MATH 1A MIDTERM 1 (PRACTICE 3)
PROFESSOR PAULIN

INSTRUCTIONS

- Do not turn over until instructed to do so.
- **Write your name and SID in the spaces provided on one side of every page of the exam.**
- This exam consists of 5 questions.
- You have 1 hours to complete this exam.
- This exam will be electronically scanned. Do not add or remove any pages from the exam.
- There is an extra blank page for scratch work on the back of the exam. It can also be used as extra space to write formal solutions as long as everything is clearly labelled.
- Calculators are not permitted.
- Show as much working as possible. Even if you don't end up with the correct answer, you may still get partial credit. Answers without justification will be viewed with suspicion and will not receive credit.
- You will find a simple formula sheet on the back of this page.

Name: _____

Student ID: _____

GSI Name: _____

Name: SID:

1. (30 points) Determine the domains of the following functions.

(a)

$$f(x) = \arccos(\frac{1}{x})$$

Solution:

1/ $x \neq 0$

and

2/ $\frac{1}{x}$ must be in domain of arccos $[-1, 1]$

$\frac{1}{x}$ in $[-1,1] \Leftrightarrow |\frac{1}{x}| \leq 1 \Leftrightarrow \frac{1}{|x|} \leq 1 \Leftrightarrow 1 \leq |x|$

$\Leftrightarrow$ x in $(-\infty, -1] \cup [1, \infty)$ ← Does not contain $x = 0$

Domain $f(x) = (-\infty, -1] \cup [1, \infty)$

(b)

$$f(x) = \sqrt{\ln(x^2 - 3)}$$

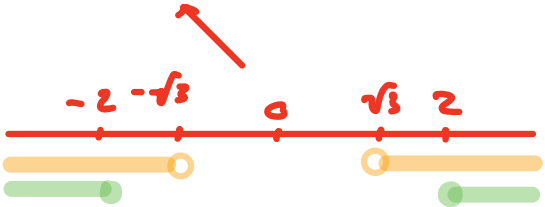
Solution:

1/ $x^2 - 3 > 0 \Leftrightarrow x^2 > 3 \Leftrightarrow |x| > \sqrt{3} \Leftrightarrow x$ in $(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

2/ $\ln(x^2 - 3) \geq 0 \Leftrightarrow x^2 - 3 \geq 1 \Leftrightarrow x^2 \geq 4 \Leftrightarrow |x| \geq 2$

$\Leftrightarrow x$ in $(-\infty, -2] \cup [2, \infty)$

$\sqrt{3} < 2 \Rightarrow$ Domain $f(x) = (-\infty, -2] \cup [2, \infty)$

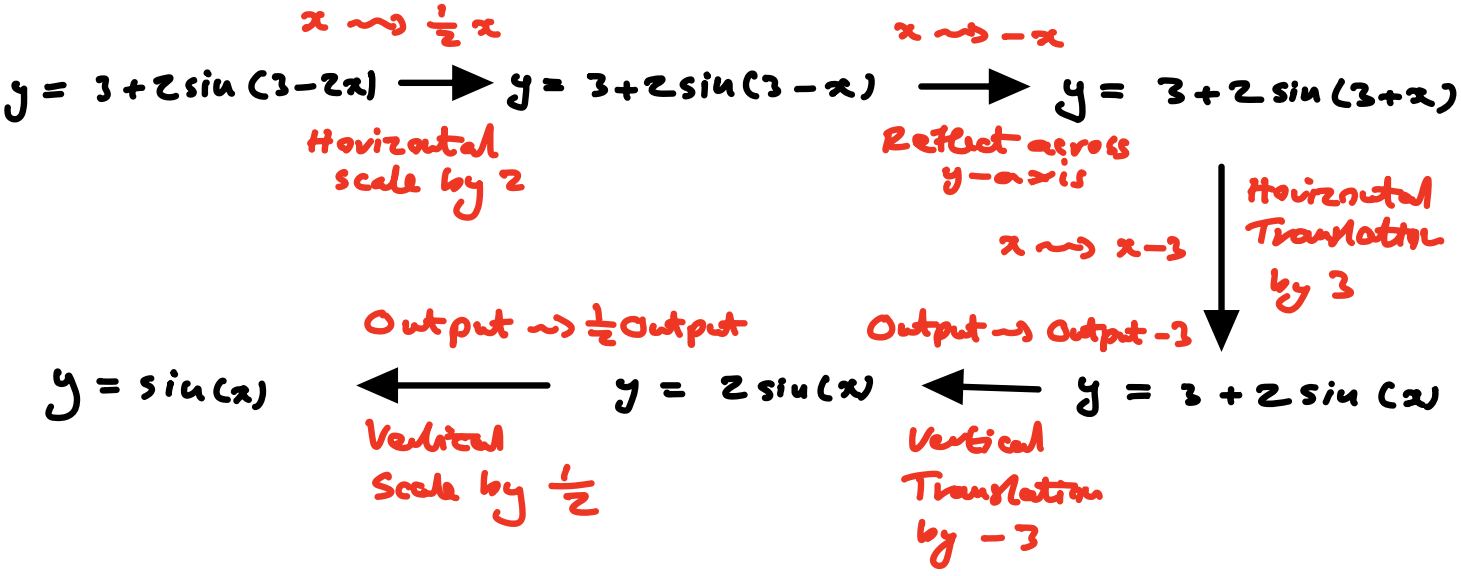


Name: SID:

2. (30 points) Describe how starting with the graph $y = 3 + 2\sin(3 - 2x)$ we can perform a sequence of transformations to draw the graph

$$y = \sin(x)$$

Solution:



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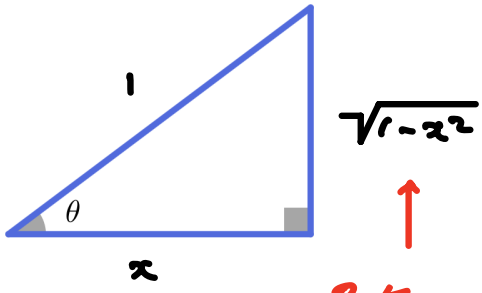
3. (30 points) Using an appropriate right-triangles express the following functions as an *algebraic* function. An algebraic function is one constructed from polynomials using basic algebraic operations and radicals.

(a)

$$f(x) = \tan(\arccos(x))$$

Solution:

$\theta = \arccos(x) \Leftrightarrow \cos(\theta) = x = \frac{x}{1}$



Pythagoras

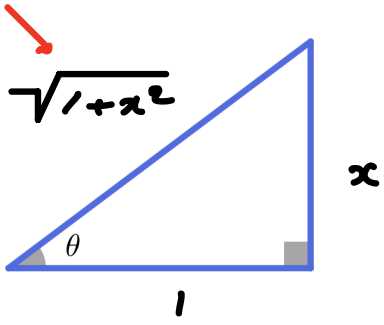
$\Rightarrow \quad \tan(\arccos(x)) = \tan(\theta) = \frac{\sqrt{1-x^2}}{x}$

(b)

$$f(x) = \sin(\arctan(x))$$

Solution:

Pythagoras



$\theta = \arctan(x) \Leftrightarrow \tan(\theta) = x = \frac{x}{1}$

$\Rightarrow \quad \sin(\arctan(x)) = \sin(\theta) = \frac{x}{\sqrt{1+x^2}}$

Name: SID:

4. (30 points) Calculate the following limits. If a limit does not exist determine if it is ∞, $-\infty$ or neither.

(a)

$$\lim_{x\to 0^+} \arctan(\ln(x))$$

Solution:

$\lim_{x\to 0^+} \ln(x) = -\infty$

$\lim_{t\to -\infty} \arctan(t) = -\frac{\pi}{2}$

$\Rightarrow \lim_{x\to 0^+} \arctan(\ln(x)) = -\frac{\pi}{2}$

(b)

$$\lim_{x\to 0} x\sin(1/x^2)$$

Solution:

If $x > 0$, $-1 \le \sin(\frac{1}{x^2}) \le 1 \Rightarrow -x \le x\sin(\frac{1}{x^2}) \le x$

Squeeze

$\lim_{x\to 0^+} -x = 0 = \lim_{x\to 0^+} x \Rightarrow \lim_{x\to 0^+} x\sin(\frac{1}{x^2}) = 0$

If $x < 0$, $-1 \le \sin(\frac{1}{x^2}) \le 1 \Rightarrow -x \ge x\sin(\frac{1}{x^2}) \ge x$

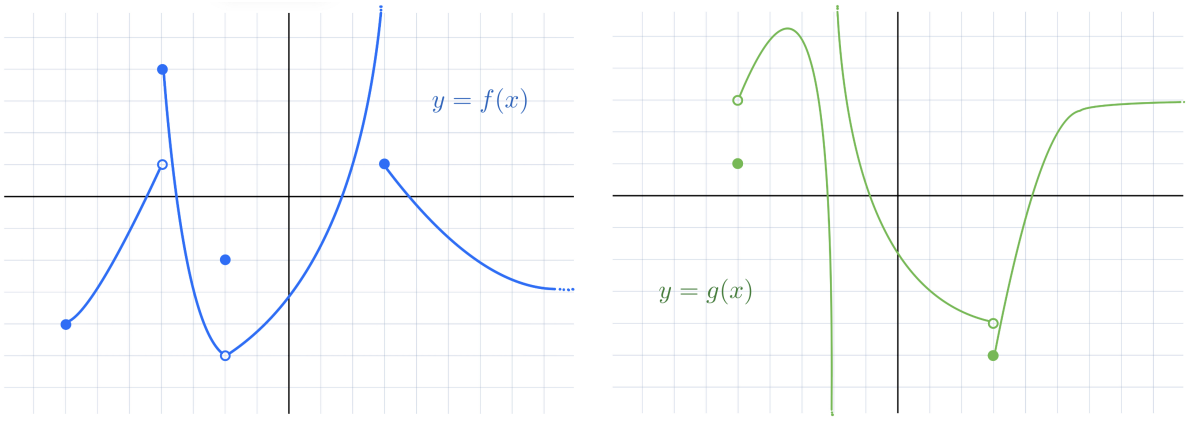
Squeeze

$\lim_{x\to 0^-} -x = 0 = \lim_{x\to 0^-} x \Rightarrow \lim_{x\to 0^-} x\sin(\frac{1}{x^2}) = 0$

Overall $\lim_{x\to 0} x\sin(\frac{1}{x^2}) = 0$

Name: SID:

5. (30 points) Imagine we have two functions with the following graphs



Using these, compute the following limits. If a limit does not exist determine if it is ∞, $-\infty$ or neither.

(a)

$$\lim_{x \to -2^+} \frac{g(x)}{f(x)}$$

$\lim_{x \to -2^+} g(x) = \infty$

$\lim_{x \to -2^+} f(x) = -5$

$\Rightarrow \lim_{x \to -2^+} \frac{g(x)}{f(x)} = -\infty$

shorthand $\frac{\infty}{(-)} = -\infty$

(b)

$$\lim_{x \to \infty} f(g(x))$$

$\lim_{x \to \infty} g(x) = 3^-$

$\lim_{t \to 3^-} f(t) = \infty$

$\Rightarrow \lim_{x \to \infty} f(g(x)) = \infty$

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