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UNIVERSITY OF CALIFORNIA
College of Engineering
Department of Electrical Engineering and Computer Sciences

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Show derivations and ***mark results*** *with box around them. Erase or cross-out erroneous attempts.* <u>*Simplify algebraic results as much as possible!*</u> *Mark your name and SID at the top of the exam and all extra sheets.*

You may need the following integrals:

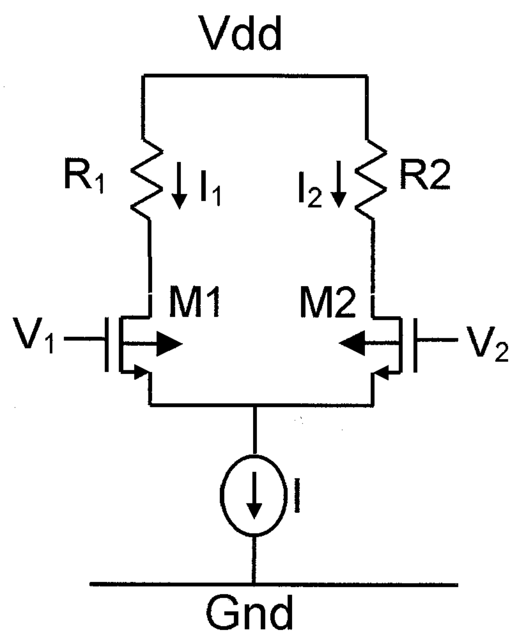
$$\int_0^\infty \left| \frac{1}{1+\frac{s}{\omega_o}} \right|^2 df = \frac{\omega_o}{4}$$

$$\int_0^\infty \left| \frac{1}{1+\dfrac{s}{\omega_o Q}+\dfrac{s^2}{\omega_o^2}} \right|^2 df = \frac{\omega_o Q}{4}$$

$$\int_0^\infty \left| \frac{\dfrac{s}{\omega_o}}{1+\dfrac{s}{\omega_o Q}+\dfrac{s^2}{\omega_o^2}} \right|^2 df = \frac{\omega_o Q}{4}$$

	Total pts	Score
Q1	20	
Q2	20	
Q3	30	
Q4	20	
Q5	10	
Total	100	

Question 1 (20pts)



In the fully-differential amplifier above:

a. **(10pts)** Calculate the input referred offset voltage for ΔV_{TH}=2mV, $\Delta(W/L)/(W/L)$=1%, $\Delta R/R$=1%. M1 and M2 have V^*=200mV.

b. **(10pts)** Assume only the following mismatches exist (different than the part a): $\Delta R/R$=1%, $\Delta gmb/gmb$=1%. Calculate the CMRR. $g_{mb} = 0.1 g_m$

a) $\Delta V_o = \Delta I \cdot R + \Delta R \cdot I$

$$\Delta I = I \cdot \frac{\Delta W/L}{W/L} + g_m \cdot \Delta V_{TH}$$

$$\Delta V_{in} = \frac{\Delta V_o}{g_m R} \qquad \frac{V_o}{V_{in}} = g_m \cdot R$$

$$\Delta V_{in} = \frac{R}{g_m R}\left[I \cdot \frac{\Delta W/L}{W/L} + g_m \cdot \Delta V_{TH} \right] + \frac{\Delta R \cdot I}{g_m \cdot R}$$

$$\Delta V_{in} = \frac{I}{g_m} \cdot \frac{\Delta W/L}{W/L} + \Delta V_{TH} + \frac{I}{g_m} \cdot \frac{\Delta R}{R}$$

$$\frac{I}{g_m} = \frac{V^*}{2} \Rightarrow \Delta V_{in} = 100mV \cdot 0.01 + 2mV + 100mV \cdot 0.01$$

$$\Delta V_{in} = 4mV$$

b) $V_{od}\Big|_{cm} = \Delta g_{mb} \cdot R \cdot V_{cm} + \Delta R \cdot g_{mb} \cdot V_{cm}$

we ignore: $\Delta R \cdot \dfrac{g_m}{\underset{\substack{\uparrow \\ \underline{\underline{\text{small}}}}}{g_m R_{Tail} + 1}}$

$$A_{d\text{-}cm} = \Delta g_{mb} \cdot R + \Delta R \cdot g_{mb} = \frac{V_{od}}{V_{cm}}$$

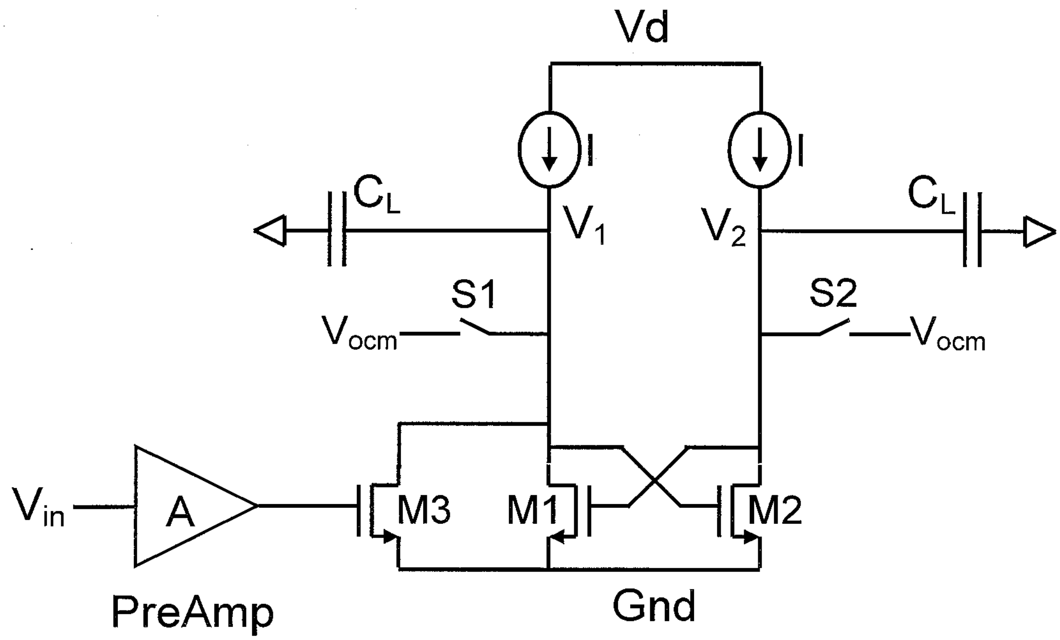
$$A_d = \frac{V_{od}}{V_{id}} = g_m \cdot R$$

$$CMRR = \frac{A_d}{A_{d\text{-}cm}} = \frac{g_m \cdot R}{\Delta g_{mb} \cdot R + \Delta R\, g_{mb}} = \frac{1}{\dfrac{\Delta g_{mb}}{g_m} + \dfrac{\Delta R}{R} \cdot \dfrac{g_{mb}}{g_m}}$$

$$g_{mb} = 0.1\, g_m \Rightarrow \Delta g_{mb} = 0.001\, g_m$$

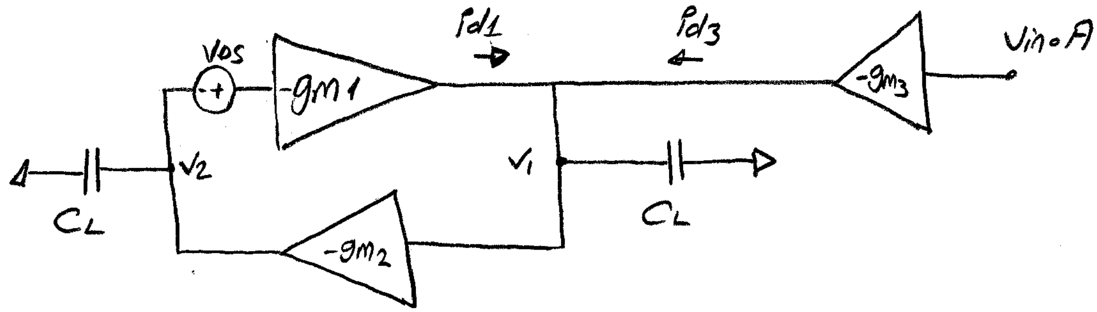
$$CMRR = \frac{1}{0.001 + 0.001} = 500 = 54\,dB$$

Question 2 (20pts)



In the comparator circuit shown above, V_1 and V_2 are initialized to V_{ocm}. Due to the implementation errors switch S2 introduces and offset voltage of V_{os}. Given the offset voltage V_{os}, what is the minimum V_{in} that this comparator can successfully detect?

Small signal model:



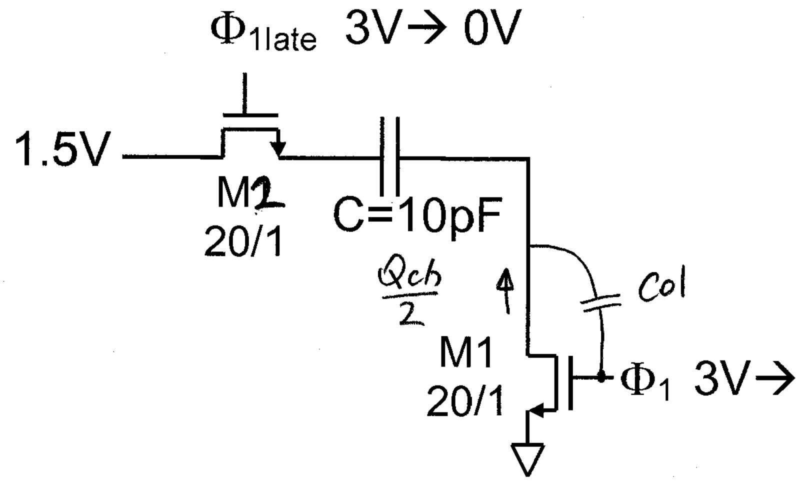
- V_2 & V_1 are initialized to 0 : small-signal
- For $V_{os} < 0$, I_{d1} cancels I_{d3}. I_{d1} must be smaller than I_{d3} for the correct transition.

$I_{d1} < I_{d3}$

$V_{os} \cdot g_{m1} < g_{m3} \cdot A \cdot V_{in}$

$V_{os} \cdot \frac{g_{m1}}{g_{m3}} \cdot \frac{1}{A} < V_{in}$ $\Rightarrow$ Pre-amp decreases the influence of the V_{os}.

Question 3 (20pts)

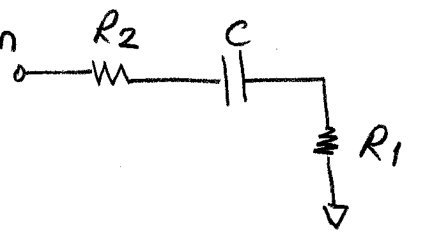


In the sampling circuit above, find the total sampling error for a sampling time of t_s=**2.5**ns. Assume fast gating and 50% charge split. Hint: $e^{-4} \sim 0.02$.

Parameter:
$V_{THN}=1V$, $\mu_n C_{ox}=200\mu A/V^2$, $C_{ox}=5fF/\mu m^2$, $C_{ol}'=0.2fF/\mu m$, $C=1pF$.
Assume square-law and ignore the body-effect.

Errors :
1-) Settling error
2-) Charge injection
3-) Ovelap cap: C_{gd_1}

1-)



$\Rightarrow \tau = C \cdot (R_1 + R_2)$

$$R_1 = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 \cdot (\underset{\substack{\uparrow \\ V_{gs}}}{3^V} - \underset{\substack{\uparrow \\ V_{TH}}}{1^V})} = \frac{1}{200\frac{\mu A}{V^2} \cdot 20 \cdot 2V} = 125\,\Omega$$

$$R_2 = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right)_2 (\underset{\substack{\uparrow \\ V_g}}{3^V} - \underset{\substack{\uparrow \\ V_s}}{1.5} - \underset{\substack{\uparrow \\ V_{TH}}}{1^V})} = \frac{1}{200\frac{\mu A}{V^2} \cdot 20 \cdot 0.5V} = 500\,\Omega \quad \Rightarrow$$

$$\tau = 10pf \cdot 625\Omega = 6.25\ nsec$$

$$\text{Settling error} = 1.5^V \cdot e^{-\frac{t_s}{\tau}} = 1.5 \cdot e^{-\frac{25ns}{6.25ns}} = 1.5 \times e^{-4} = 1.5^V \cdot 0.02$$

$$= 30\,mV \,//$$

2-) Charge injection : Bottom plate sampling → We'll consider M_1 only.

$$Q_{ch} = W_1 \cdot L_1 \cdot C_{ox}\,(V_{gs} - V_{TH})$$

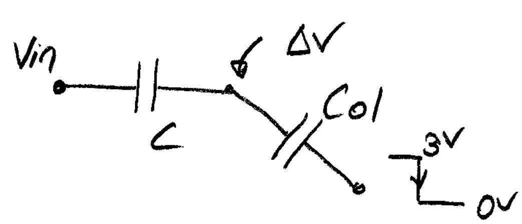
$$= 20\mu m \cdot 1\mu m \cdot 5\frac{fF}{\mu m^2} \cdot (3-1) = 20 \cdot 5 \cdot 2\ fF \cdot V$$

$$= 200\,fF \cdot V$$

$$\text{Charge injection error} = \frac{Q_{ch}}{2} \cdot \frac{1}{C} = \frac{200\,fF \cdot V}{2} \cdot \frac{1}{10pF}$$

$$= 10\,mV \,//$$

3-) Overlap cap :



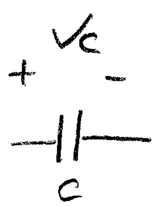
$$\Delta V \cong \frac{C_{ol}}{C} \cdot 3^V$$

$$C_{ol} = W_1 \cdot C_{ol} = 20\mu m \cdot 0.2\frac{fF}{\mu m} = 4fF$$

$$\Delta V = \frac{4fF}{10pF} \cdot 3^V = 1.2\,mV \,//$$

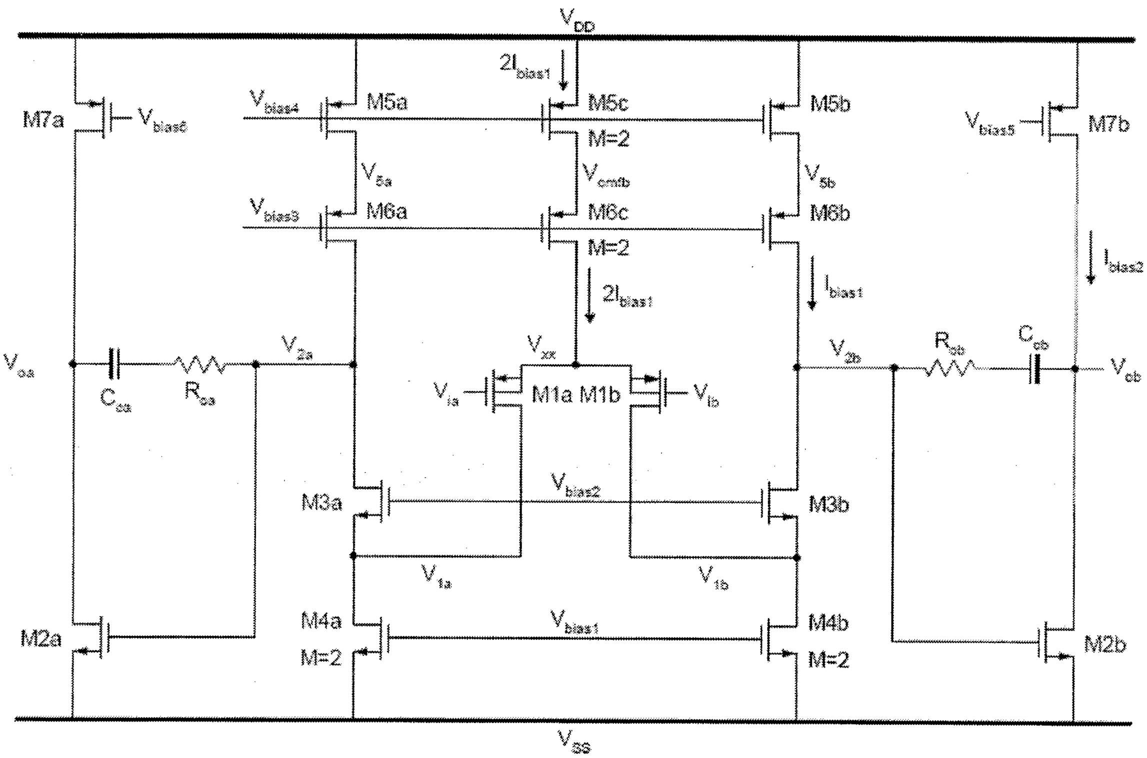
$$\text{Total sampling error} = -30mV + 10mV + 1.2mV$$

$$= -18.8\,mV$$



Charge injection and overlap cap increases Vc, while settling error decreases it.

Question 4 (30pts)

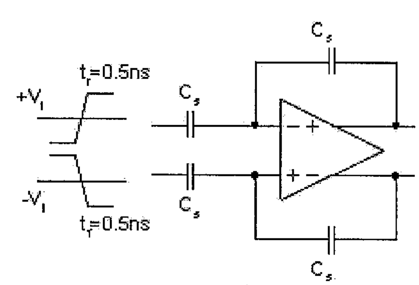


In the OTA above, all channel lengths are the same, all parasitic caps are 0 and $2I_{bias1}=I_{bias2}=I$.

a. **(10pts)** Calculate the ratio of W_{1a}/W_{2a} that sets the non-dominant pole to of the OTA to 4X the unity gain frequency ω_u of the open loop OTA . You can assume the 2nd pole in the Miller compensation is at -[gm of the 2nd stage]/C_c.

b. **(5pts)** Calculate the value of R_{ca}, in terms if the appropriate gm, that sets the RHP zero to infinity.

c. **(10pts)** Assuming the amplifier is in the feedback loop shown below, estimate the total output voltage noise in terms of V^*s. For this part you can ignore the effects of the non-dominant pole and the noise from the second stage.



d. **(5pts)** Using the feedback amplifier in part c, find the slewing time (t_{slew}) when the amplifier is driven by a differential input of $+8V_{1a}^*$.

Q4

a) Miller compensation:

$$\omega_u \cong \frac{g_{m1a}}{C_c} \qquad p_2 = \frac{g_{m2a}}{C_c} \text{ (given)} \qquad \frac{g_{m2a}}{C_c} = \frac{g_{m1a}}{C_c} \cdot 4$$

$$\frac{g_{m1a}}{g_{m2a}} = \frac{1}{4} \qquad g_{m1a} = \sqrt{2 \cdot \left(\frac{W}{L}\right)_{1a} \mu_n C_{ox} I_{D1a}}$$

$$g_{m2a} = \sqrt{2 \left(\frac{W}{L}\right)_{2a} \mu_n C_{ox} I_{D2a}}$$

$$\frac{g_{m1a}}{g_{m2a}} = \sqrt{\frac{(W/L)_{1a}}{(W/L)_{2a}} \cdot \frac{I_{D1a}}{I_{D2a}}} = \frac{1}{4} \qquad I_{D1a} = I/2 \qquad I_{D2a} = I$$

$$L_{1a} = L_{2a}$$

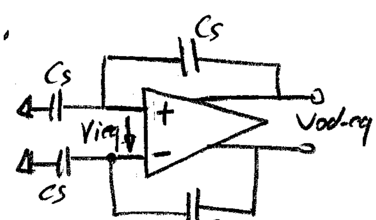
$$\frac{W_{1a}}{W_{2a}} \cdot \frac{1}{2} = \frac{1}{16} \Rightarrow \frac{W_{1a}}{W_{2a}} = \frac{1}{8} \,//$$

b) $$z = \frac{g_{m2a}}{C_c} \rightarrow \text{with } R_z \qquad z = \frac{1}{\left(\frac{1}{g_{m2}} - R_c\right) C_c}$$

$$z \rightarrow \infty \qquad R_c = \frac{1}{g_{m2}} \,//$$

c)
$$\underbrace{\frac{\overline{V_{ieq}^2}}{\Delta f}}_{\text{input equivalent noise}} = \left(\frac{1}{g_{m1}}\right)^2 \cdot 4kT\gamma\left(g_{m1} + g_{m4} + g_{m5}\right) \cdot \underbrace{2}_{\text{differential}}$$

In the given feedback configuration $F = \frac{1}{2}$.



$$\left(\frac{\overline{V_{ieq}^2}}{\Delta f} \rightarrow \frac{\overline{V_{od\text{-}eq}^2}}{\Delta f}\right) = \left(\frac{1}{F}\right)^2 \cdot \left|\frac{1}{1 + \frac{s}{\omega_u \cdot F}}\right|^2$$

$$\frac{\overline{V_{od\text{-}eq}^2}}{\Delta f} = \underbrace{\left(\frac{1}{g_{m1}}\right)^2 \cdot 4kT\gamma\left(g_{m1} + g_{m4} + g_{m5}\right) \cdot 2}_{\frac{\overline{V_{ieq}^2}}{\Delta f}} \cdot \underbrace{\left(\frac{1}{F}\right)^2 \left|\frac{1}{1 + \frac{s}{\omega_u \cdot F}}\right|^2}_{\left|\frac{V_{od}}{V_{id}}\right|^2}$$

Noise integral = $\dfrac{\omega_u \cdot F}{4} = \dfrac{1}{4}\dfrac{g_{m1}}{C_c}\cdot F$

$$\Rightarrow \overline{V_{od\text{-}eq}^2} = 2\left(\frac{1}{g_{m1a}}\right)^2 4kT\gamma\, g_{m1a}\left(1+\frac{g_{m4a}}{g_{m1a}}+\frac{g_{m5a}}{g_{m1a}}\right)\cdot\left(\frac{1}{F}\right)^2\cdot\frac{1}{4}\frac{g_{m1a}}{C_c}\cdot F$$

$$\overline{V_{od\text{-}eq}^2} = 2\cdot\frac{kT}{C_c}\cdot\frac{1}{F}\cdot\gamma\left(1+\frac{g_{m4a}}{g_{m1a}}+\frac{g_{m5a}}{g_{m1}}\right)$$

$$g_{m4a} = \frac{2\cdot I}{V_4^*}$$

$$g_{m1a} = \frac{2\cdot I/2}{V_1^*}$$

$$\overline{V_{od\text{-}eq}^2} = 2\cdot\frac{kT}{C_c}\cdot\frac{1}{F}\cdot\gamma\left(1+2\cdot\frac{V_{1a}^*}{V_{4a}^*}+\frac{V_{1a}^*}{V_{5a}^*}\right)$$

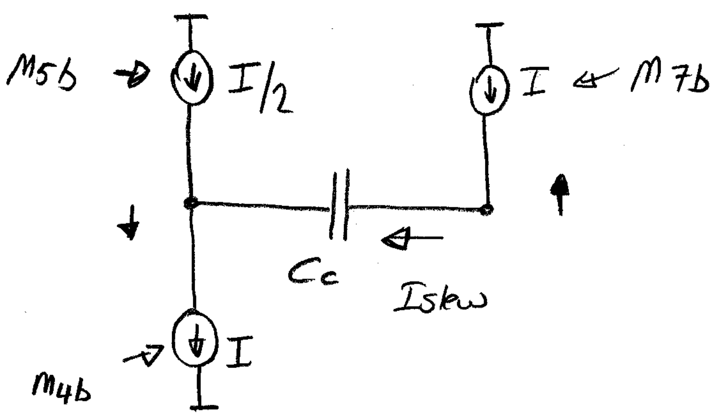
$$\frac{g_{m4a}}{g_{m1a}} = 2\cdot\frac{V_1^*}{V_4^*}$$

d) $\Delta V_{od} = 8V_{1a}^*$

(Signal gain = 1)

$\Delta V_{od\text{-}linear} = \dfrac{V_{1a}^*}{F} = 2V_{1a}^* \quad \left(F=\dfrac{1}{2}\right)$

$\Delta V_{od\text{-}slew} = 6\,V_{1a}^*$



I slew is limited by the folded cascode and it is $\dfrac{I}{2}$

$$\frac{I}{2}\cdot t_{slew}\cdot\frac{1}{C_c} = 6\cdot V_{1a}^* \Rightarrow t_{slew} = 6\cdot C_c\cdot\frac{V_{1a}^*}{I/2}$$

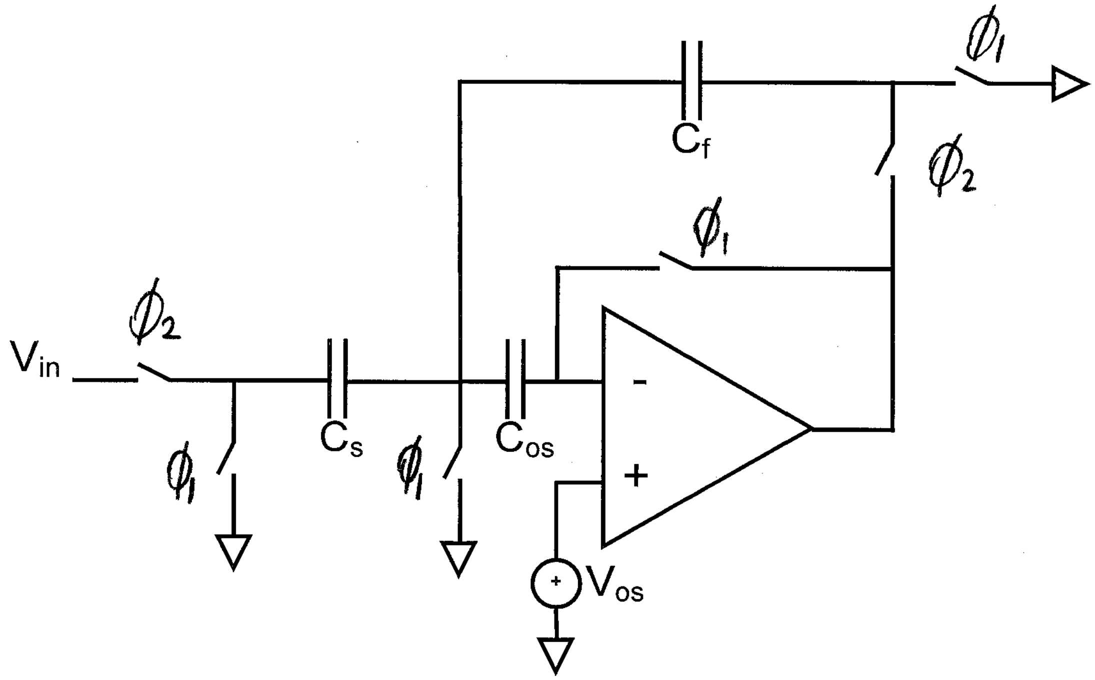
$I/2 \leftarrow I_{D1a}$

$$t_{slew} = 12\cdot\frac{C_c}{g_{m1a}}$$

$$t_{slew} = \frac{12}{\omega_u}$$

ω_u ← open loop (not the loop gain)

Question 5 (10pts)



In the switch cap gain circuit above, there are two non-overlapping phases available: Φ_1, Φ_2. In order to have an inverting gain stage with offset cancellation, assign appropriate phases (Φ_1 or Φ_2) to switches in the circuit.