E115 FALL 2003

ENGINEERING THERMODYNAMICS
UNIVERSITY OF CALIFORNIA
COLLEGE OF ENGINEERING
DEPARTMENT OF MATERIALS SCIENCE AND ENGINEERING

NAME: [redacted]

MIDTERM I

SEPTEMBER 26TH, 2003

THIS EXAM ACCOUNTS FOR UP TO 20% OF YOUR FINAL GRADE.

YOU MAY USE ONE 3 BY 5 INDEX CARD WITH NOTES.

PROBLEM #	POSSIBLE POINTS	SCORE
1	50	19
2	25	12
3	25	20
TOTAL	100	51

1) *(50 points)*

Consider one mole of a monatomic ideal gas at an initial state (P_1, V_1, T_1). The gas is taken to its final state (P_2, V_2, T_2) through an intermediate state (P_A, V_A, T_A). The two processes by which the gas is taken from state I to state II are:

1. An *isothermal doubling of the volume* from state I to state A.
2. A *reversible adiabatic expansion* from state A to state II. State II lies on an isotherm $T_2 = \frac{T_1}{4}$.

These processes are shown on the plot below:

$\boxed{V_A = 2 \cdot V_1}$

$\boxed{T_A = T_1}$ (isothermal)

$\frac{P_A \overset{2V_1}{V_A}}{T_A = T_1} = \frac{P_1 V_1}{T_1}$ $\boxed{P_A = \frac{P_1}{2}}$

$\gamma = \frac{5}{3}$ (mono-ideal)

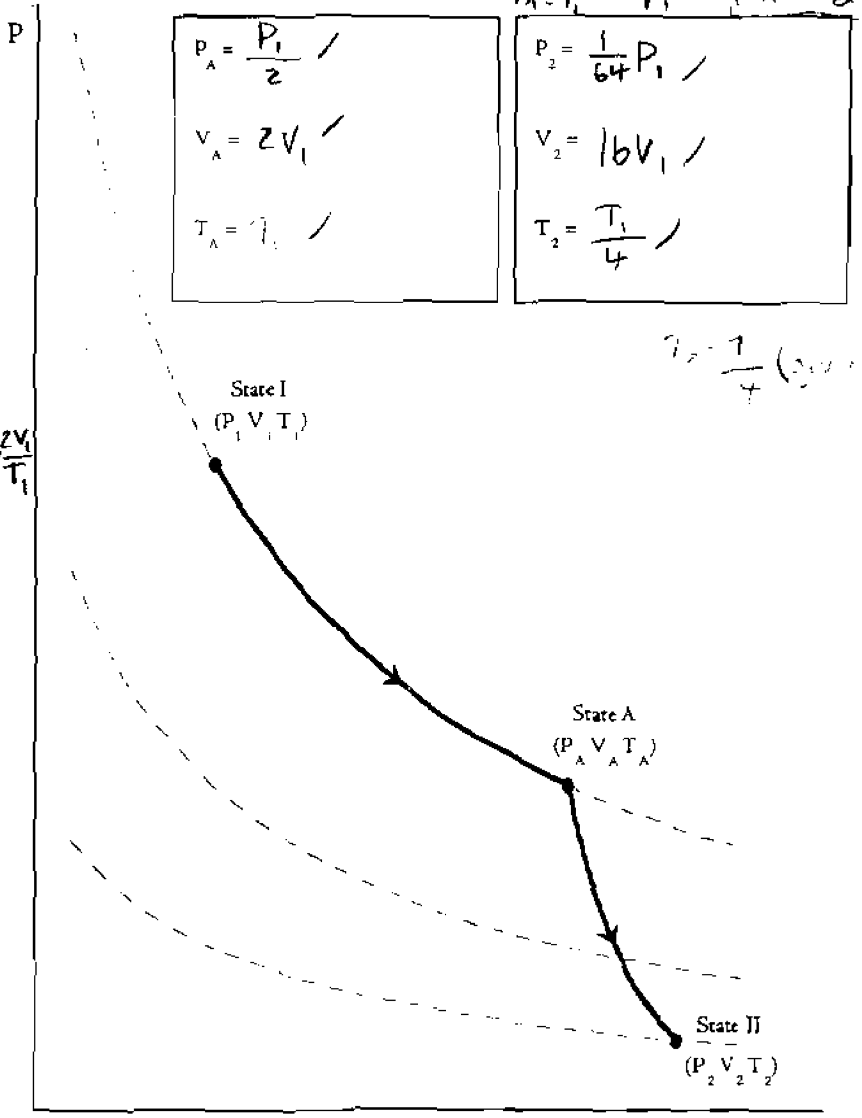
$\frac{\overset{T_A}{T_1}}{\left(\frac{T_1}{4}\right)} = \left(\frac{\overset{V_A}{2V_1}}{V_2}\right)^{\gamma - 1}$

$V_2 = (4)^{\frac{1}{\gamma-1}} \cdot 2V_1$

$\boxed{V_2 = 16 V_1}$

$P_2 = \frac{T_2}{V_2} \cdot \frac{P_A V_A}{T_A} = \frac{T_1}{4} \frac{1}{16V_1} \cdot \frac{P_1}{2} \cdot \frac{2V_1}{T_1}$

$\boxed{P_2 = \frac{1}{64} P_1}$



$P_A = \frac{P_1}{2}$ ✓

$V_A = 2V_1$ ✓

$T_A = T_1$ ✓

$P_2 = \frac{1}{64} P_1$ ✓

$V_2 = 16V_1$ ✓

$T_2 = \frac{T_1}{4}$ ✓

$T_2 = \frac{1}{4}$ ([illegible])

a) (*12 points*)
Solve for the pressure, volume, and temperature at state A and state II in terms of P_1, V_1, and T_1. Write your answers in the space provided on the plot. Show all work for full credit. (12)

work on page 1

b) (*10 points*)
Calculate the work, heat, and internal energy change going from state I to state II in terms of T_1 and the gas constant R. ③

$\underline{I \to II}$

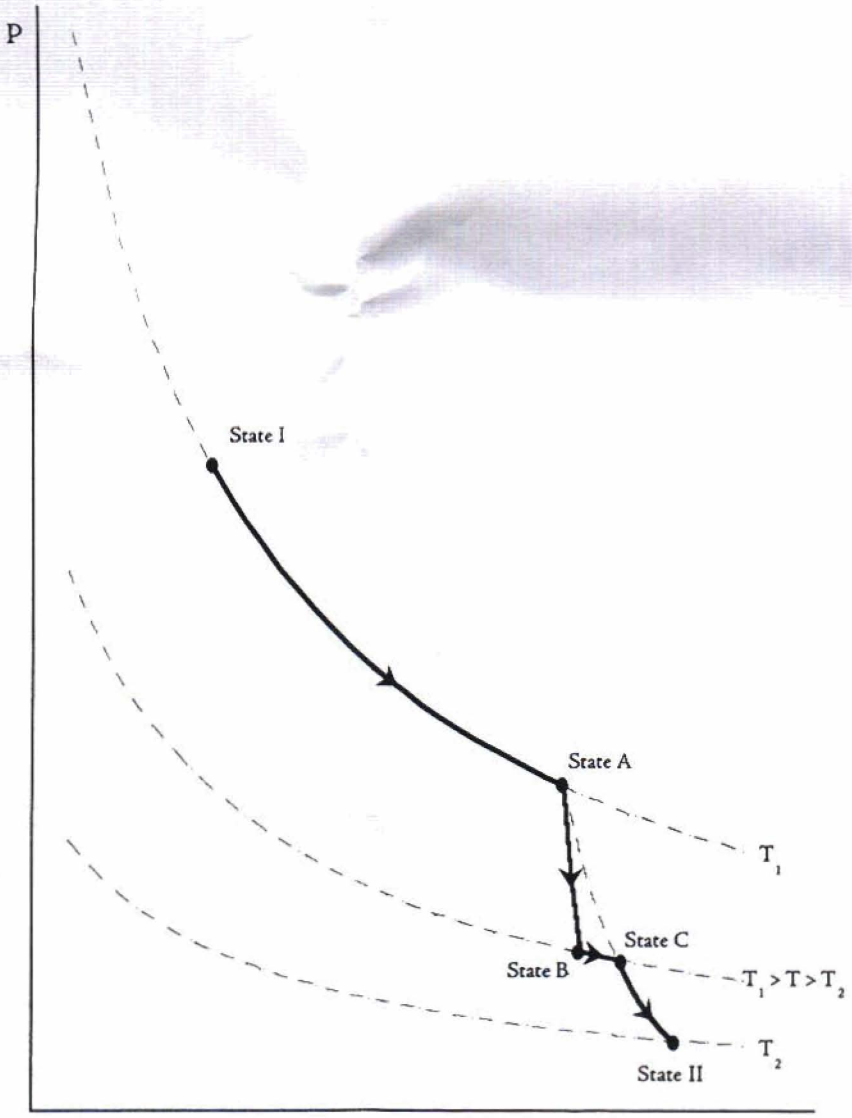
internal E = $\Delta U = 0$ (isothermal)

$q = w = RT_1 \ln\left(\frac{V_2}{V_1}\right) = RT_1 \ln(2) = \boxed{.693\, RT_1}$ = heat = work $\Big\}$ isotherm only

Now consider the process where instead of going from state A to state II *via* a reversible adiabatic expansion, the gas is taken from state A to state II *via:*

1. An expansion by a *polytropic path* given by $PV^N = \text{constant}$ from state A to state B, where state B lies on an intermediate isotherm between T_1 and T_2.
2. An *isothermal expansion* along this intermediate isotherm to state C, which lies on the adiabat that connects state A and state II.
3. A *reversible adiabatic expansion* along this adiabat to state II.

These processes are shown on the plot below:



c) (8 *points*)

If $V_C = \left(4\sqrt{2}\right)V_1$, solve for the intermediate temperature, T_C, in terms of T_1. ⓪

$\Delta U_{\text{old path}} = \Delta U_{\text{new path}}$

$nC_V(T_2 - T_1) =$

$\frac{3}{2}R\left(\frac{T_1}{4} - T_1\right) =$

$\frac{9}{8}RT_1 =$

$PV^N = c$

$\frac{T_1}{T_C} = \left(\frac{V_1}{V_C}\right)^{N-1}$? state 1 and state C ?

$$\boxed{T_C = T_1 \Big/ \left(\frac{V_1}{4\sqrt{2}V_1}\right)^{N-1}}$$

$$* \cancel{\boxed{T_C = \left(4\sqrt{2}\right)^{N-1} T_1}} *$$

d) *(5 points)*

Determine the internal energy change going from state I to state II for this process in terms of T_1 and the gas constant R. Explain your answer. ②

A→II

$$\Delta U = nC_V \Delta T$$

$$\Delta U = C_V (T_2 - T_1)$$

$$\Delta U = \frac{3}{2}R\left(\frac{T_1}{4} - T_1\right)$$

$$\Delta U = \frac{-9}{8}RT_1$$

I→A

$\Delta U = 0$ (isothermal)

$$\boxed{\Delta U_{(I\to II)} = \frac{-9}{8}RT_1}$$

state function, so same as in b)

e) (*12 points*)

If $q_{A\to B} = -2w_{A\to B}$, solve for the exponent N for the path $PV^N = \text{constant}$ going from state A to state B. ①

$$\xrightarrow{} 1(\text{mono ideal})?$$

$$q_{A\to b} = \frac{\cancel{C_P - N C_v}}{(1-N)}(T_2 - T_1) \qquad W_{A\to B} = \left.\frac{\frac{R}{T_2} - \frac{R}{T_1}}{(1-(N))}\right\} ?$$

$$\frac{-\frac{2R}{T_2} + \frac{2R}{T_1}}{1-N} = \frac{T_2 - T_1}{1-N}$$

$$\Delta U = -3w$$

$$T_2 - T_1 = \frac{2R}{T_1} - \frac{2R}{T_2}$$

$$N = 1 - -3 = \boxed{4}$$

$$T_2 + \frac{2}{T_2} = T_1 + \frac{2}{T_1}$$

$$T_2 = T_1$$

f) (*3 points*)
If the gas is expanded from state I to state II by some irreversible process, what is the internal energy change? What is the relationship between w_{rev} and w_{irr}? What is the relationship between q_{rev} and q_{irr}? Explain. ①

No, the internal energy will not change, because it is a state function. ✓

$W_{irr} \cancel{>} W_{rev}$ & $q_{irr} \cancel{>} q_{rev}$; so $q_{irr} = W_{irr}$ which allows ΔU to hold its value.

$w_{rev} = w_{max}$!

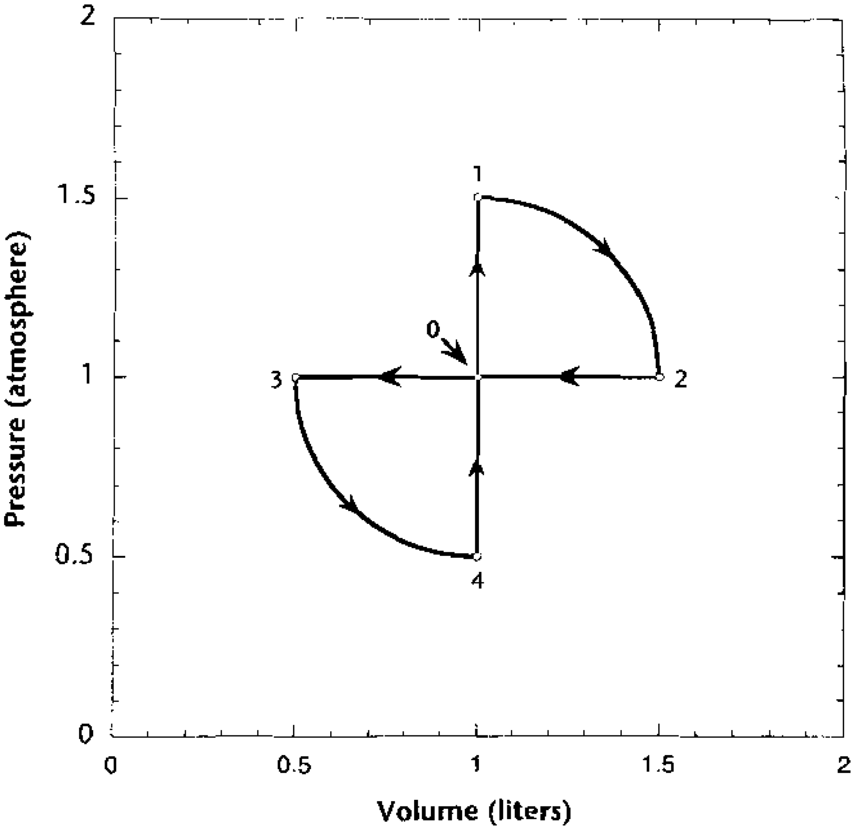
2. (*25 points*)

Stanford Junior University is deservedly famous for its vast output of nanoengineers and nanoscientists. Some argue that this has been going on much longer than the current interest in nanomaterials.

At the Bill Gates Institute for GNP (GigaNanoProgress), an enterprising Stanford Ph.D., Dr. Demetrius Witt, known as Dimi by his friends, has developed a new cyclic process, which is illustrated below.

An ideal monatomic gas fills a container with an initial volume of 1.0 liter so that the pressure is 1.0 atmosphere at 298K. This defines the point 0 on the plot, and the origin of the STANFORD CYCLE. From this starting point, the following processes are conducted.

1) A reversible isochoric increase in pressure to 1.5 atmospheres (point 1),
2) A reversible expansion along a "circular" path to point 2, during which the pressure drops back to 1 atmosphere, and the volume increases to 1.5 liters,
3) A reversible isobaric compression to a volume of 0.5 liters (point 3),
4) A reversible expansion along a "circular" path to point 4, during which the pressure drops to 0.5 atmospheres, and the volume increases to 1.0 liter, and
5) An isochoric increase in pressure that returns the system to point 0, and completes the cycle.



$P = \sqrt{(.5)^2 - V}$

$P^2 + V^2 = (.5)^2$

a) *(15 points)*

Before reducing the size of this engine to nanodimensions using the latest nanomachining technology, a Berkeley graduate suggested that the efficiency of the STANFORD CYCLE be evaluated. Using the given information, determine the net work performed by the cycle, and the efficiency of the cycle, η. Show your work, and/or explain your reasoning.

$\frac{P_0}{T_0} = \frac{P_1}{T_1} \quad T_1 = \frac{P_1}{P_0} T_0$

$0 \to 1 \; \begin{cases} W_{0\to 1} = 0 \text{ (isochoric)} \\ P_0 = 1\text{atm} \quad P_1 = 1.5\text{ATM} \quad T_0 = 298 \quad T_1 = 298\text{K} \cdot \frac{1.5\text{atm}}{1.0\text{atm}} = 447\text{K} \\ q_{0\to 1} = \frac{3}{2}R(447\text{k} - 298\text{k}) = 223.5R \end{cases}$

$1 \to 2 \; \{ W = \int_{V_1}^{V_2} \sqrt{.25 - V^2}\, dV$ (No time to integrate

$2 \to 3 \; \{ W = P\,dV = 1\text{atm}\,(.5 - 1.5) = -1$

$3 \to 4 \; \{ \; \parallel \; 1 \to 2$

$4 \to 5 \; \{ W = 0$

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$\eta = \frac{W_{Net}}{q_{IN}} = 0\%$ efficient (he goes to STANFORD?) !!

b) *(10 points)*

The temperature of the ideal gas varies continuously along the path. Points 1 and 2 lie on an isotherm, and the ***maximum*** temperature is reached along the path from 1 to 2. Points 3 and 4 also lie on an isotherm, and the ***minimum*** temperature is reached along the path from 3 to 4. The *maximum PV* product is 1.832 liter•atm, and the *minimum PV* product is 0.4179 liter•atm. Using this information, determine the efficiency of a Carnot cycle operating between these maximum and minimum temperatures.

3. *(25 points)*

One mole of solid aluminum at 600°C is placed in an adiabatic container along with two moles of solid titanium at 25°C at constant pressure. The two metals are in thermal contact (can exchange heat with one another), but as a result of being adiabatically contained can exchange no heat with the surroundings. The system reaches thermal equilibrium at some temperature T. In contrast to ideal gases that have heat capacities that are independent of temperature, the heat capacities of real gases, liquids, and solids do vary with temperature.

For solid aluminum and titanium, the molar constant pressure heat capacities (in joules/deg) are:

$$c_P(\mathrm{Al}) = 20.7 + \left(12.4\times10^{-3}\right)T$$

$$c_P(\mathrm{Ti}) = 22.1 + \left(10.5\times10^{-3}\right)T$$

a) *(15 points)*

Using this information, determine the equilibrium temperature, T.

⑮

$$1\,C_P(\mathrm{Al})\,dT = 2\,C_P(\mathrm{Ti})\,dT$$

$$\int_{T_e}^{600}(20.7 + 12.4E{-}3T)\,dT = \int_{25}^{T_e}(44.2 + 21.0E{-}3T)\,dT$$

$$20.7T + 6.2E{-}3T^2\Big|_{T_e}^{600} = 44.2T + 10.5E{-}3T^2\Big|_{25}^{T_e}$$

$$20.7(873) + 6.2E{-}3(873)^2 - 20.7T - 6.2E{-}3T_e^2 = 44.2T_e + 10.5E{-}3T_e^2 - (44.2)(2$$

$$-(10.5E{-}3)(298)^2$$

$$18071.1 + 4725.2 + 13171.6 + 932.442 = 64.9T_e + .0167T_e^2$$

$$.0167T^2 + 64.9T_e - 36900.242 = 0$$

$$\frac{-64.9}{2(.0167)} \pm \frac{\sqrt{64.9^2 - 4(.0167)(-36900.242)}}{2(.0167)} =$$

$$-1943.11 \pm 2446.48 = 503.37\,K$$

$$= \boxed{230.4°C}$$ ✓

b) *(10 points)*

Using your answer to part a), determine the entropy change of the system that accompanies this irreversible process.

⑤

If and only if you did not get an answer to part a), assume a final temperature of 250°C, and determine the entropy change of the system.

$$dS = \frac{C_v}{T}dT + \frac{R}{V}\cancel{dV}\to^{0}$$ (for a solid)

$$dS = \frac{q\,dT}{T} = C_v \ln\left(\frac{T_2}{T_1}\right)$$

process is at constant pressure so $\delta q_{rev} = C_p dT$

does not equal C_v for a solid. We discussed this in class.

$$dS = (C_p - R)\left(\ln\frac{T_2}{T_1}\right) = \frac{(C_p(Al) - R)}{T}dT + \frac{2(C_p(Ti) - R)}{T}dT$$

[illegible] - 8.3141

$$\Delta S = \int_{503.4}^{673}\left(\frac{12.386}{T} + 12.4E\text{-}3\right)dT + 2\int_{503.4}^{298}\left(\frac{3.786}{T} + 10.5E\text{-}3\right)dT$$

$$\Delta S = -6.82 \quad .588 + 2(7.217 + 2.1525)$$

$$\Delta S = -11.408 + 18.7[illegible]$$

$$\boxed{\Delta S = 7.331\,[illegible]}$$ it would be J/K for this case $n \neq 1$