p p_0) = q \mu_p p_0 = 1.6 \times 10^{-19} C \cdot 300 \frac{cm^2}{V\text{-}s} \cdot 5 \times 10^{17} / cm^3$$

$$= 24 \ S/cm$$

$\sigma_0 =$ 24 S/cm

e) [6 pts] This sample is illuminated by a steady state light which generates hole-electron pairs uniformly throughout its bulk, and the conductivity of the sample is found to increase by 1% (that is, to 1.01 $\sigma_0$). What are the excess hole and electron concentrations, $\Delta p$ and $\Delta n$, in the illuminated sample, assuming that the illumination has been on for a long time?

$$\boxed{\Delta n = \Delta p}$$

$$1.01 \sigma_0 = q \mu_n (n_0 + \Delta n) + q \mu_p (p_0 + \Delta p) = \sigma_0 + q \mu_n \Delta n + q \mu_p \Delta p$$

$$0.01 \sigma_0 = q \mu_n \Delta p + q \mu_p \Delta p$$

$$\Delta p = \Delta n = \frac{0.01 \sigma_0}{q \mu_n + q \mu_p}$$

$\Delta n =$ $3.488 \times 10^{14}$ $cm^{-3}$

$\Delta p =$ $3.488 \times 10^{14}$ $cm^{-3}$

$$\Delta p = \Delta n = \frac{0.01 \cdot 24 \ S/cm}{1.6 \times 10^{-19} C \left(300 \frac{cm^2}{Vs} + 4000 \frac{cm^2}{V\text{-}s}\right)} = \boxed{3.488 \times 10^{14} \ cm^{-3}}$$

2. Temperature Dependence of Carrier Concentrations and Mobility [15 pts]

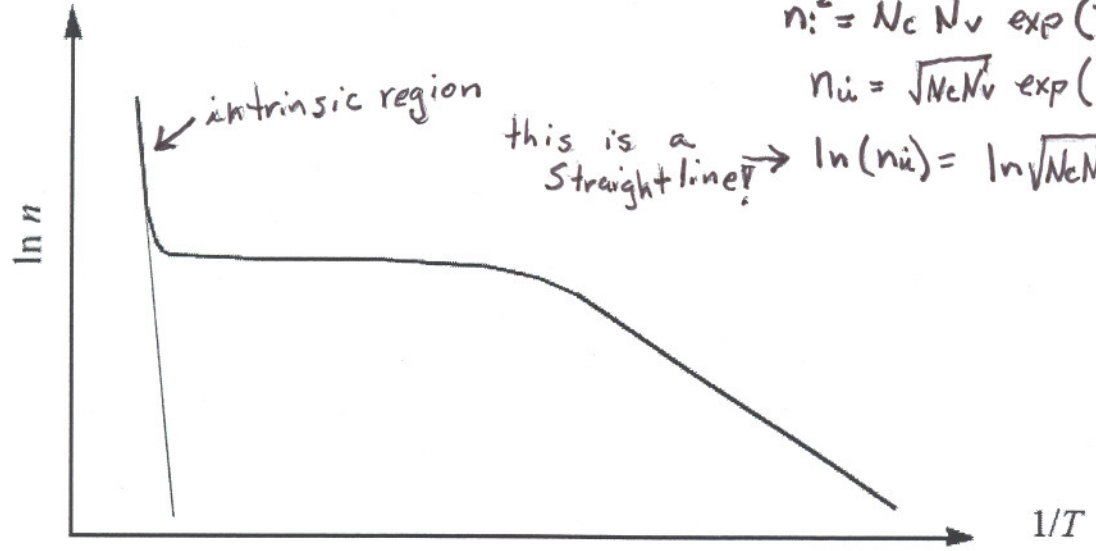
A silicon wafer is moderately doped with arsenic. The plots in parts (a)-(c) show the relationship between ln(*n*) and 1/*T* for this Si wafer, where n is the electron density in the conduction band and T is the temperature. In each case, clearly mark any pertinent shift in the curve and/or the slopes of the two non-flat regions as various properties of the semiconductor is changed.

a) [5 pts] Draw a second curve that would correspond to an intrinsic (undoped) Si wafer.

$n_i^2 = N_C N_V \exp(-E_g/kT)$

$n_i = \sqrt{N_C N_V} \exp(-E_g/2kT)$

this is a Straight line! → $\ln(n_i) = \ln\sqrt{N_C N_V} - E_g/2kT$

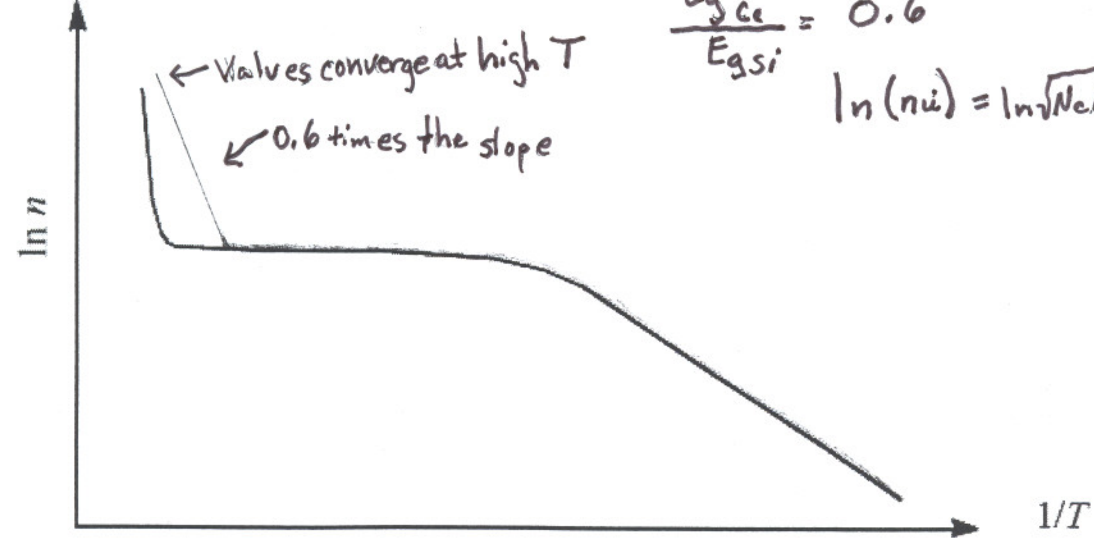


b) [5 pts] Draw a second curve that would correspond to using a Ge ($E_g$~0.67) wafer with the same dopant density instead of a Si ($E_g$~1.1) wafer. Assume the same ($E_D - E_C$).

$\frac{E_{g\,Ge}}{E_{g\,Si}} = 0.6$

$\ln(n_i) = \ln\sqrt{N_C N_V} - E_{g\,Ge}/2kT$

↑ Smaller Slope



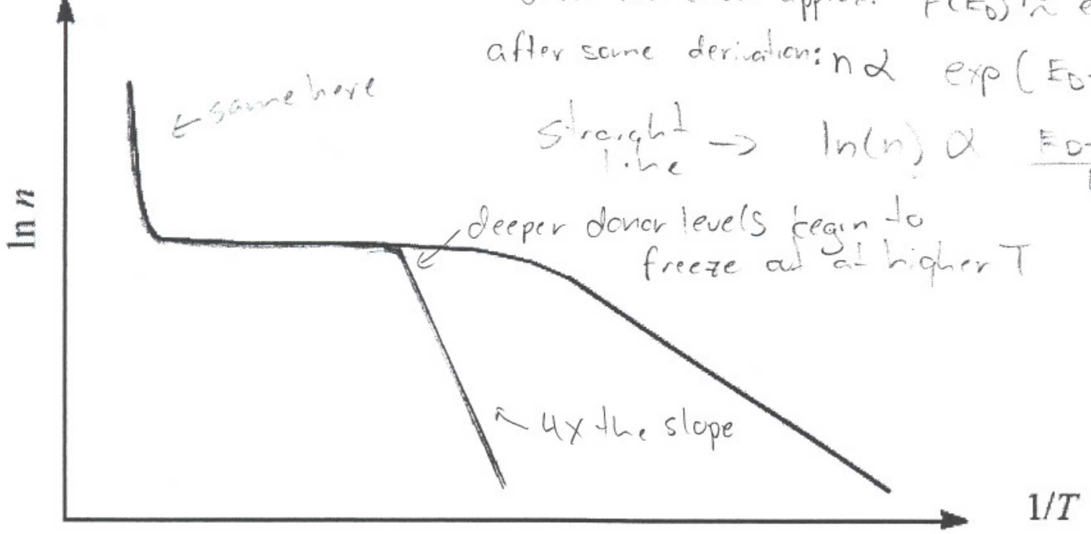
c) [5 pts] Draw a second curve that would correspond to another Si wafer, but doped with a different donor such that $(E_D - E_C)_{NEW\_DONER} = 4 \times ((E_D - E_C)_{As}$, where $E_D$ is the donor energy level.

Probability a donor site is filled is $f(E_D) = \frac{1}{1+e^{\frac{E_D-E_F}{kT}}}$

Under Boltzmann approx. $f(E_D) \approx \exp(E_F-E_D/kT)$

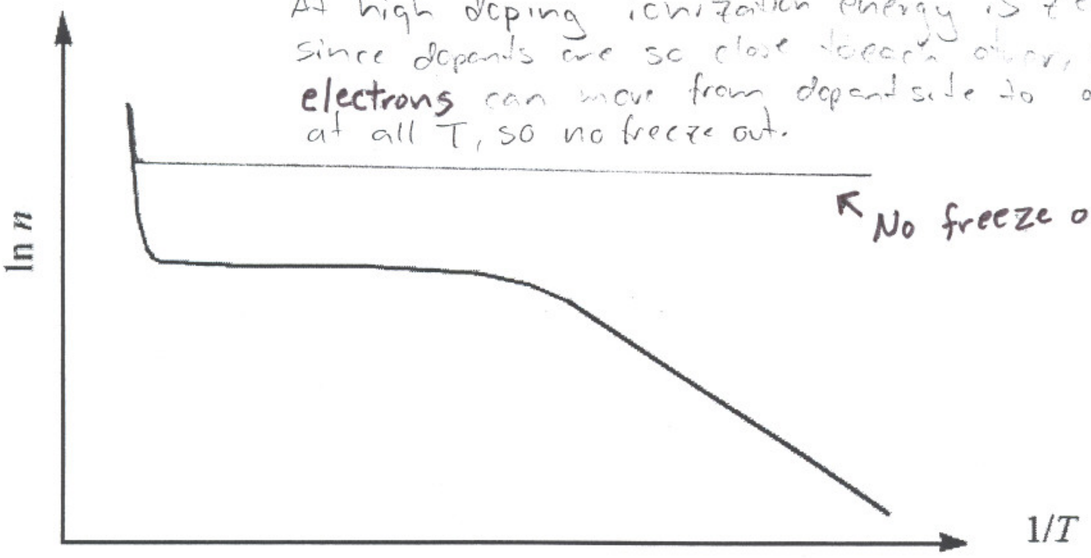
after some derivation: $n \propto \exp(E_D-E_C/kT)$

straight line $\rightarrow$ $\ln(n) \propto \frac{E_D-E_C}{kT}$



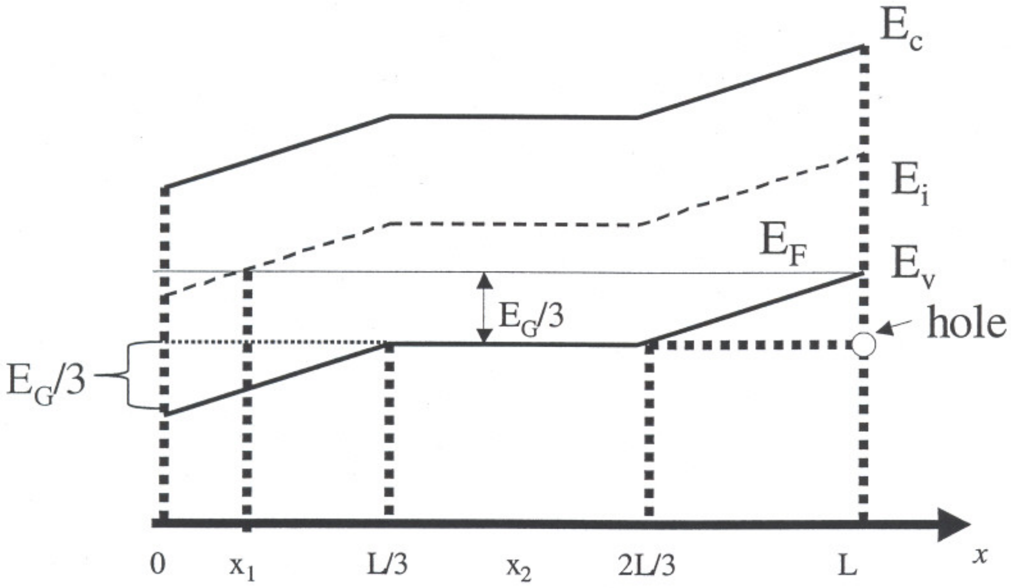
d) [*5pts extra credit*] Draw a second curves that would correspond to a heavily doped Si wafer. Hint: when doping density is high, the impurity energy level splits into a band of available states due to Pauli exclusion principle. This impurity band crosses $E_c$.

At high doping ionization energy is zero since dopants are so close to each other, extra **electrons** can move from dopant site to dopant site at all T, so no freeze out.



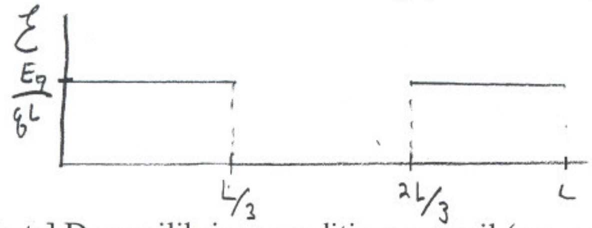
3. Band Model [28 pts]

A silicon device maintained at 300 K is characterized by the following band diagram. Use the cited energy band diagram in answering parts (a)-(e)



(a) [8 pts] Sketch the electric field inside the semiconductor.

$\mathcal{E} = \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{E_g/3}{L/3} = \frac{E_g}{q\cdot L}$ between $x = 0$ & $L/3$ and $x = 2L/3$ & $L$



(b) [5 pts] Do equilibrium conditions prevail (yes, no, or cannot determine)?

Yes. $E_F$ is constant and continuous. $\frac{dE_F}{dx} = 0$

(c) [5 pts] Is the semiconductor degenerate at any point? If so, specify one point where this is the case.

Yes.

Degeneracy occurs when $E_F - E_V < 3kT$

This is clearly the case at $x = L$

(d) [5 pts] What is the electron current density ($J_N$) flowing at $x = x_1$?

By definition, under equilibrium $\boxed{J_n = 0}$

(e) [5 pts] What is the kinetic energy of the hole shown in the diagram?

$E_v - E_{HOLE} = \boxed{\frac{E_g}{3}}$

4. [27 pts] Assume a Si PN junction with the following dopant density profiles for the two segments:

| **N** | **P** |
|---|---|
| $N_D = 2 \times 10^{16}\ \mathrm{cm}^{-3}$<br>$N_A = 1 \times 10^{16}\ \mathrm{cm}^{-3}$ | $N_A = 1 \times 10^{17}\ \mathrm{cm}^{-3}$<br>$N_D = 1 \times 10^{13}\ \mathrm{cm}^{-3}$ |

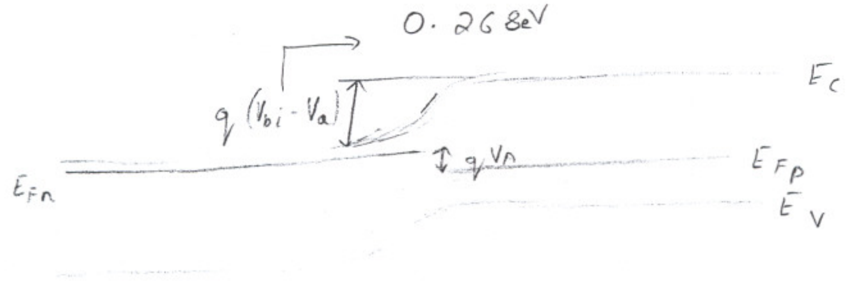
a) [6 pts] Find $V_{bi}$.

| | |
|---|---|
| $N_D - N_A$ | $N_A - N_D$ |

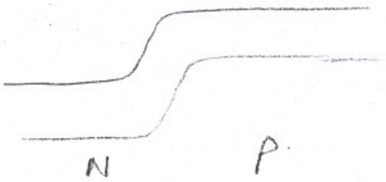
$$V_{bi} = \frac{kT}{q} \ln\left(\frac{(N_D - N_A)(N_A - N_D)}{n_i^2}\right)$$

$V_{bi}$= 0.768 V

b) [7 pts] Draw a band diagram for the structure with a forward bias of $V_A$=0.5 V. Label $V_A$, $V_{bi}$, $E_v$, $E_c$, and Fermi (or quasi-Fermi) levels.



c) [4 pts] For part b, using arrows, indicate direction of $I_{n,diff}$, $I_{p,drift}$, $I_n$, and $I_{total}$ (Redraw the band diagrams from b here).



$e^-$ diffusion →

⇒ $I_n$ diff ←

hole drift →

⇒ $I_p$ drift →

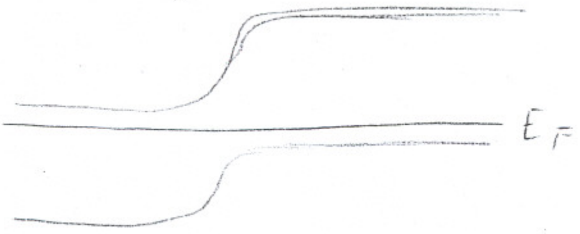
$I_n$ ←

$I_{total}$ ←

(P to N for fwd bias)

d) [10 pts] So far, we have been assuming that there is no series resistance (and therefore, no potential drop) in the neutral P and N regions of our diodes. However, when lightly doped (~<5e16cm$^{-3}$), the resistivity of the P and N type regions are often high, leading to series resistance or potential drop in the P and N regions under an applied voltage. Draw a band diagram for this PN junction in equilibrium and then under forward bias, this time including the effect of the series resistance (qualitatively) of the N segment. Hint: assume the series resistance is constant throughout the N segment.

Equi



→ V(x)

→ Series Res ⇒ potential drop

⇒ V(x)

Fwd Bias