Solutions – EE20n Midterm 1

Problem 1 (24 points)

Please indicate whether the following statements are true or false. You will receive 0 points for a wrong answer, 1.5 points for no answer, and 3 points for a correct answer. There will be no partial credit.

(a) If A and B are sets with one element, then $A \times B$ is a set with two elements.

Answer: False

(b) If $|A| = 1$ then $|P(A)| = 2$, where $|A|$ is the number of elements in the set A, and $P(A)$ is the powerset of A.

Answer: True

(c) The statement "If A then B" is logically equivalent to the statement "If not A then not B".

Answer: False

(d) If a feedback composition is well formed, then it must be true that all the component state machines in the composition have state-determined outputs.

Answer: False

(e) If a non-deterministic machine A simulates another non-deterministic machine B and B also simulates A, then A and B are bisimilar.

Answer: False

(f)

$$(A \cup B)^c = A^c \cup B^c$$

where A, B are two sets that are subsets of a universe set U and A^c, B^c contain the elements in U but outside of A, B respectively.

Answer: False

(g) If $A \subset B$, then

$$\{\text{graph}(f) | (f \in [A \to A])\} \subset \{\text{graph}(f) | (f \in [A \to B])\}.$$

Answer: True

(h)

$$[\text{Naturals} \to \text{Reals}] \subset [\text{Reals} \to \text{Reals}]$$

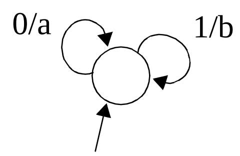
Answer: False

Problem 2 (30 points)

(a) Suppose we input the periodic sequence “0101010101...” into a *finite* state machine. For each of the infinite sequences below, either construct a state machine (specify the state transition diagram and the initial state) that will yield the sequence as output, or argue that it is impossible to do so.

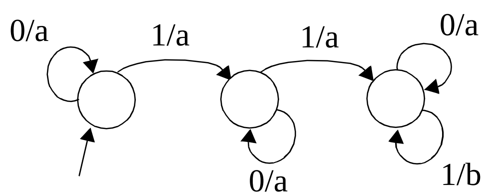
(i) abababab...

Answer:



(ii) aaaaababababababab...

Answer:

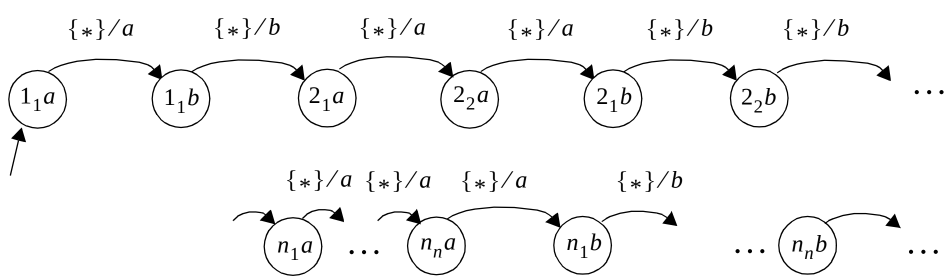


(iii)abaabbaaabbbaaaabbbb...

Answer: Impossible to construct a finite state machine that will output this sequence. In any step, in order to know how many a’s and b’s to produce next, we must know how many a’s or b’s have already been produced. We need an infinite number of states as the number of a’s and b’s approaches infinity.

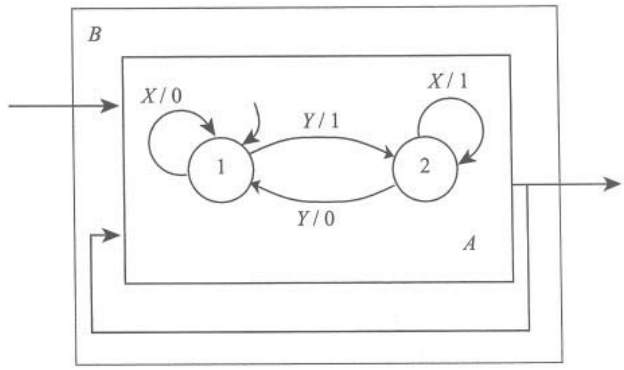
(b) For those sequences above that cannot be generated by a finite state machine, can you do so if an infinite number of states are allowed? If so, construct a machine that can do so.

Answer:



Problem 3 (20 points)

Consider the feedback composition shown below.



Assume that state machine A has input alphabet

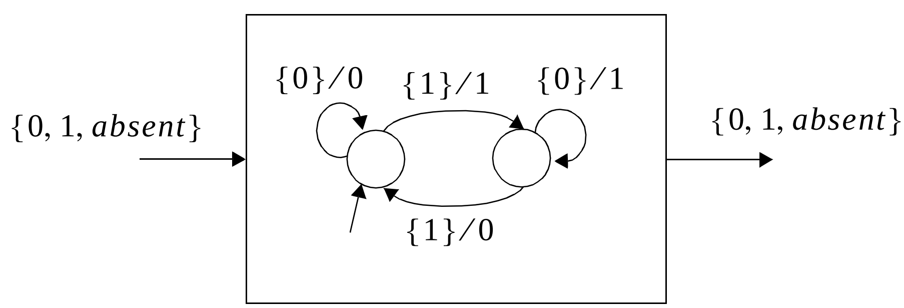
$$Inputs = \{(0, 0), (0, 1), (1, 0), (1, 1), absent\}$$

and recall that by convention the first element of each of these tuples is the input symbol provided to the upper input port. The output alphabet is $Outputs = \{0, 1, absent\}$. In the figure, the guards are give by the sets

$$X = \{(0, 0), (0, 1)\}$$
$$Y = \{(1, 0), (1, 1)\}$$

Either show that this feedback composition is not well formed or give the system B as a simple state machine (with no feedback or other composition) with as few states as possible. It is sufficient to define the input and output alphabets of B, its initial state, and its state transition diagram.

Answer:



Problem 4 (40 points)

In this question, we look at systems whose domain and range are both $[\text{Naturals}_0 \to \text{Reals}]$.

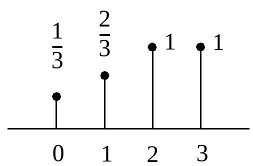
(a) Consider the system with input-output relationship given by:

$$y(n) = \frac{1}{K} \sum_{m = max(n-K+1,\,0)}^{n} x(m) \qquad \text{for all } n \in \text{Naturals}_0$$

where $max(a, b)$ is defined to be a if $a \geq b$ and b otherwise.

[4] (i) Plot the output for $K = 3$ and the specific input $x(n) = 1$ for all $n \in \text{Naturals}_0$.

Answer:



[4] (ii) Given an arbitrary input x, what is the qualitative effect of increasing the parameter K on the output y?

Answer: It increases the number of samples averaged, $x(n)\ldots x(n-K+1)$ so smoother output.

[4] (iii) For $K = 3$, design a state machine that implements this system, by giving the *Inputs, Outputs and States* sets, the *InitialState*, and the *nextState* and *output* functions.

Answer: Inputs = Reals   Outputs = Reals

States: $(s_1, s_2) \in \text{Reals}^2$

InitialState = (0,0)

nextState: $s_1(n+1) = s_2(n)$

$s_2(n+1) = x(n)$

output: $y(n) = \frac{1}{3}x((n) + s_1(n) + s_2(n))$

[2] (iv) Is this a finite state machine? If it is finite, how many states does it have?

Answer: Infinite

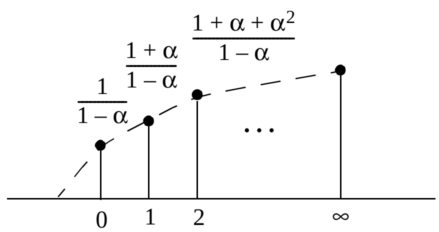
(b) Now consider the system:

$$y(n) = \frac{1}{1-\alpha}\sum_{m=0}^{n}\alpha^m x(n-m) \qquad \text{for all } n \in \text{Naturals}_0$$

where α is a real number strictly between 0 and 1.

[8] (i) Sketch the output of this system in response to the input $x(n) = 1$ for all $n \in \text{Naturals}_0$ and for general α. There is no need to compute exactly the output at each and every time, except the limiting value when $n \to \infty$.

Answer:



[6] (ii) Given an arbitrary input x, what is the qualitative effect of increasing the parameter α on the output y?

Answer: It weighs past samples more in the sum and scales curve up.

[12](iii)Design the simplest possible state machine that implements this system, by giving the *Inputs*, *Outputs* and *States* sets, the *InitialState*, and the *nextState* and *output* functions. (Hint: You can get this by using a machine with *States* = Reals.)

Answer: Inputs = Reals Outputs = Reals

States: $s \in$ Reals Initial State: $s = 0$

nextState: $s(n+1) = x(n) + \alpha s(n)$

output: $y(n) = \frac{1}{1-\alpha}[x(n) + \alpha s(n)]$