Berkeley
UNIVERSITY OF CALIFORNIA

ME40: Thermodynamics

Midterm 2

April 9th, 2025

NAME:

STUDENT ID:

Problem and Point Summary:

Remarks:

- Show your work on the worksheets provided.
- Use a new sheet for each problem.
- Write your name and student ID number on each sheet.
- You are required to interpolate values from tables where necessary.
- Required accuracy for results is +-1%.
- Multiple or illegible solution attempts will not be graded.
- Unsubstantiated solution attempts will receive no points.
- To get full credit you must show your work:
    - Write down every intermediate step and result (including units).
    - For each new equation that you put, using keywords, describe what it represents and why it applies.
    - If terms in the full equation are neglected, indicate why.
    - State assumptions and idealization that you make.
    - Specify units and the table number for tabular values.
    - Include units on properties and final answer.
- The UC Berkeley honor code applies.

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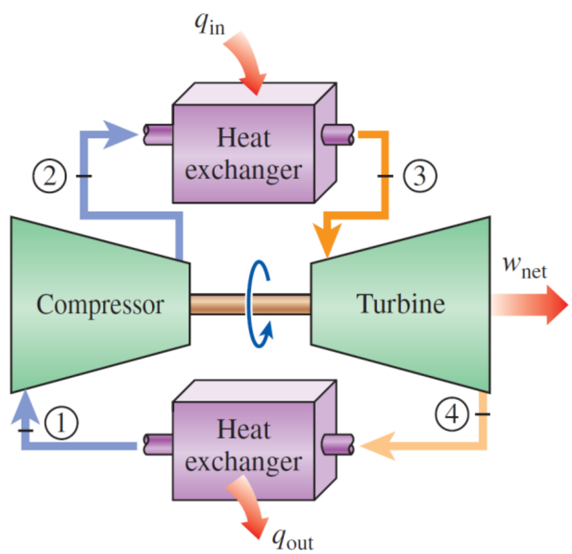
Problem A: Designing a Gas Power Plant (34 pts)

You are tasked with designing a gas power plant to provide enough power for the UC Berkeley campus (89 MW). The campus task force has identified a Brayton Cycle with air as working fluid as a promising candidate. Your colleague has come up with a first draft model of the cycle illustrated in the Figure below and described in the following. The cycle operates at steady state conditions. Air enters the compressor at $T_1 = 25°\text{C}$ and $p_1 = 120$ kPa. The compressor operates with an isentropic efficiency of $\eta_C = 0.9$ and a pressure ratio of 15. The air is then heated in the combustion chamber to $T_3 = 800°\text{C}$, and is subsequently expanded in the turbine. The turbine operates with an isentropic efficiency of $\eta_T = 0.8$. Assume the combustion chamber 2-3 is isobaric and the air exhaust is modelled as an isobaric heat removal 4-1. Describe air as an ideal gas and use the cold air assumption with constant properties at 300 K with $c_v = 0.718$ kJ/kgK, $c_p = 1.005$ kJ/kgK, $R = 0.287$ kJ/kgK, and $k = 1.4$. Neglect any changes in kinetic and potential energy.



(a) Sketch the cycle in a T-s diagram. Make the diagram large and clear; unclear process steps will not earn points. (6 pts)

(b) Determine the pressure (in kPa) and temperature (in K) for each state 1 to 4 of the cycle. (12 pts)

(c) Determine the mass specific rate of heat transfer *q* and work *w* for each of the process steps 1-2, 2-3, 3-4, 4-1. (8 pts)

(d) Find the air flow rate $\dot{m}$ needed to achieve the required net power output of $W_{net} = 89$ MW to provide sufficient power to the UC Berkeley campus. (3 pts)

(e) Determine the thermal efficiency η_{th} of the power plant. (3 pts)

(f) Compare the thermal efficiency of your Brayton cycle to an ideal Brayton cycle with the same pressure ratio. (2 pts)

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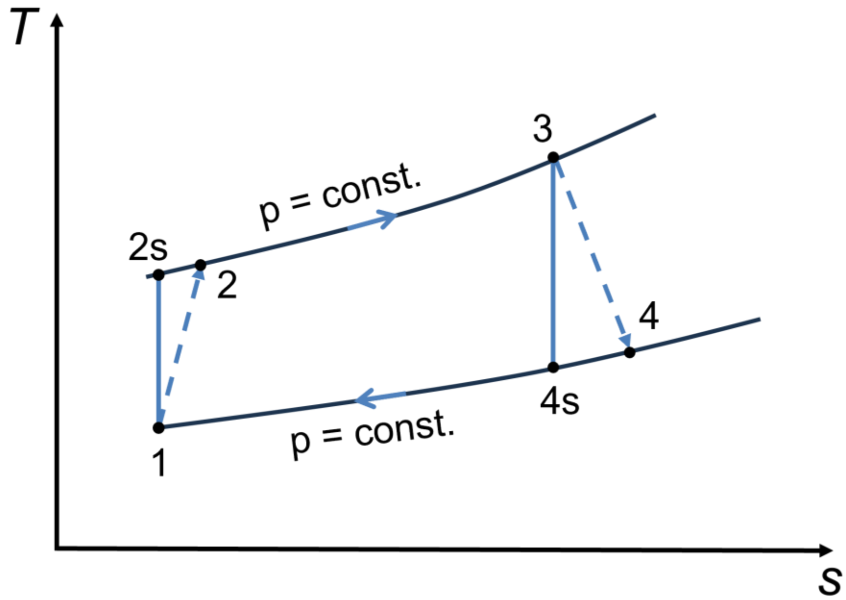
Solution:

Part (a)



Part (b)

At state 1: $\boldsymbol{p_1 = 120}$ **kPa,** $\boldsymbol{T_1 = 298}$ **K**

Process 1-2: Isentropic compression

$$\frac{p_2}{p_1} = 15$$

$$p_2 = 15p_1 = 15 \times 120 = \boldsymbol{1800\ kPa}$$

$$T_{2s} = T_1 \left(\frac{p_2}{p_1}\right)^{(k-1)/k} = 298 \times (15)^{0.4/1.4} = 646.02\ K$$

$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{c_p(T_{2s} - T_1)}{c_p(T_2 - T_1)}$$

$$T_2 = T_1 + \frac{T_{2s} - T_1}{\eta_C} = 298 + \frac{646.02 - 298}{0.9} = \boldsymbol{684.69\ K}$$

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At state 2: $\boldsymbol{p_2 = 1800}$ **kPa,** $\boldsymbol{T_2 = 684.69}$ **K**

<u>Process 3-4: Isentropic expansion</u>

$$\frac{p_3}{p_4} = 15$$

$$p_3 = 15p_4 = 15p_1 = 15 \times 120 = \mathbf{1800\ kPa}$$

$$T_{4s} = T_3\left(\frac{p_4}{p_3}\right)^{(k-1)/k} = 1073 \times \left(\frac{1}{15}\right)^{0.4/1.4} = 494.96\ K$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{c_p(T_3 - T_4)}{c_p(T_3 - T_{4s})}$$

$$T_4 = T_3 - \eta_T(T_3 - T_{4s}) = 1073 - 0.8 \times (1073 - 494.96) = \mathbf{610.57\ K}$$

At state 3: $\boldsymbol{p_3 = 1800}$ **kPa,** $\boldsymbol{T_3 = 1073}$ **K**

At state 4: $\boldsymbol{p_4 = p_1 = 120}$ **kPa,** $\boldsymbol{T_4 = 610.57}$ **K**

Part (c)

Applying the first law to each process and assuming compression and expansion processes are adiabatic**.**

<u>Process 1-2: Isentropic compression</u>

$$w_{in} = h_2 - h_1 = c_p(T_2 - T_1) = 1.005 \times (684.69 - 298)$$

$\boldsymbol{w_{in} = 388.62}$ **kJ/kg**

<u>Process 2-3: Constant pressure heat addition</u>

$$q_{in} = h_3 - h_2 = c_p(T_3 - T_2) = 1.005 \times (1073 - 684.69)$$

$\boldsymbol{q_{in} = 390.25}$ **kJ/kg**

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Process 3-4: Isentropic expansion

$$w_{out} = h_3 - h_4 = c_p(T_3 - T_4) = 1.005 \times (1073 - 610.57)$$

$$\boldsymbol{w_{out} = 464.74}\ \textbf{kJ/kg}$$

Process 4-1: Constant pressure heat rejection

$$q_{out} = h_4 - h_1 = c_p(T_4 - T_1) = 1.005 \times (610.57 - 298)$$

$$\boldsymbol{q_{out} = 314.13}\ \textbf{kJ/kg}$$

Part (d)

Net power output is given by:

$$\dot{W}_{net} = \dot{m}_{air} w_{net}$$

$$\dot{m}_{air} = \frac{\dot{W}_{net}}{w_{net}} = \frac{\dot{W}_{net}}{w_{out} - w_{in}} = \frac{89 \times 10^3}{464.74 - 388.62}$$

$$\boldsymbol{\dot{m}_{air} = 1169.2}\ \textbf{kg/s}$$

Part (e)

Thermal efficiency of the cycle is:

$$\eta_{th} = \frac{w_{net}}{q_{in}}$$

$$\eta_{th} = \frac{w_{out} - w_{in}}{q_{in}} = \frac{464.74 - 388.62}{390.25}$$

$$\boldsymbol{\eta_{th} = 0.195 = 19.5}\ \%$$

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Part (f)

Thermal efficiency of the ideal Brayton cycle with the same pressure ratio is:

$$\eta_{th,ideal} = 1 - \frac{1}{r_p^{(k-1)/k}} = 1 - \frac{1}{15^{0.4/1.4}}$$

$$\boldsymbol{\eta_{th,ideal} = 0.539 = 53.9}\ \%$$

The efficiency of the given Brayton cycle is less than the ideal Brayton cycle due to the presence of irreversibilities.

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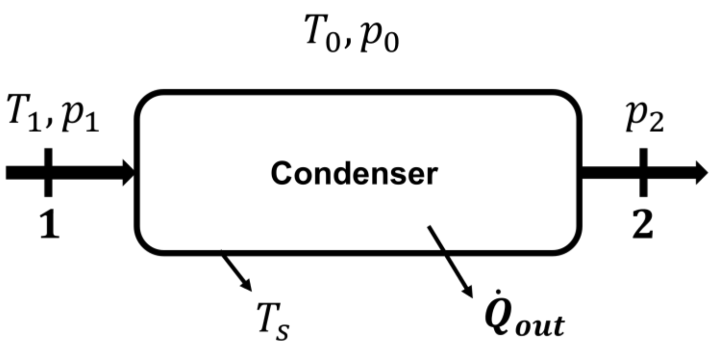
Problem B: Heat Exchanger (16 pts)

Superheated steam is condensed in a dry air heat exchanger by rejecting heat to the surrounding air at environmental conditions $T_0 = 30°\text{C}$ and $p_0 = 100$ kPa. The steam enters the condenser at $p_1 = 1200$ kPa and $T_1 = 200°\text{C}$ with a mass flow rate of 2 kg/s and exits at $p_2 = 1200$ kPa as a saturated liquid. Heat transfer occurs at the outer surface of the condenser, which is maintained at $T_s = 188°\text{C}$.



Determine

(a) The rate of heat rejected $\dot{Q}_{out}$ in the condenser. (5 pts)

(b) The exergy of the inlet stream of superheated steam. (5 pts)

(c) The rate of exergy destruction in the condenser. (5 pts)

(d) Instead of condensing the superheated steam, you would like to employ it for power generation. What is the maximum amount of useful power $\dot{W}_{max}$ you can generate from the stream? (1 pt)

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Solution:

Part (a)

State 1 (Superheated Steam at 1200 kPa and 200°C): Using table A-6

$h_1 = 2816.1\ kJ/kg, \qquad s_1 = 6.5909\ kJ/kgK$

State 2 (Saturated Liquid at 1200 kPa): Using table A-5

$h_2 = h_f = 798.33\ kJ/kg, \qquad s_2 = s_f = 2.2159\ kJ/kgK$

The rate of heat rejected in the condenser:

$\dot{Q}_{out} = \dot{m}(h_1 - h_2) = 2 \times (2816.1 - 798.33)$

$\boldsymbol{\dot{Q}_{out} = 4035.54}$ **kW**

Part (b)

The exergy per unit mass is given by: $\psi = (h - h_0) - T_0(s - s_0)$

Assuming reference state at $T_0 = 30$°C and $p_0 = 100$ kPa, using table A-4:

$h_0 = h_{f@30^oC} = 125.74\ kJ/kg, \qquad s_0 = s_{f@30^oC} = 0.4368\ kJ/kgK$

Total exergy rate at the inlet stream is:

$\dot{X}_1 = \dot{m} \times \psi_1 = \dot{m}[(h_1 - h_0) - T_0(s_1 - s_0)]$

$\dot{X}_1 = 2 \times [(2816.1 - 125.74) - 303.15 \times (6.5909 - 0.4368)]$

$\boldsymbol{\dot{X}_1 = 1649.49\ \mathrm{kW}}$

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Part (c)

The rate of exergy destruction is given by: $\dot{X}_{destroyed} = T_0\dot{S}_{gen}$

where

$$\dot{S}_{gen} = \dot{m}(s_2 - s_1) + \frac{\dot{Q}_{out}}{T_s}$$

$$\dot{S}_{gen} = 2 \times (2.2159 - 6.5909) + \frac{4035.54}{461.15} = 0.001035 \text{ kW/K}$$

Hence, the rate of exergy destruction in the condenser is:

$$\dot{X}_{destroyed} = T_0\dot{S}_{gen} = 303.15 \times 0.001035$$

$$\boldsymbol{\dot{X}_{destroyed} = 0.314\ \mathrm{kW}}$$

Part (d)

The maximum useful power that can be extracted from the steam instead of condensing it, is the exergy available in the stream at state 1:

$$\boldsymbol{\dot{W}_{max} = \dot{X}_1 = 1649.49\ \mathrm{kW}}$$