EECS 20N: Structure and Interpretation of Signals and Systems MIDTERM 4
Department of Electrical Engineering and Computer Sciences 8 May 2008
UNIVERSITY OF CALIFORNIA BERKELEY

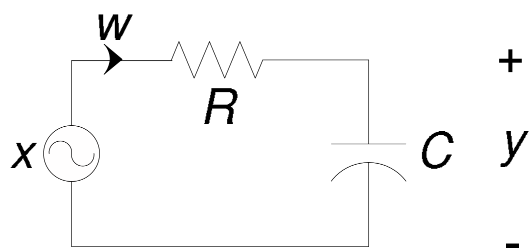
LAST Name ____________________ FIRST Name ____________________

Lab Time ____________________

- **(10 Points)** Print your name and lab time in legible, block lettering above AND on the last page where the grading table appears.
- This exam should take up to 70 minutes to complete. You will be given at least 70 minutes, up to a maximum of 80 minutes, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except four double-sided $8.5" \times 11"$ sheet of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The exam printout consists of pages numbered 1 through 8.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the eight numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

MT4.1 (30 Points) You may tackle the two parts of this problem in either order. Specifics of one part do not necessarily carry over to the other.

(a) Consider the first-order RC-circuit shown below. The signal x is the input voltage. The output signal y is the voltage across the capacitor. The signal w is the loop current that passes through the resistor and the capacitor. The resistance R and the capacitance C are real constants having units of Ohms and Farads, respectively.



The differential equation governing the input x and the output y, and the corresponding frequency response H, are given below:

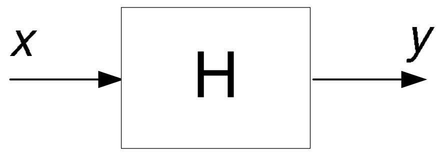
$$RC\,\frac{dy}{dt}(t) + y(t) = x(t)\,, \qquad H(\omega) = \frac{1}{1 + i\omega RC}\,.$$

Determine a simple expression for the output y if the input x is

$$x(t) = \cos\left(\frac{t}{RC}\right).$$

Your expression for $y(t)$ must be in the form $y(t) = A_0 \cos(\omega_0 t + \phi_0)$. Determine the appropriate *numerical* values of A_0 and ϕ_0.

(b) Consider a continuous-time LTI system H having input x, output y, frequency response H, and *real-valued* impulse response h.



If the input signal is characterized by the sinusoid, $x(t) = \cos(\omega_0 t)$, determine a simple expression for the output y in the form $y(t) = A_0 \cos(\omega_0 t + \phi_0)$. Determine A_0 and ϕ_0 in terms of the frequency response H.

You may use the blank space below for scratch work. Nothing written below this line on this page will be considered in evaluating your work.

MT4.2 (30 Points) A continuous-time LTI filter H has impulse response

$$h(t) = \frac{\sin 20\pi t}{\pi t} \cos(200\pi t).$$

(a) Provide a reasonably well-labeled sketch of the impulse response h.

(b) Provide a well-labeled sketch of the filter's frequency response $H(\omega)$. What type of filter is H: low-pass, band-pass, or high-pass? Explain your reasoning succinctly, but clearly and convincingly.

(c) Determine the output y of the filter in response to the input

$$x(t) = \cos(100\pi t) + \frac{1}{\sqrt{2}} \sin(200\pi t).$$

MT4.3 (45 Points) You may tackle each of the parts of this problem independently.

(a) A discrete-time LTI filter H has impulse h and frequency response H characterized by

$$H(\omega) = \frac{1}{1 - \frac{1}{2}e^{-i\omega}}.$$

A related filter G has impulse response g characterized by

$$g(n) = (-1)^n h(n), \quad \forall n.$$

Determine the frequency response G and provide a well-labeled plot of the magnitude response $|G(\omega)|$.

(b) A causal, discrete-time LTI filter has frequency response

$$H(\omega) = \frac{1 - \dfrac{1}{3}e^{-i\omega}}{\left(1 - \dfrac{1}{2}e^{-i\omega}\right)\left(1 + \dfrac{1}{4}e^{-i\omega}\right)}.$$

(i) Determine the linear, constant-coefficient difference equation governing the input x and output y of this filter.

(ii) Which of the following *could* be the impulse response of this filter (for some appropriate values of constants α, β, and γ)? Explain your reasoning succinctly, but clearly and convincingly.

(I) $h_{\mathrm{I}}(n) = \alpha\left(\frac{1}{2}\right)^n u(n) + \beta\left(-\frac{1}{4}\right)^n u(n)$.

(II) $h_{\mathrm{II}}(n) = \alpha\left(\frac{1}{2}\right)^n u(n) + \beta\left(-\frac{1}{4}\right)^n u(n) + \gamma\left(\frac{1}{3}\right)^n u(n)$.

(III) $h_{\mathrm{III}}(n) = \alpha\left(-\frac{1}{2}\right)^n u(n) + \beta\left(\frac{1}{4}\right)^n u(n)$.

(IV) $h_{\mathrm{IV}}(n) = \alpha\left(-\frac{1}{2}\right)^n u(n) + \beta\left(\frac{1}{4}\right)^n u(n) + \gamma\left(-\frac{1}{3}\right)^n u(n)$.

(V) None of the above.

(c) Consider the continuous-time signal x described by

$$x(t) = \begin{cases} \cos(\omega_0 t) & |t| \leq T \\ 0 & |t| > T, \end{cases}$$

where $\omega_0 \gg \pi/T$.

(i) Provide a sketch of $x(t)$.

(ii) Determine and provide a reasonably well-labeled sketch of $X(\omega)$, the CTFT of x.

LAST Name ______________________ FIRST Name ______________________

Lab Time ______________________

Problem	Points	Your Score
Name	10	
1	30	
2	30	
3	45	
Total	**115**	