ME85 Final Examination

May 14th, 800–1000

NAME : ______________________________

SID : ______________________________

Problem 1: _____ /16 points

Problem 2: _____ /15 points

Problem 3: _____ /21 points

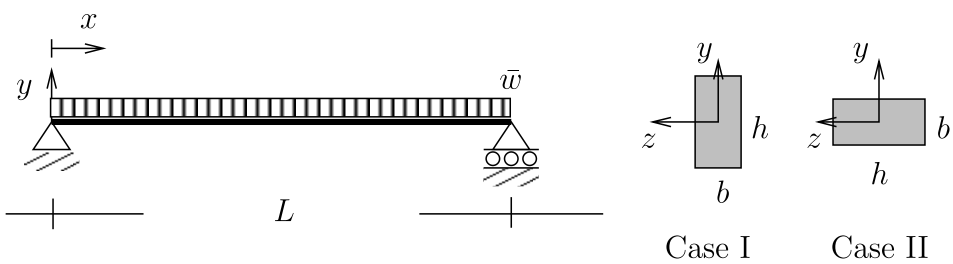
Problem 4: _____ /17 points

Notes:
1. Write your name and SID on the cover page.
2. Turn off your cell phone.
3. Record your answers only in the pages provided.
4. You may not ask questions during the exam.

With my signature below, I agree to take this examination without the help of any other individual.

Problem 1 (4+4+4+2+2 points)

Consider a simply-supported beam of length L which is subjected to a uniform load $\bar{w}$ per unit length. The beam has rectangular cross section of size $b \times h$, where $b < h$, and is placed with the longer side aligned to the y-axis (Case I) or the shorter side aligned to the y-axis (Case II), as in the figure below.



(a) Confirm that the maximum shear (in absolute value) along the beam is $V_{\max} = \bar{w}L/2$ and the maximum moment is $M_{\max} = \bar{w}L^2/8$.

(b) For Case I, find the maximum normal stress $\sigma_{\max,\mathrm{I}}$ and the maximum shear stress $\tau_{\max,\mathrm{I}}$ as a function of $\bar{w}$, L, b, and h.

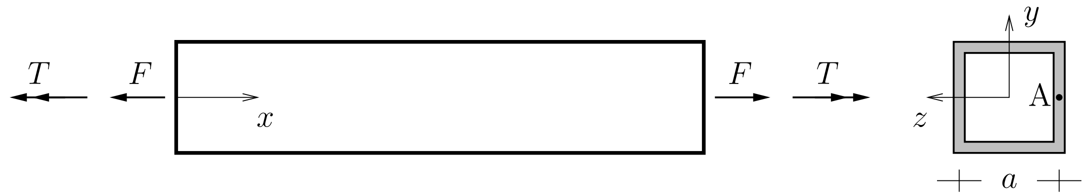
(c) For Case II, find again the maximum normal stress $\sigma_{\max,\mathrm{II}}$ and the maximum shear stress $\tau_{\max,\mathrm{II}}$ as a function of $\bar{w}$, L, b, and h.

(d) Suppose one is asked to design the beam based on normal stress only. Use the results of parts (b) and (c) to argue which of the two beam configurations (Case I or Case II) would be more advantageous.

(e) Suppose one is asked to design the beam based on shear stress only. Use again the results of parts (b) and (c) to argue which of the two beam configurations (Case I or Case II) would be more advantageous.

Problem 2 (3+5+4+3 points)

Consider a tube of thin-walled square cross-section with side a and thickness t ($t \ll a$) that is subject to a constant axial force F and torque T, as shown in the figure.



(a) Find the average tensile stress due to F.

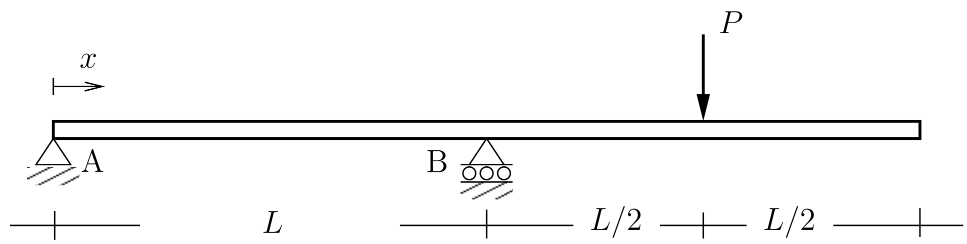
(b) Find the average shear stress due to T and show the corresponding shear flow on a sketch of the cross-section.

(c) Taking into account the stress components determined in parts (a) and (b), find the principal stresses and the angle of rotation θ_{p1}(relative to the (x, y)-axes) due to F and T at the point A of the cross-section.

(d) Find the maximum shear and the angle of rotation θ_{s1} (relative to the (x, y)-axes) due to F and T at the point A of the cross-section.

Problem 3 (3+6+6+2+4 points)

Consider a simply-supported beam with an overhang that is subject to a point load P acting on the overhang section, as in the figure below. Assume that the beam is made of a homogeneous linearly elastic material with Young's modulus E and that its uniform cross-section has second moment of area I.



(a) Find the reactions at the restrained points A and B.

(b) Draw the shear and moment diagrams.

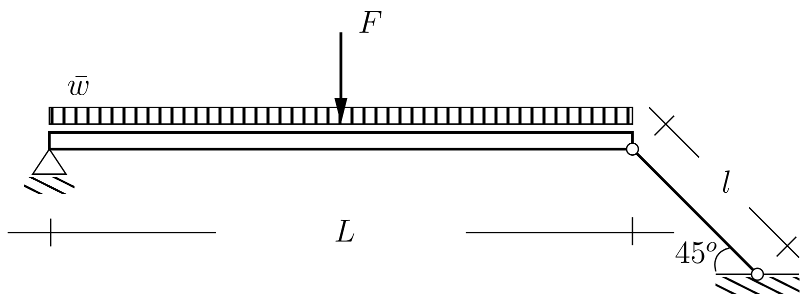
(c) Find an expression for the deflection v as a function of the position x (hint: use singularity functions!).

(d) Sketch the deformed shape of the beam.

(e) Find the points of the maximum positive (upward) and negative (downward) deflection, as well as the values of these extremum deflections.

Problem 4 (3+2+6+6 points)

Consider a rigid beam of length $L = 4$ m which is supported by a pin and a deformable bar of length $l = 1$ m, as shown in the figure. The beam is loaded by a uniform downward force $\bar{w} = 100$ N per meter and by a concentrated force F acting at its mid-point. The bar has a circular cross-section with radius $r = 1$ cm and is made of a linearly elastic material with Young's modulus $E = 10$ GPa. Also, the bar can sustain maximum allowable axial stress $\sigma_{\text{all}} = 50$ MPa.



(a) Draw the free-body diagram of the beam.

(b) Find the force acting on the bar as a function of F.

(c) Find the maximum value of the force F when designing the bar for both compression and buckling. For the latter, consider the bar to be practically simply-supported.

(d) Determine the change of length Δl of the bar and the angle θ by which the beam rotates from the horizontal direction due to the combined effect of the distributed load $\bar{w}$ and the force F found in part (c).