EECS 126 Probability and Random Processes — UC Berkeley, Spring 2025
Kannan Ramchandran — February 26, 2025

Midterm 1

Last Name	First Name	SID

Left Neighbor First and Last Name	**Right Neighbor** First and Last Name

Rules.

- **Unless otherwise stated, all your answers need to be justified and your work must be shown. Answers without sufficient justification will get no credit.**
- **All work you want graded should be on the front or back of the sheets in the space provided. Scratch paper will not be scanned/graded.**
- You have a **5 minute** reading period followed by **80 minutes** to complete the exam. You may **not begin writing** until the 5-minute reading period is over. DSP students with $X\,\%$ time accommodation should spend $5 \cdot X\,\%$ time reading and $80 \cdot X\,\%$ time on completing the exam.
- This exam is closed-book. You may reference **one** double-sided **handwritten** sheet of paper. No calculator or phones allowed.
- Collaboration with others is strictly prohibited. If you are caught cheating, you may fail the course and face disciplinary consequences.
- Unless otherwise stated, you **may** use results from **previous** subparts of a problem without proof, and you **may not** use results from **future** subparts of a problem without proof.

Problem	SID	1	2	3	4	5	Total
out of	1	40	25	15	20	25	126

1 Potpourri [40 points]

1.1 Multiple Choice [6 points]

For each of the following statements, bubble True if the statement is always true, or bubble False if the statement can be false. No justification is required.

Each subpart is worth 2 points.

(a) Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space. If we have two events $E_1, E_2 \in \mathcal{F}$ and $E_1 \cap E_2 = \emptyset$, then E_1, E_2 are independent.

◯ True ◯ False

(b) The sum of two normally distributed random variables is always normally distributed.

◯ True ◯ False

(c) Let X be a non-negative random variable with finite mean and variance. Applying Markov's bound to this random variable will sometimes give a stronger bound than Chebyshev's.

◯ True ◯ False

1.2 Conditional Geyser [10 points]

You are waiting for the Old Faithful geyser at Yellowstone National Park to erupt. However, you can wait only from (continuous time) $t = 0$ to $t = 100$. There is a 50% chance that the geyser will erupt in this time window, and if it does erupt, its eruption time is continuous uniformly distributed between $t = 0$ and $t = 100$.

Given that you have been waiting until $t = 70$, what is the probability that the geyser will erupt in the remaining time?

1.3 Exponential Concentration [10 points]

Let $X \sim \text{Exponential}(1)$. Use the Chernoff bound to find the tightest possible bound on $P(X > 8)$.

1.4 Estimate the Expectation [14 points]

Let $X_1, X_2, \ldots$ be independent and identically distributed random variables with $\mathbb{E}[X_i] = \mu$ and $\text{Var}(X_i) = 16$ for all i. Define the running average of this process to be $S_n = \frac{1}{n}\sum_{i=1}^{n} X_i$. Use Chebyshev's inequality to give a lower bound on the number of samples n until the running average is within 10^{-3} of the true mean with probability greater than 0.8.

2 Exponentially Crowded Concert [25 points]

After taking the midterm, you decide to go to a concert. There are only two workers to let fans into the venue. Once a fan reaches a worker, the time it takes for the fan to be let in to the concert is $\stackrel{iid}{\sim}$ Exponential(λ_1) for worker 1 and $\stackrel{iid}{\sim}$ Exponential(λ_2) for worker 2, where λ_1 and λ_2 are the serving rates of the two workers. When you arrive, both of the workers are busy with a fan in front of each of them, and there is a single line of $(n-1)$ fans in front of you waiting to get to the first available worker.

What is the expected amount of time that it will take for you to get let into the concert venue, starting from the time you enter the line? (Please give your answer in terms of λ_1, λ_2 and n.)

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3 Expectation Enigma [15 points]

Suppose $Z \sim \text{Unif}[0, 1]$, $Y \sim \text{Unif}[0, Z]$, $X \sim \text{Unif}[Y, Z]$. Compute $\mathbb{E}[X]$.

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4 Probability Proofs [20 points]

Suppose A and B are both events in a probability space $(\Omega, F, \mathbb{P})$.

In each part, state whether or not the statement is true.

If the statement is always true, give a formal proof. If the statement can be false, give a counter-example.

(a) (10 points) If A, B are independent then A, B^c are independent.

(b) (10 points) If X, Y are random variables and $\text{Cov}(X, Y) = 0$, then X and Y are independent.

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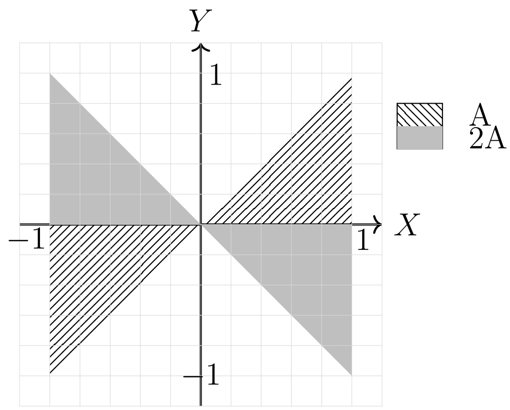
5 Graphical Density [25 points]

Two random variables X and Y have a joint density function shown in the graph below.



(a) (5 points) What must be the value of A so that the above graph is a valid joint density?

(b) (10 points) What is the marginal PDF of X?

(c) (10 points) What is $E[X|Y > \frac{1}{2}]$?

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Here is some extra workspace for you. If you wish to use it, please indicate which problem it is for and indicate on the original problem page that it leads to here.

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