EECS 20N: Structure and Interpretation of Signals and Systems   MIDTERM 3
Department of Electrical Engineering and Computer Sciences   15 November 2007
UNIVERSITY OF CALIFORNIA BERKELEY

LAST Name __De Cay__   FIRST Name __Ekspo__

Lab Time __No time for that!__

- **(10 Points)** Print your name and lab time in legible, block lettering above AND on the last page where the grading table appears.

- This exam should take up to 70 minutes to complete. You will be given at least 70 minutes, up to a maximum of 80 minutes, to work on the exam.

- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except three double-sided 8.5" × 11" sheets of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff— *including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.

- **The exam printout consists of pages numbered 1 through 10.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the ten numbered pages. If you find a defect in your copy, notify the staff immediately.

- Please write neatly and legibly, because *if we can't read it, we can't grade it.*

- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*

- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.

- We hope you do a *fantastic* job on this exam.

You may use this page for scratch work only.
Without exception, subject matter on this page will *not* be graded.

MT3.1 (25 Points) Consider a well-structured interconnection H of discrete-time LTI systems A and B, as shown in the figure below. Each of the individual systems is a function defined on $[\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$.

Note the signal r's presence.
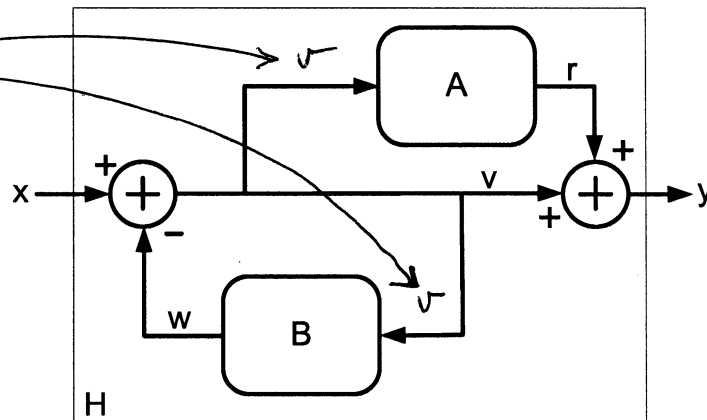
Let the respective frequency responses be A and B, defined on $\mathbb{R} \to \mathbb{C}$.

NOTE: The intermediate signals r, v, and w have been labeled on the diagram for your convenience.

(a) (15 Points) Determine the composite system's frequency response $H : \mathbb{R} \to \mathbb{C}$ in terms of the frequency responses A and B of the individual components. Reduce your expression to the simplest form possible.

Redraw the diagram to reveal the camouflaged structure:
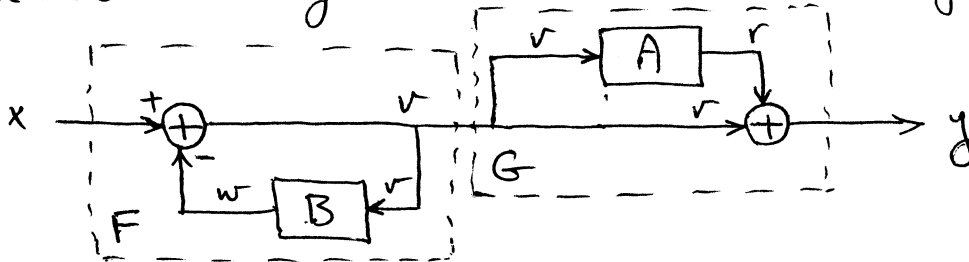
Clearly, H is the cascade of F and G. Hence,

$$H(\omega) = F(\omega)\, G(\omega)$$

But $F(\omega) = \dfrac{1}{1 + B(\omega)}$, because it is a textbook-style feedback interconnection with a forward gain of 1 and a loop gain of $-B(\omega)$.

As for G, it is a parallel interconnection: $G(\omega) = 1 + A(\omega)$

Therefore,

$$H(\omega) = \frac{1 + A(\omega)}{1 + B(\omega)}$$

(b) (10 Points) Suppose $A(\omega) = 1 + e^{-i\omega}, \forall\omega$, and let

$$b(n) = \frac{1}{2}\delta(n-1)$$

denote the impulse response of system B.

Determine a reasonably simple expression for the output values $y(n)$ in response to the input signal characterized by $x(n) = (-1)^n$.

Method 1: $x(n) = (-1)^n = e^{i\pi n} \implies y(n) = H(\pi) e^{i\pi n} = H(\pi)(-1)^n$.

$H(\pi) = \dfrac{1 + A(\pi)}{1 + B(\pi)}$; but $A(\pi) = 1 + e^{-i\pi} = 0$ (you can toss out A)

$b(n) = \frac{1}{2}\delta(n-1) \implies B(\omega) = \frac{1}{2}e^{-i\omega} \implies B(\pi) = \frac{1}{2}e^{-i\pi} = -\frac{1}{2}$

Therefore, $y(n) = H(\pi)(-1)^n = \dfrac{1}{1 - \frac{1}{2}}(-1)^n = 2(-1)^n \implies y(n) = 2(-1)^n$

Method 2: Even if you didn't get $H(\omega)$ in part (a) correctly, you could have argued this way... A & B are LTI, so if $x(n) = (-1)^n = e^{i\pi n}$, then every signal on every branch will be some multiple of $(-1)^n \triangleq e^{i\pi n}$, including the signal v, which is the input to system A; Let $v(n) = V_0 e^{i\pi n} \triangleq V_0 (-1)^n, \exists V_0 \in \mathbb{C}$.
Then $r(n) = A(\pi) V_0 (-1)^n$, but $A(\pi) = 1 + e^{-i\pi} = 0 \implies r(n) = 0 \; \forall n$ (i.e., $r \equiv 0$).
So it is as if we can eliminate A from the diagram, in which case we are left

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You may use the blank space below for scratch work. Nothing written below this line on this page will be considered in evaluating your work.

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with

X $\xrightarrow{+}\oplus \longrightarrow$ y   for $\omega = \pi$

which means $H(\pi) = \dfrac{1}{1 + B(\pi)}$

$b(n) = \frac{1}{2}\delta(n-1) \implies B(\omega) = \frac{1}{2}e^{-i\omega} \implies B(\pi) = -\frac{1}{2}$

$H(\pi) = \dfrac{1}{1 - \frac{1}{2}} = 2 \implies y(n) = H(\pi)(-1)^n = 2(-1)^n$

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MT3.2 (40 Points) Some traders on Wall Street treat stock price data as discrete-time signals. They make buy, sell, and hold decisions at least in part based on certain indicators that they compute from the data.

One indicator of stock price movement is an *M-point exponentially-weighted moving average (EWMA)*. We can think of the *M*-point EWMA as the output of a DT-LTI filter. A model of the relationship between the filter's input and output is given by the equation

$$y(n) = C \sum_{m=0}^{M-1} \alpha^m x(n-m). \tag{1}$$

The input signal x and the output signal y denote the stock price and the EWMA values, respectively. The integer constant M is the number of data points used in the averaging $(1 < M)$; the constant C is a normalization factor $(0 < C)$; and the constant α is the exponential weighting factor $(0 < \alpha < 1)$.

The sample $n \in \mathbb{Z}$ denotes an instance; it can be the index representative of a day, an hour, a minute, or whatever time scale the trader is interested in.

NOTE: Although the answers to some of the following parts may be useful in solving others, it is possible to tackle each part independently.

Also, in one or more parts of this problem, you may find the following finite-sum identity helpful. If α is a real number, and a and b are integers such that $a \le b$, then

$$\sum_{m=a}^{b} \alpha^m = \begin{cases} b - a + 1 & \text{if } \alpha = 1 \\ \dfrac{\alpha^{b+1} - \alpha^a}{\alpha - 1} & \text{if } \alpha \ne 1. \end{cases}$$

(a) (7 Points) Determine the normalization constant C (in terms of α and M) so that if the input signal is constant, $x(n) = K$, then the output signal is the same constant, $y(n) = K$, where $K \in \mathbb{R}$. In other words, design the parameter C into the filter so the filter does not alter the DC value of any input signal.

$x(n) = K \implies y(n) = C \sum_{m=0}^{M-1} \alpha^m K$. But we want $y(n) = K$,

So it must be that $K = CK \sum_{m=0}^{M-1} \alpha^m$. If $K = 0$, the equality

holds trivially, and C can be anything. But what if $K \ne 0$?

Then we have $C \sum_{m=0}^{M-1} \alpha^m = 1 \implies C = \dfrac{1}{\sum_{m=0}^{M-1} \alpha^m} = \dfrac{1}{\dfrac{\alpha^M - 1}{\alpha - 1}} = \dfrac{\alpha - 1}{\alpha^M - 1}$

$$C = \dfrac{\alpha - 1}{\alpha^M - 1}$$

This ensures that the DC gain of the filter is unity.

(b) (7 Points) For this part only, assume $M = 3$ and provide a well-labeled delay-adder-gain block diagram implementation of the filter of Equation (1). Use the generic symbol C in your diagram, not the value you found in part (a).
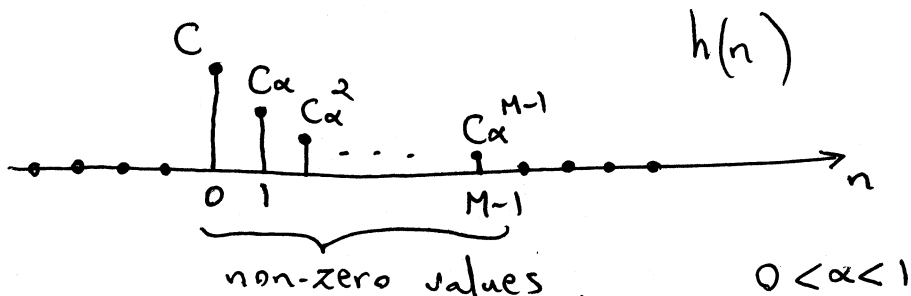
$$M = 3 \implies y(n) = C \sum_{m=0}^{2} \alpha^m x(n-m) = C\left[x(n) + \alpha x(n-1) + \alpha^2 x(n-2)\right]$$
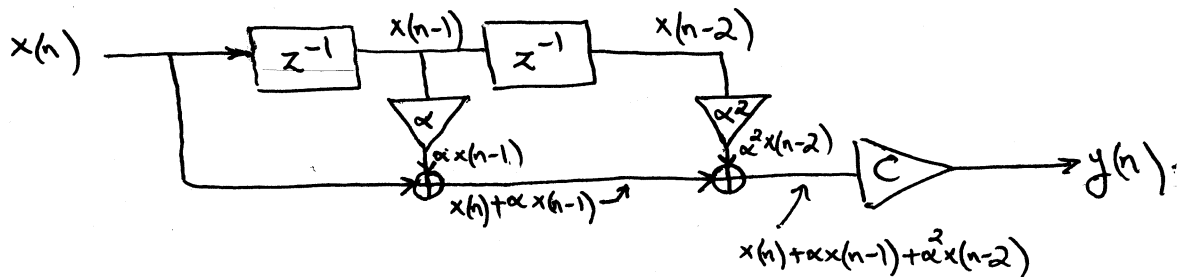
(c) (7 Points) Determine a simple expression for, and provide a well-labeled sketch of, $h(n)$, the impulse response values of the M-point EWMA filter. Use generic values for C and M.

$$\text{Let } x(n) = \delta(n) \implies h(n) = C \sum_{m=0}^{M-1} \alpha^m \delta(n-m) \implies$$

$$h(n) = C\left[\delta(n) + \alpha \delta(n-1) + \alpha^2 \delta(n-2) + \cdots + \alpha^{M-1} \delta(n-M+1)\right]$$

$0 < \alpha < 1$

(d) (7 Points) Determine a simple expression for the frequency response $H(\omega)$, $\forall \omega \in [-\pi, \pi]$.

Method 1: $h(n) = C \sum_{m=0}^{M-1} \alpha^m \delta(n-m) \implies H(\omega) = C \sum_{m=0}^{M-1} \alpha^m e^{-i\omega m}$, based on the knowledge that $\delta(n-m) \longleftrightarrow e^{-i\omega m}$

$\underset{\text{"time" domain}}{\uparrow} \quad \underset{\text{frequency domain}}{\uparrow}$. $\quad H(\omega) = C \dfrac{\alpha e^{-i\omega M} - 1}{\alpha e^{-i\omega} - 1}$

Method 2: $x(n) = e^{i\omega n} \implies y(n) = H(\omega) e^{i\omega n}$. Plug into Eqn 1 and solve for $H(\omega)$: $\quad H(\omega) e^{i\omega n} = C \sum \alpha^m e^{i\omega(n-m)} \implies H(\omega) \uparrow$

(e) (7 Points) We want to compare the M-point EWMA filter of Equation (1) with the M-point moving average (MA) filter described below:

$$y(n) = \frac{1}{M} \sum_{m=0}^{M-1} x(n-m). \tag{2}$$

Which filter responds more quickly to instantaneous changes in the input signal x: the M-point EWMA filter (for an appropriately-chosen value of C) or the M-point MA filter? Explain your reasoning succinctly, but clearly and convincingly.

In computing $y(n)$, the EWMA filter weights $x(n)$ most heavily; it weights $x(n-1)$ less; $x(n-2)$ even less; $\cdots$. The MA filter, however, weights $x(n), x(n-1), \cdots, x(n-M+1)$ all equally. A sudden change in x from $x(n-1)$ to $x(n)$ is captured more quickly by the EWMA filter b/c $x(n)$ is weighted most heavily.

(f) (5 Points) Suppose C and M are fixed, and α is the only tunable design parameter. We want to make the EWMA filter more quickly responsive to changes in the input signal. Which direction should be tweak α? Toward zero ($\alpha = 0$) or toward one ($\alpha = 1$).

Toward $\alpha = 0$. This makes the weights of input values at previous samples $n-1, n-2, \cdots, n-M+1$ vanish more sharply.

$\lim_{\alpha \to 1} \text{EWMA Filter} = \text{MA Filter.}$

MT3.3 (40 Points) In this problem you will explore aspects of discrete-time filter design. Parts (a) and (b) are mutually independent and may be tackled in either order.

(a) (20 Points) A discrete-time LTI filter G has frequency response values $G(\omega)$ and impulse response values $g(n)$. We wish to design a new filter H based on G in the following manner. The impulse response values $h(n)$ of filter H are related to those of the filter G as follows:

$$h(n) = \begin{cases} 1 - g(0) & \text{if } n = 0 \\ -g(n) & \text{if } n \neq 0. \end{cases}$$

    (i) Determine a reasonably simple expression for $H(\omega)$, the frequency response values of filter H, in terms of $G(\omega)$.

$$H(\omega) = \sum_{n=-\infty}^{\infty} h(n) e^{-i\omega n} = h(0) + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} h(n) e^{-i\omega n} = 1 - g(0) - \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} g(n) e^{-i\omega n}$$

$$= 1 - \sum_{n=-\infty}^{\infty} g(n) e^{-i\omega n} = 1 - G(\omega) \implies \boxed{H(\omega) = 1 - G(\omega)}$$

    (ii) True or False? If G is a low-pass filter, then H *must* be a high-pass filter. If your answer is "True," explain your reasoning succinctly, but clearly and convincingly. If your answer is "False," then give a counter-example, where G is a low-pass filter, but H is not a high-pass filter.
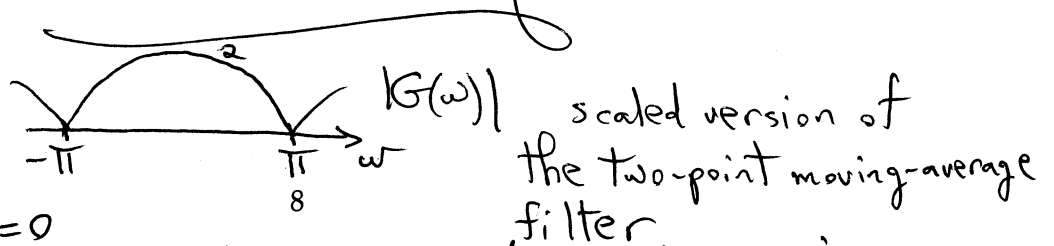
False! Consider the two-point FIR filter G w/ impulse response:

$$g(n) = \delta(n) + \delta(n-1) \implies G(\omega) = 1 + e^{-i\omega}$$
$$\implies G(\omega) = 2\cos\left(\frac{\omega}{2}\right) e^{-i\omega/2} \qquad = \left(e^{+i\omega/2} + e^{-i\omega/2}\right) e^{-i\omega/2}$$

$$|G(\omega)| = 2\left|\cos \frac{\omega}{2}\right|$$

$|G(\omega)|$ scaled version of the two-point moving-average filter.

$$h(n) = \begin{cases} 1 - g(0) = 0 & n = 0 \\ -g(n) & n \neq 0 \end{cases} \implies h(n) = -\delta(n-1) \left(\implies H(\omega) = -e^{-i\omega} \implies \right.$$
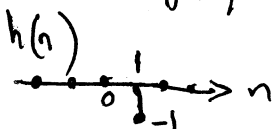
$|H(\omega)| = 1$ an all-pass filter, not a high-pass filter.

(b) (20 Points) A discrete-time LTI filter G has frequency response values $G(\omega)$ and impulse response values $g(n)$. We wish to design a new filter F based on G in the following manner. The frequency response values $F(\omega)$ of filter F are related to those of the filter G as follows:
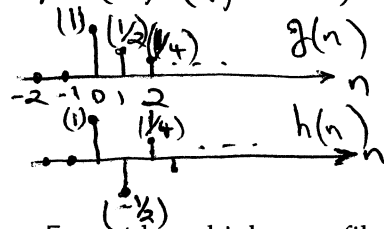
$$F(\omega) = G(\omega - \pi), \quad \forall \omega.$$

(i) Determine a reasonably simple expression for $f(n)$, the impulse response values of filter F, in terms of $g(n)$.

$$F(\omega) = G(\omega - \pi) = \sum_{n=-\infty}^{\infty} g(n) e^{-i(\omega-\pi)n} = \sum_{n=-\infty}^{\infty} g(n) \underbrace{e^{+i\pi n}}_{=(-1)^n} e^{-i\omega n} \implies$$

$$F(\omega) = \sum_{n=-\infty}^{\infty} \underbrace{(-1)^n g(n)}_{\text{must be } f(n)} e^{-i\omega n} \implies \underline{f(n) = (-1)^n g(n)}$$

(ii) Suppose $g(n) = \left(\frac{1}{2}\right)^n u(n)$ for all n. Determine a reasonably simply expression for $f(n)$.

$$f(n) = (-1)^n g(n) = (-1)^n \left(\frac{1}{2}\right)^n u(n) \implies$$

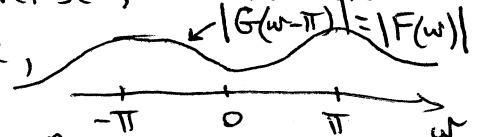$$\underline{f(n) = \left(\frac{-1}{2}\right)^n u(n)}$$

(iii) True or False? If G is a low-pass filter, then F *must* be a high-pass filter. If your answer is "True," explain your reasoning succinctly, but clearly and convincingly. If your answer is "False," then give a counter-example, where G is a low-pass filter, but F is not a high-pass filter.

True! The frequency response F is a π-shifted version of the frequency response G. If G is low-pass, it favors frequencies in the neighborhood of $\omega=0$ and disfavors frequencies in the vicinity of $\omega=\pi$.
For example,
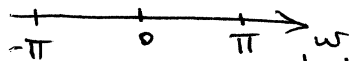
When this behavior is shifted by π, the situation between frequencies around $\omega=0$ and $\omega=\pi$ is reversed, and the new filter F becomes high-pass. For example,

You can test this also w/ $g(n) = \left(\frac{1}{2}\right)^n u(n)$ and $f(n) = \left(\frac{-1}{2}\right)^n u(n)$ in part (b)(ii) above.

LAST Name _De Cay_   FIRST Name _Ekspo_
Lab Time _No time for that!_

Problem	Points	Your Score
Name	10	10
1	25	25
2	40	40
3	40	40
Total	115	115