EECS 40, Fall 2006
Prof. Chang-Hasnain
Midterm #1

September 27, 2006
Total Time Allotted: 50 minutes
Total Points: 100

1. This is a closed book exam. However, you are allowed to bring one page (8.5" x 11"), single-sided notes
2. No electronic devices, i.e. calculators, cell phones, computers, etc.
3. SHOW all the steps on the exam. Answers without steps will be given only a small percentage of credits. Partial credits will be given if you have proper steps but no final answers.
4. Draw BOXES around your final answers.
5. **Remember to put down units.** Points will be taken off for answers without units.

Last (Family) Name: Perfect

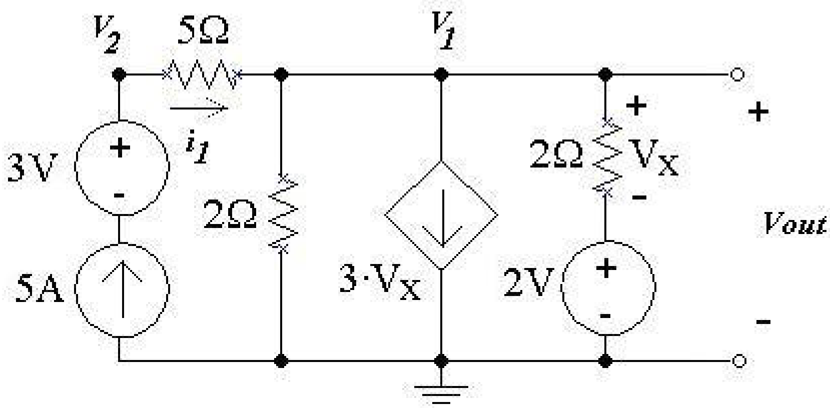
First Name: Peter

Student ID: 00000001 Discussion Session: 000

Signature: ____________________

Score:	
Problem 1 (50 pts)	
Problem 2 (50 pts):	
Total	

1. (50 pts) Equivalent circuit.



(a) (5 pts) What is the current i_1 through the 5 Ohm resistor?

$$i_1 = 5A$$

(b) (5 pts) Use KVL, write down the equation for V_x in terms of V_1 and/or V_2

$$V_x = V_1 - 2$$

(c) (5 pts) Use KCL, write down the equation for V_1 and solve for V_1

$$-5 + \frac{V_1}{2} + 3 \cdot V_x + \frac{V_x}{2} = 0$$
$$-10 + V_1 + 6 \cdot V_x + V_x = 0$$

$$-10 + V_1 + 7 \cdot V_x = 0$$
$$-10 + V_1 + 7 \cdot (V_1 - 2) = 0$$
$$-24 + 8 \cdot V_1 = 0$$
$$V_1 = 3V$$

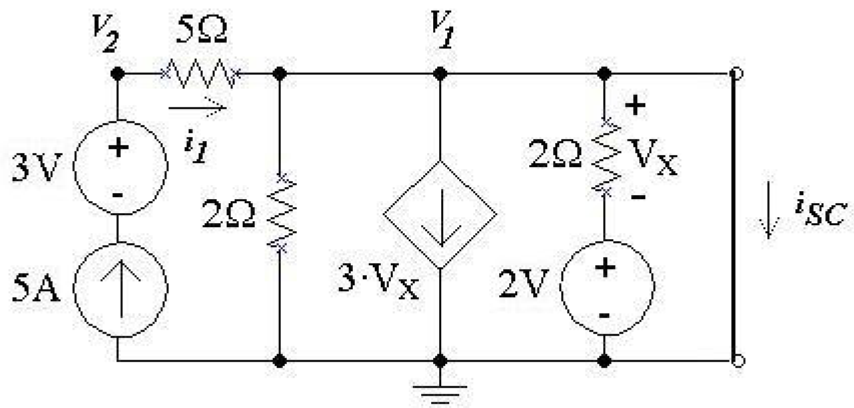
(d) (5 pts) Use KCL, write down the equation for V_2 and solve for V_2

$$-5 + \frac{V_2 - V_1}{5} = 0$$
$$-25 + V_2 - V_1 = 0$$
$$V_2 = 25 + V_1$$
$$V_2 = 28V$$

(e) (5 pts) Solve for V_{out} (this is simply the Thevenin Voltage)

$$V_{out} = V_1$$
$$V_{out} = V_1 = 3V$$

(f) Now we short the two end terminals.



(5 pts) What is V_x ?

$$V_x = V_1 - 2$$
$$V_1 = 0$$
$$V_x = 0 - 2$$
$$V_x = -2V$$

(g) (5 pts) What is V_1 ?

$$V_1 = 0$$

(h) (5 pts) What is I_{sc}?

$$-5+3\cdot V_x+I_{sc}+\frac{V_x}{2}=0$$

$$-5+3\cdot(-2)+I_{sc}+\frac{-2}{2}=0$$

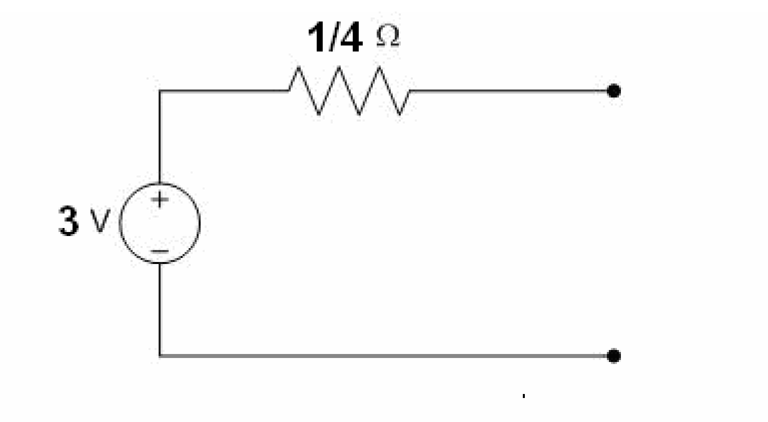
$$I_{sc}=12A$$

(i) (5 pts) what is the Thevenin Resistance?

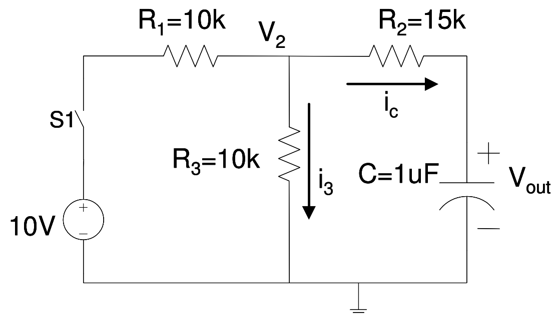
$$R=\frac{V_{oc}}{I_{sc}}$$

$$R=\frac{3V}{12A}=\frac{1}{4}\Omega$$

(j) (5 pts) Draw the Thevenin Equivalent Circuit.



2. For $t<0$, the switch was open and $V_{out}=0$. At t = 0s, S1 closes. NOTE: $\mu=10^{-6}$; $k=10^3$; $e^{-1}=0.37$; $e^{-2}=0.14$ **Remember to put down units.**



(a) (12 pts) Construct the differential equation of V_{out} in terms of all the given quantities. *Hint:you may solve this use Mesh or Nodal analysis, or, even simpler, Thevnin equivalent circuit. Write all your steps.*

Thevenin Equivalence:
Rewrite the 10V source and R1 into a Nodal Equivalent Circuit:
10V source becomes 1A source
R1 is now in parallel with the 1A source.
Combine R1 and R3 together to create a 5k ohm resister.
Rewrite the 1A source and 5k ohm resister into Thevenin Equivalent Circuit.
1A source becomes 5V source
5k ohm resister is in series with the 5V source.
Combine R1||R3 with R2 to yield 20k ohm resister.
We now have a 5V source in series with a 20k ohm resister in series with a 1uF capacitor.
Using the predetermined equations, we can fill in the variables and obtain the equation show below.

Nodal Analysis:

$$\frac{V_2 - V_{in}}{10k} + \frac{V_2}{10k} + \frac{V_2 - V_{out}}{15k} = 0$$

$$\frac{V_{out} - V_2}{15k} + C\frac{dV_{out}}{dt} = 0$$

$$\textit{multiply both sides by } 30k$$

$$3V_2 - 3V_{in} + 3V_2 + 2V_2 - 2V_{out} = 0$$

$$8V_2 - 3V_{in} - 2V_{out} = 0$$

$$V_2 = \frac{3V_{in} + 2V_{out}}{8}$$

$$\frac{V_{out}}{15k} - \frac{1}{15k}(\frac{3}{8}V_{in} + \frac{1}{4}V_{out}) + C\frac{dV_{out}}{dt} = 0$$

$$V_{out} - \frac{3}{8}V_{in} - \frac{1}{4}V_{out} + 15k \cdot C\frac{dV_{out}}{dt} = 0$$

$$\frac{3}{4}V_{out} + 15k \cdot C\frac{dV_{out}}{dt} = \frac{3}{8}V_{in}$$

$$V_{out} + 20k \cdot C\frac{dV_{out}}{dt} = \frac{1}{2}V_{in}$$

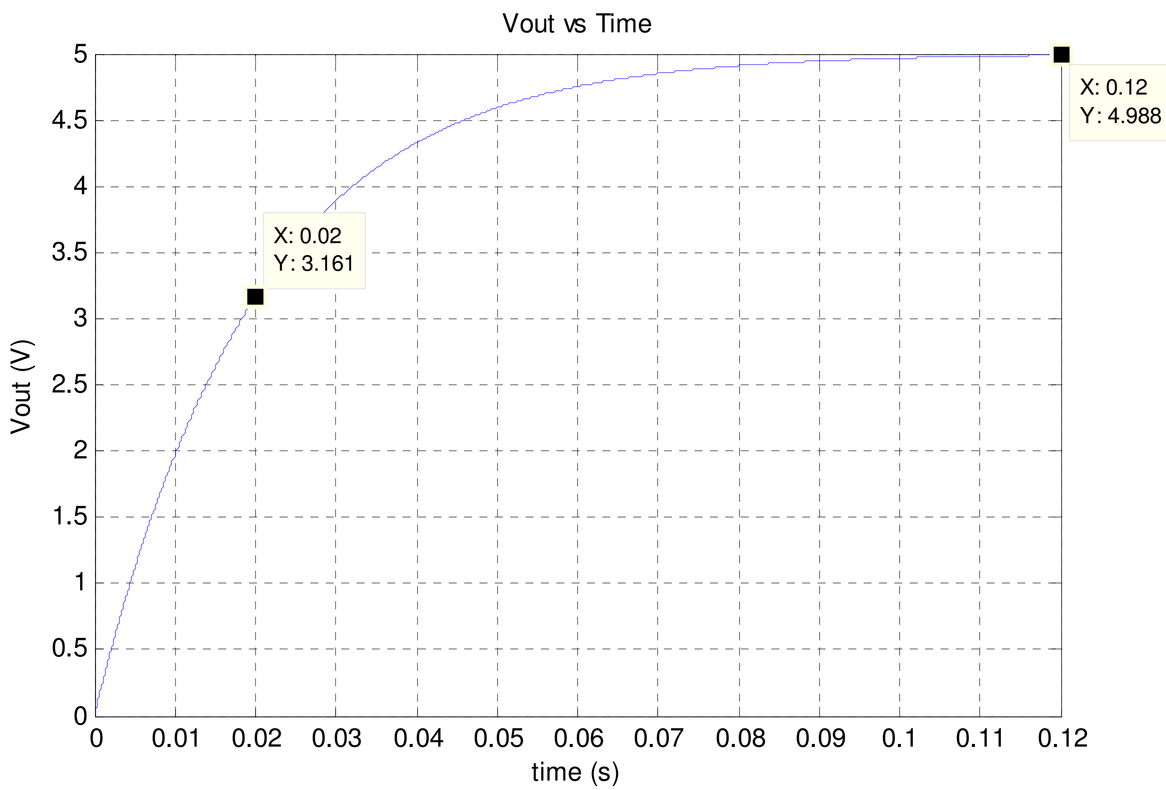
$$V_{out} + 15k \cdot 1uF\frac{dV_{out}}{dt} = 5$$

$$V_{out} + 20ms\frac{dV_{out}}{dt} = 5V$$

(b) (5 pts) Write a closed-form expression for $V_{out}(t)$ for t>0

$$Vout = 5(1 - e^{-t/20ms})$$

(c) (8 pts) Plot V_{out} as a function of time t = 0 to t = 100ms. **Label the y-axis and all key points:** starting value, 1 time constant value, value at infinity.



(Note at 20ms, Vout = 3.15 using the above approximation for e^{-1})
(Note at infinity, Vout should approach 5V)

(d) (5 pts) As t approaches infinity, what value will i_3 approach?

Because at infinity, the capacitor becomes an open,

$$I = \frac{V}{R} = \frac{10}{R1 + R2} = \frac{10}{20k} = \frac{1}{2} mA$$

(e) (5 pts) Now, suppose someone disturbed the circuit and S1 is re-opened at 40 ms again! Construct the new differential equation.

If switch S1 is open, R1 becomes irrelevant because it is connected to an open circuit. Therefore we combine R2 and R3 to yield a 25k ohm resister.
Again we have a predetermined form and therefore the equation is

$$V_{out} + RC\frac{dV_{out}}{dt} = 0$$

$$V_{out} + 25k \cdot 1uF\frac{dV_{out}}{dt} = 0$$

$$V_{out} + 25ms\frac{dV_{out}}{dt} = 0$$

(f) (6 pts) What is the new time constant? What is the new expression for $V_{out}(t)$ for t>40 ms.

$$\tau = RC = 25ms$$

$$Vout = Ke^{-t/25ms}$$

$$Vout(t = 40ms) = 5(1 - e^{-40ms/20ms}) = Ke^{-0/25ms} = 4.3$$

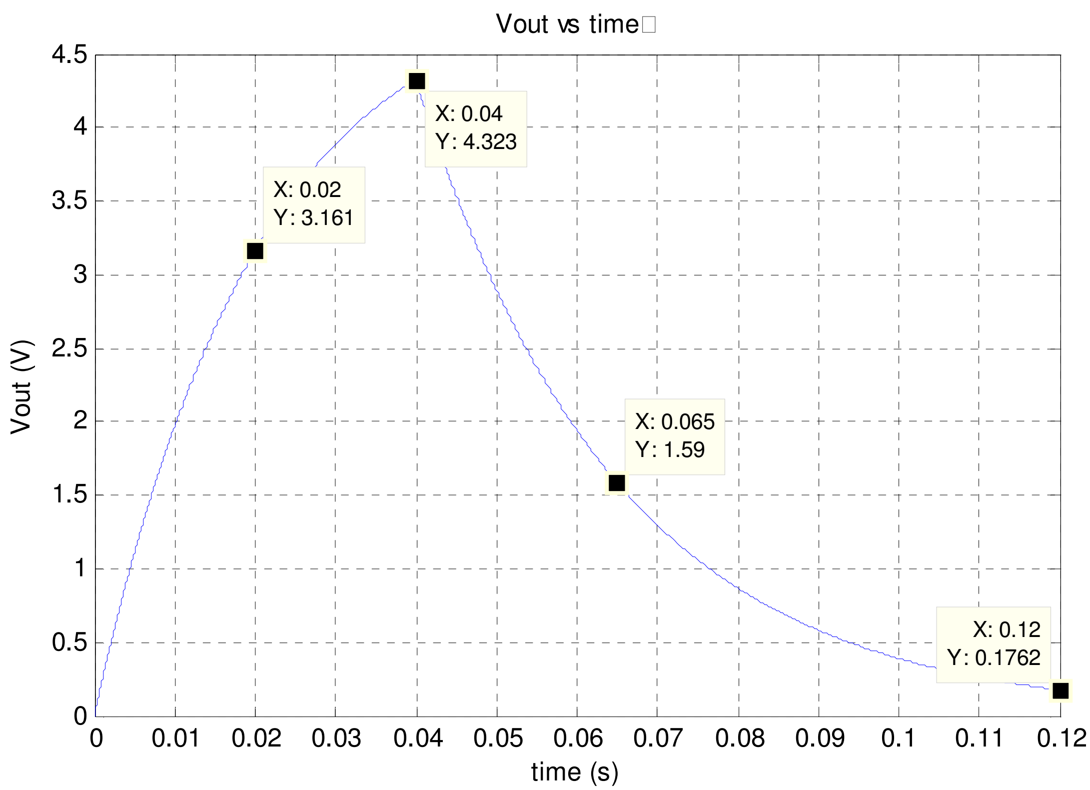
$$K = 4.3$$

$$Vout = 4.3Ke^{-t/25ms}$$

$$with\ a\ 40ms\ timeshift$$

$$Vout = 4.3e^{-(t-40ms)/25ms}$$

(g) (5 pts) Plot the new V_{out} from t=0ms to 100 ms to include the re-opening of the switch at 40 ms. **Label the y-axis and all key points:** starting value, value at switching point, 1 time constant values, value at infinity.



(Note that at 20ms, Vout = 3.15V, using approximation)

(Note that at 40ms, Vout = 4.3V, using approximation)

(Note that at 65ms, Vout = 1.591V, using approximation)

(Note that at infinity, Vout approaches 0V)

(h) (5 pts) In this case, as t approaches infinity, what value will i_3 approach?

$I_3 = 0A$