University of California
College of Engineering
Department of Electrical Engineering
and Computer Sciences

Jan M. Rabaey

TuTh 2-3:30

Thursday, September 28, 6:30-8:00pm

EECS 105: FALL 06 — MIDTERM 1
SOLUTIONS

NAME	Last First

SID	

Problem 1 (8):

Problem 2 (12):

Problem 3 (10):

Total (30)	

PROBLEM 1: Circuit Analysis (8 pts)

In the lab of EE105, you are given a special device "X", shown in the Figure 1a to analyze large and small signal behavior. Your measurements reveal that the device has the I-V relationship of Figure 1b, which can be expressed as $I = -(V\text{-a})^2 + \text{a}^2$ with a = 1 (for $0 \le V \le 2$).

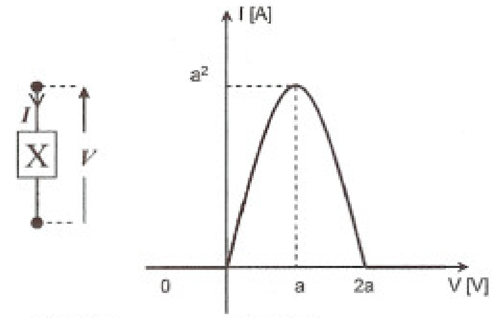


figure 1a   figure 1b

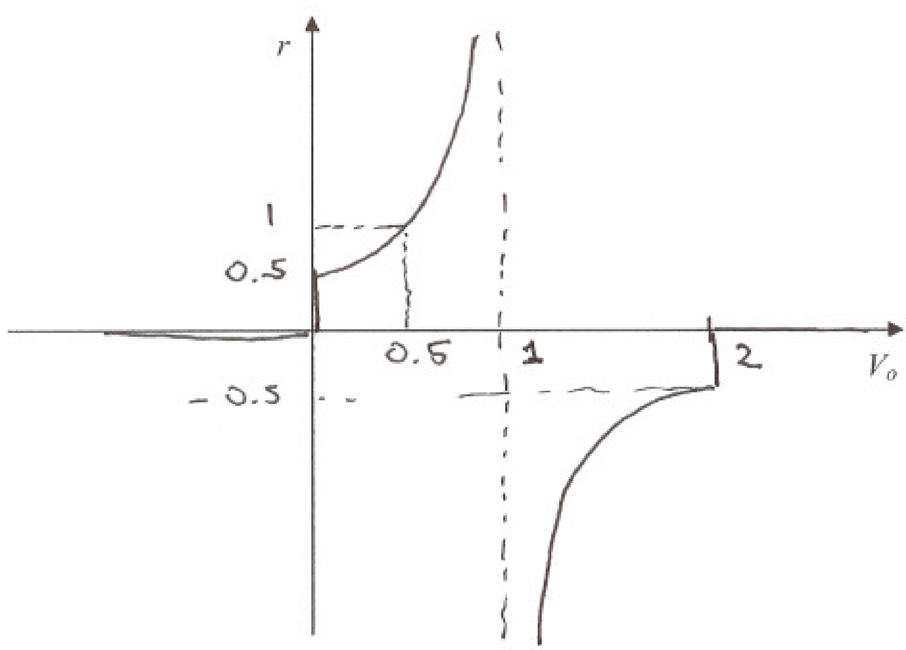
a. Determine the current I_o for $V = V_o = 0.5\text{V}$. **(1 pt)**

$I = 1 - (v-1)^2$

for $V_o = 0.5V \rightarrow I_o = 3/4\ A$

$I_o =$ 3/4 A

b. Draw the small signal resistance r vs V_o. Note that I = 0[A] outside the region $0[\text{V}] \le V_o \le 2\text{a}[\text{V}]$. **(2 pt)**



$r = 1/g = 1/\left(\frac{dI}{dv}\right) = \frac{1}{2(1-V_o)}$

c. Now you slightly change the voltage from the bias point established in (a), increasing it with $\Delta v = +0.1$V. **Using the results from (b)**, determine the current change Δi. **(2pt)**

$$\Delta i = \frac{\Delta v}{r} = \frac{0.1V}{1\Omega} = 0.1\,A$$

$$r\,(V_0 = 0.5) = 1\,\Omega$$

Δi = 0.1A

d. We now hook up X between the reference voltage $VDD = 2$[V], and an input current source $I_{in} = I_o + 0.1\,\sin(2\pi 1000t)$ as shown in Figure 2 (with I_o as computed in **a**). Draw V_{out} as a function of time. **(3 pt)**

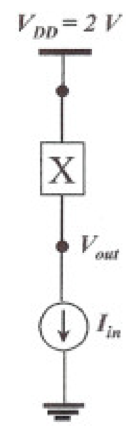
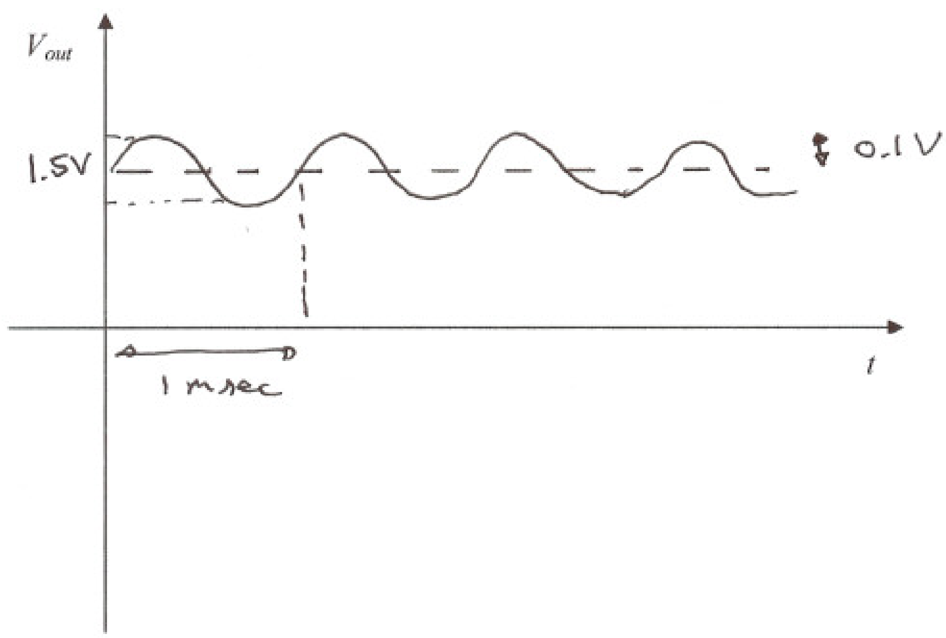
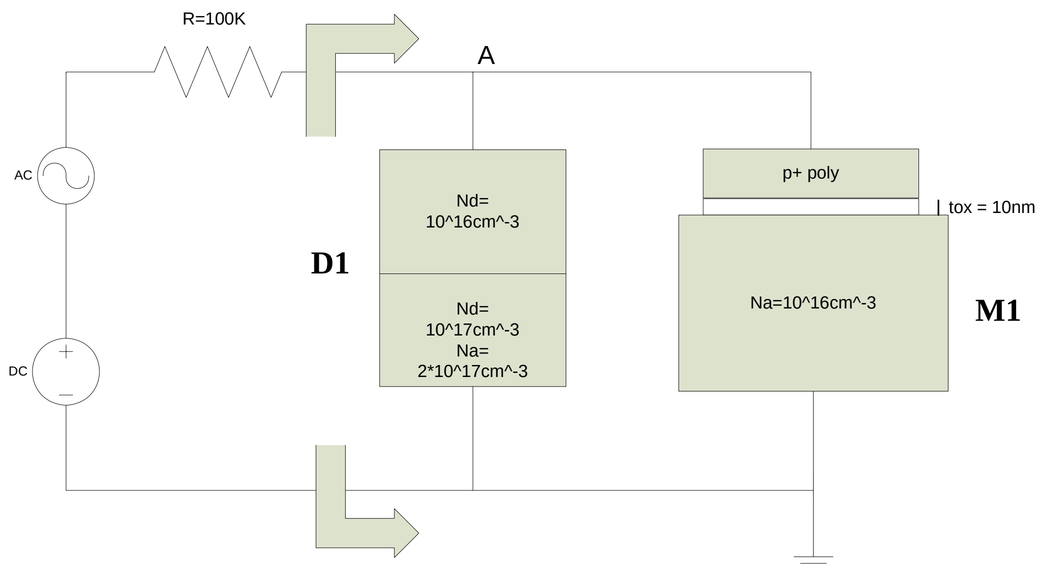


Figure 2



PROBLEM 2: Diode (12 pts)

Consider the circuit picture in the Figure below. For this problem, you may assume the following values: $\varepsilon_s = 1.035\ 10^{-12}$ F cm^{-1}; ε_{ox} = 3.45 10-13 F cm^{-1}. Contact potentials should be ignored throughout this question. Also, D1 and M1 are identical in area.



a. The pn-diode D1 has the following doping profile:

p-type material:

$Na = 2*10^{17}$ cm^{-3}

$Nd = 1*10^{17}$ cm^{-3}

n-type material:

$Nd=10^{16}$ cm^{-3}

Determine the depletion capacitance per unit area of D1 in thermal equilibrium. **(2 pts)**

$$p0 = Na - Nd = 2*10^{17}cm^{-3} - 10^{17}cm^{-3} = 10^{17}cm^{-3}$$

$$n0 = Nd = 10^{16}cm^{-3}$$

$$\phi_B = \phi_n - \phi_p = (60mV\log(\frac{n0}{ni})) - (-60mV\log(\frac{p0}{ni})) = 780mV$$

$$Xd0 = \sqrt{\frac{(2*\varepsilon_s*\phi_B)}{q}*(\frac{1}{p0}+\frac{1}{n0})} = 3.33*10^{-5}cm$$

$$Cjo = \frac{\varepsilon_s}{Xd0} = \frac{11.7*8.85*10^{-14}F/cm}{3.33*10^{-5}cm} = 3.1*10\text{^}-8\frac{F}{cm^2}$$

b. The second component M1 is a MOSCAP with the following characteristics: Gate = p+ material with ϕ_{p+} = -550 mV; tox = 10 nm; substrate is doped with Na = 10^{16} cm^{-3}; and $\mathbf{V_T = 1.05V}$. Determine the flatband voltage V_{FB} of M1. **(2pts)**

$$\phi_{p+} = -550mV$$

$$\phi_p = 60mV\log\left(\frac{10^{16}}{10^{10}}\right) = 360mV$$

$$\phi_B = -550mV - (-360mV) = -190mV$$

$$V_{fb} = 190mV$$

$$\boxed{\mathbf{V_{FB} = 190\ mV}}$$

c. Determine the minimum and maximum small signal capacitance per unit area of M1. **(3 pts)**

$$C_{\max} = C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{3.45*10^{-13}}{10*10^{-7}cm} = 3.45*10^{-7}\frac{F}{cm^2}$$

$$C\min = \frac{Cox*C(Vth)}{Cox+C(Vth)}$$

where

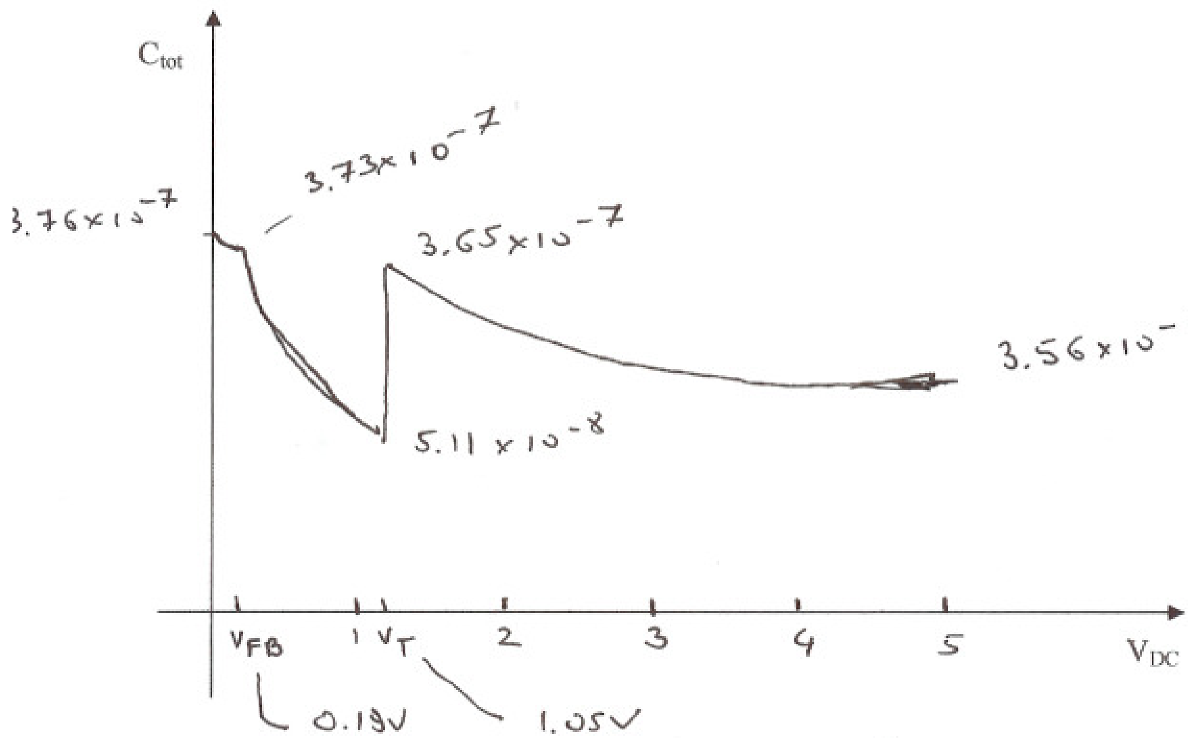
$$C(Vth) = \frac{\varepsilon_s}{Xd(Vth)}$$

$$Xd(Vth) = \sqrt{\frac{2*\varepsilon_s*(2\phi_p)}{q*Na}} = 3.05*10^{-5}cm$$

$$C(Vth) = \frac{1.035*10^{-12}F/cm}{3.05*10^{-5}cm} = 3.39*10^{-8}F/cm\text{^}2$$

$$C\min = 3.09*10^{-8}$$

d. Plot the total small signal capacitance per unit area as seen between node A and the ground (as indicated by the arrows in the Figure) when sweeping the bias voltage V_{DC} between 0 and 5 V. Your plot should show the individual components and should include numerical values for the important breakpoints in the graphs. **(5 pts)**



Parallel connection of two capacitors.

$V=0$: $C = C_0 + C_{ox}$

$= 3.11\times10^{-8} + 3.45\times10^{-7} = 3.76\times10^{-7}$ F/cm²

V_{fb} : $C_1 = \dfrac{C_0}{\sqrt{1+\frac{0.19}{0.78}}} = 2.8\times10^{-8}$

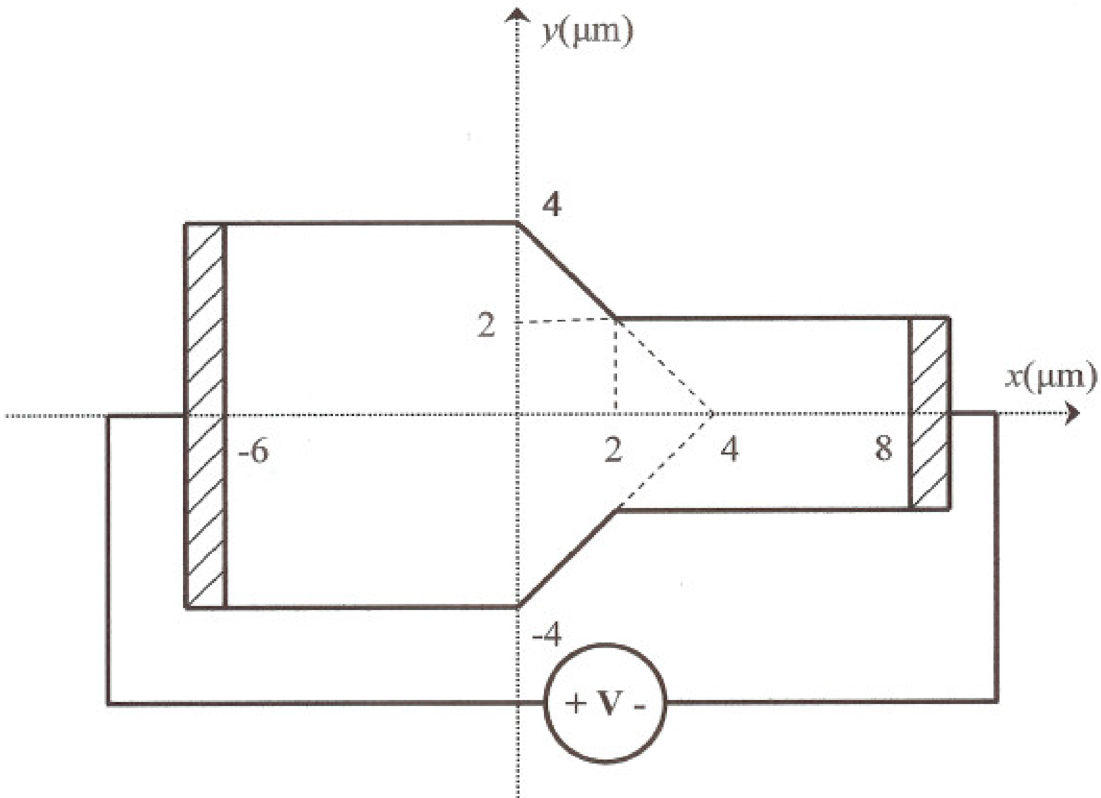
$C = C_1 + C_{ox} = 3.73\times10^{-7}$ F/cm²

V_{th} : $C_{d2} = \dfrac{C_0}{\sqrt{1+\frac{1.05}{0.78}}} = 2.03\times10^{-8}$ F/cm²

$C = C_{min} + C_d = 5.11\times10^{-8}$ F/cm²

5V : $C_3 = 1.14\times10^{-8}$: $C = C_3 + C_{ox} = 3.56\times10^{-7}$ F/cm²

PROBLEM 3: Semiconductor Physics (10 pts)
Given an ion-implanted silicon region with dimension as shown in figure below. The arsenic dose implanted per unit area equals $Q_d = 10^{13}\text{cm}^{-2}$, and the post-anneal thickness $t = 1\mu\text{m}$. You may ignore the contact potential effect and assume room temperature. Also use Figure 2.8 in the text book to derive mobilities.



(a) Compute the doping concentration N_d, and the carrier concentrations n_0 and p_0 under thermal equilibrium. (2 pts)

$N_d = Q_d / t = 10^{17}\text{cm}^{-3}$
$N_d >> n_i$, $n_o \approx N_d = 10^{17}\text{cm}^{-3}$
Mass-action law, $p_o = n_i^2 / n_o = 10^3\text{cm}^{-3}$

$N_d =$	10^{17} cm^{-3}
$n_0 =$	10^{17} cm^{-3}
$p_0 =$	10^{3} cm^{-3}

(b) Viewing the silicon as three regions (-6 to 0, 0 to 2, and 2 to 8 μm), compute the resistance of each region as well as the total resistance of the strip. **(4 pts)**

$R_\square = 1/(q N_d \mu_n t) = 823\Omega$ $\quad (\mu_n = 760\ cm^2/Vs$ − from 2.8)

$R_1 = 6/8\ R_\square = 617\Omega$

$R_3 = 6/4\ R_\square = 1235\Omega$

$R_2 = R_\square \int_0^2 \frac{dx}{y} = R_\square \int_0^2 \frac{dx}{-2x+8} = R_\square\ 0.5 \ln 2 = 0.34\ R_\square = 274\Omega$

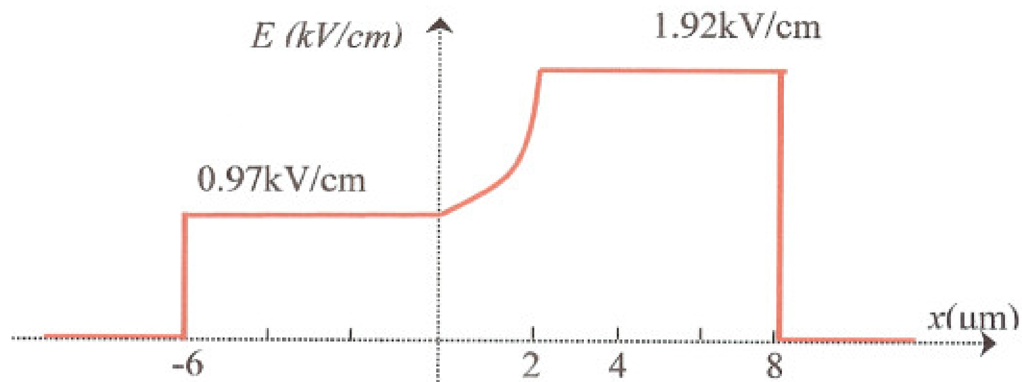
or approx as square

$R_2 = 2/6\ R_\square = 274\Omega$

R1 =	617Ω
R2 =	274Ω
R3 =	1235Ω
Rtot =	2126Ω

$R_{tot} = R_1 + R_2 + R_3$

(c) If $V = 2V$, compute the electric field and sketch it. You may ignore the variation effect along the y-axis. **(4 pts)**



$E_1 = \frac{V_1}{L_1} = \frac{V R_1}{R_{tot} L_1} = 967\ V/cm$

$E_3 = \frac{V_3}{L_3} = \frac{V R_3}{R_{tot} L_3} = 1936\ V/cm$

E_2 gradually changes from E_1 to E_3