EE 120 Signals and Systems — UC Berkeley, Spring 2025
Laura Waller — Feb 27, 2025

Midterm 1

Last Name	First Name	SID

Left Neighbor Full Name	**Right Neighbor** Full Name	**Room Number**

***Rules*.**

- **Write all numerical answers clearly. Answers that are illegible may not get credit.**
- You have 110 minutes to complete the exam. DSP students with $X\,\%$ time accommodation will get $110 \cdot X\,\%$ time on the exam.
- This exam is not open book. You may reference one double-sided handwritten sheets of paper. No calculator or phones allowed.
- All problems **require** work and explanations, **including** the multiple choice problems. Write your explanations succinctly and clearly. If your work is illegible or otherwise not clear, you may get no credit for the problem.
- For multiple choice problems, select one of the choices by filling in the bubble completely.
- Collaboration with others is strictly prohibited. If you are caught cheating, you will receive a 0 on the exam and will face disciplinary consequences.
- Write in your SID on every page to receive 1 point.

Problem	points earned	out of
SID		1
Problem 1		40
Problem 2		31
Problem 3		23
Problem 4		25
Total		120

1 Potpourri (12 + 10 + 18 points)

(a) **Periodic Table of Signals (4 points each)**

Determine whether or not each of the following discrete- or continuous-time signals is periodic. If the signal is periodic, determine its fundamental frequency **in radians per second**.

(i) $x[n] = e^{\frac{i\pi n}{\sqrt{2}}}$

(ii) $x(t) = \cos^2(2\pi t)$

(iii) $x[n] = \sin\left(\frac{8}{15}\pi n\right) + \sin\left(\frac{2}{15}\pi n\right)$

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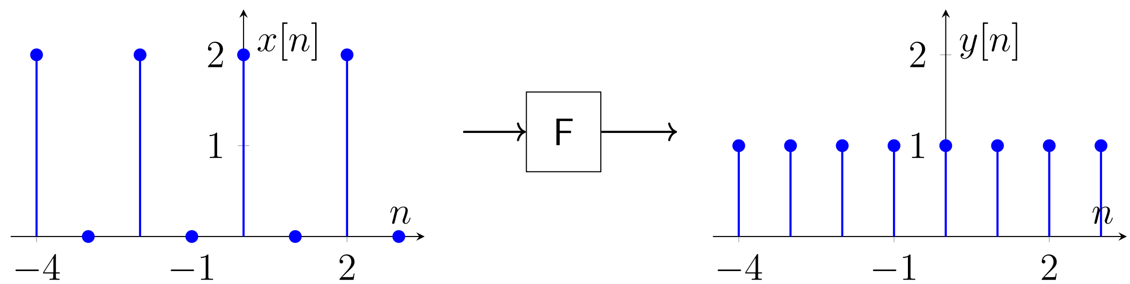
(b) **Look, That's Incredible! (5 points each)**

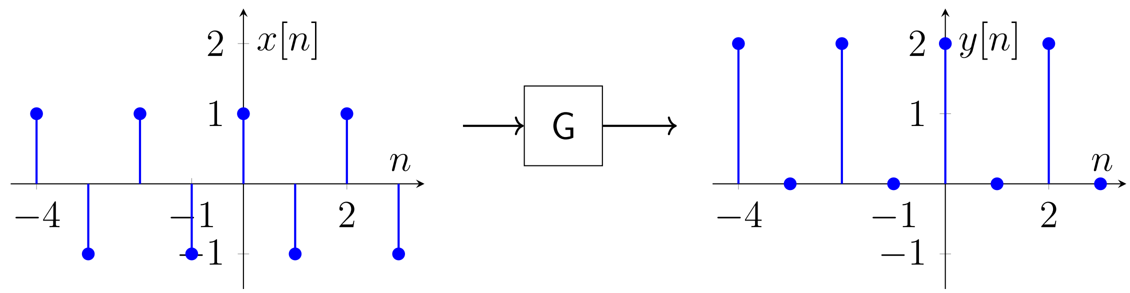
(i) Consider a discrete-time system F with the input-output pair shown below. Both the input $x[n]$ and the output $y[n]$ **repeat periodically** outside of the region shown.



Choose one of the following options and **explain your reasoning**.

- ○ The system must be LTI.
- ○ The system can be LTI, but does not have to be.
- ○ The system must not be LTI.

(ii) Consider a **different** discrete-time system G with the input-output pair shown below. Both the input $x[n]$ and the output $y[n]$ **repeat periodically** outside of the region shown.



Choose one of the following options and **explain your reasoning**.

○ The system must be LTI.

○ The system can be LTI, but does not have to be.

○ The system must not be LTI.

(c) **A Horse Walks Into a System... (6 points each)**

The following are the impulse responses or input-output equations of various discrete-time systems. Let $u[n]$ denote the unit step function. Determine whether each system is causal and/or BIBO stable. **Justify your reasoning.**

(i) Impulse response: $h[n] = u[n]$
Note: This system is LTI.

○ Causal ○ Not causal

○ BIBO stable ○ Not BIBO stable

(ii) Input-Output Equation: $y[n] = u[-n]x[n]$

○ Causal ○ Not causal

○ BIBO stable ○ Not BIBO stable

(iii) Impulse response: $h[n] = (\frac{3}{2})^n u[-n-5]$

Note: This system is LTI.

○ Causal ○ Not causal

○ BIBO stable ○ Not BIBO stable

2 Leaky Integrator (7 + 8 + 8 + 8 points)

A **causal**, discrete-time LTI system is described by the difference equation

$$y[n] \;=\; \alpha\, y[n-1] \;+\; (1-\alpha)\, x[n], \quad 0 < \alpha < 1,$$

.

(a) Find the impulse response $h[n]$ of the system.

(b) Using the impulse response from part (a), derive the output $y[n]$ of this system when the input is the signal $x[n] = c$ for $n \geq 0$ and $x[n] = 0$ for $n < 0$, **and** quantitatively explain the system's response to a constant input signal as n goes to infinity.

(c) Prove that if **any** LTI system is stable and its input $x[n]$ is periodic with period N, then its output $y[n]$ is also periodic with the same period.

(d) Recall the causal, discrete-time LTI system:

$$y[n] \;=\; \alpha\, y[n-1] \;+\; (1-\alpha)\, x[n], \quad 0 < \alpha < 1,$$

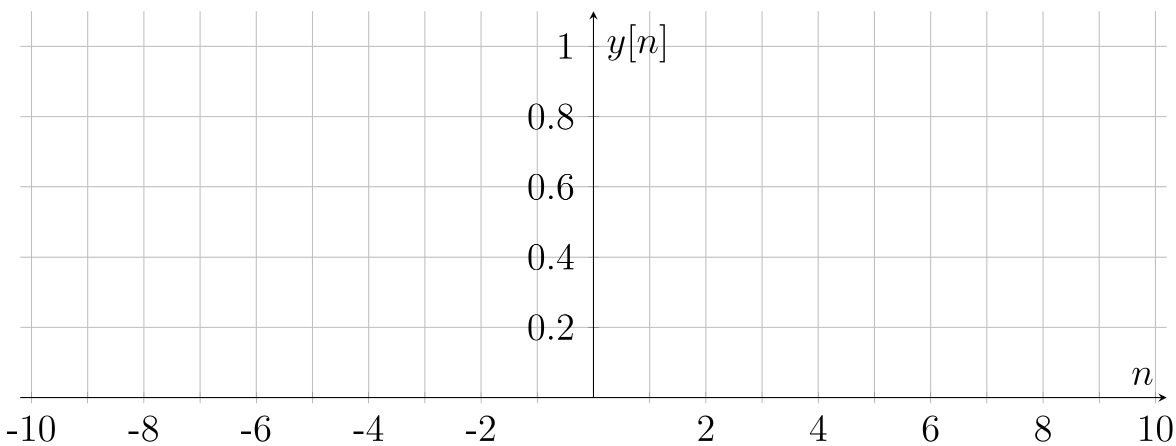
Let the input be a pulse train:

$$x[n] \;=\; \sum_{k=-\infty}^{\infty} \delta\big[n - kN\big],$$

where N is a positive integer (i.e. the period of the impulse train).

Sketch the output $y[n]$ of the original system on the plot below when $\alpha = 0.1$ and $N = 5$ for $n \in \{-10, -9, -8, \ldots, 10\}$. **Is the output a periodic signal?**

Note: Do not attempt to calculate the exact value at each time point, just qualitatively draw the plot roughly to scale.



3 Fourier's Favorite Trick (6 + 12 + 5 points)

Consider a discrete-time LTI system H with the following frequency response $H(\omega)$:

$$H(\omega) = \frac{3}{2 - 2e^{-i4\omega}}$$

(a) Find the LCCDE representation of this system.

(b) Determine the output of the system $y[n]$ when the input is $x[n] = \sin\left(\frac{\pi}{8}n\right)$. Express $y[n]$ using sine or cosine, **not** with complex exponentials.

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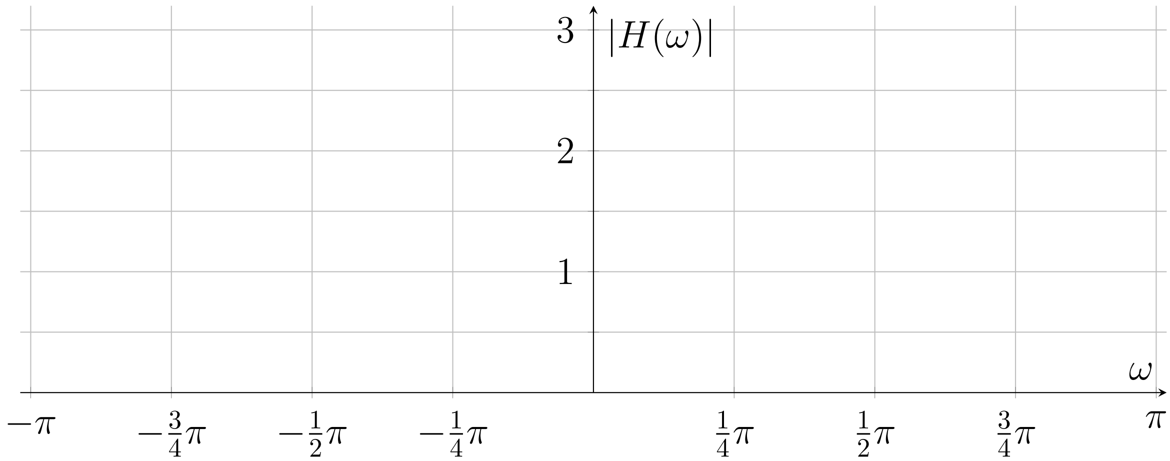
(c) Plot the magnitude response $|H(\omega)|$ of this system for $-\pi \leq \omega \leq \pi$ on the axes below.



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4 DAG Basics (6 + 14 + 5 points)

Note that each subpart are **distinct** systems and have no relation to each other.

(a) System F has the LCCDE

$$y[n] + 2y[n-1] = \frac{1}{3}x[n] - x[n-2]$$

Draw a DAG block diagram for this system with the minimum number of delay blocks.

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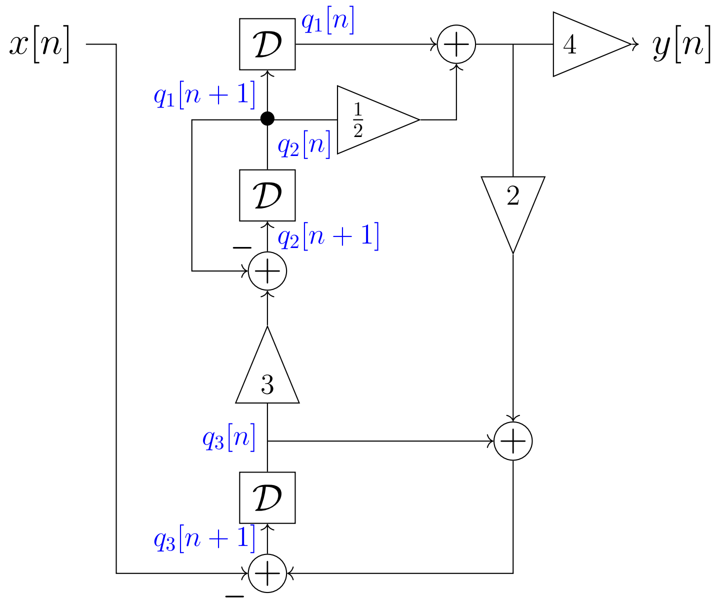
(b) System G has the following DAG block diagram.



Find the state-space representation of this system, where $\mathbf{q}[n] = \begin{bmatrix} q_1[n] & q_2[n] & q_3[n] \end{bmatrix}^\top$. Explicitly identify the values A, B, C, and D, where

$$\mathbf{q}[n+1] = A\mathbf{q}[n] + Bx[n]$$

$$y[n] = C\mathbf{q}[n] + Dx[n]$$

Note: The junction with a dot indicates all paths are connected.

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(More space for part (b))

(c) Now consider the discrete-time LTI system H with the following state-space representation:

$$\mathbf{q}[n+1] = \begin{bmatrix} 1-i & 1 \\ 0 & \frac{1}{2} \end{bmatrix} \mathbf{q}[n] + \begin{bmatrix} -1 \\ 2 \end{bmatrix} x(n)$$
$$y[n] = \begin{bmatrix} i & 1 \end{bmatrix} \mathbf{q}[n]$$

Is system H internally stable? Show your work.