The formula for linear expansion is Delta L = alpha * L0 * Delta T.
It is important to recognize that the relevant Delta T is 40C-18C=22C, the production temperature: Shrinking tiles would be irrelevant for the calculation of when there is buckling.
Furthermore, two tiles expand into a gap. But also, each tile expands in two directions. So these two effects cancel, and the "effective L0" that has to be used is just the length of one tile, L0=16m.
Plugging in these values yields that the gap has to be Delta L=3.168e-3m wide.

# Midterm 1 Q2 Solution

October 10, 2024

## Q2 Solution

1. Using energy conservation,

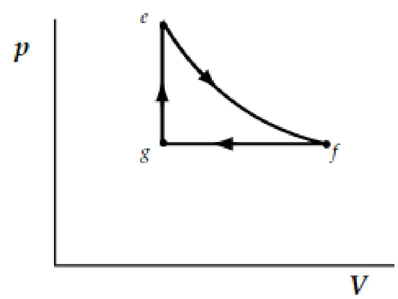
$$MgH = Q = m_{water}c\Delta T$$
$$\Delta T = \frac{MgH}{cM/6}$$
$$\Delta T = \frac{6gH}{c}$$

2. The potential energy of the mass is converted to rotational energy of the paddles. These paddles rotating in the water is then converted to the kinetic energy of the water. The increase in the kinetic energy of the water molecules correspond to an increase in temperature. The friction between the paddles and the water also dissipates some of the energy into heat. The work is being done by the gravity on the block.

3. To bring the energy back to its initial state, we need to remove the energy gained by the water during the process of part 1 $\implies |Q| = MgH$.

4. The heat put into the water in part 3 must be negative (or the heat received from the water is positive).

1. The process was the same one as in the lab assignment Let $e$ be position




NOTE: $pV$ is constant along path $ef$

   1, $f$ be position 2, and $g$ be position 3, such that, $P_1$, $V_1$, and $T_1$ refer to the pressure, volume, and temperature at $e$ or position 1, and likewise for positions 2 and 3.

2. Along the isothermal path, $T_1 = T_2$ so

$$P_1 V_1 = NKT_1 \tag{1}$$
$$P_2 V_2 = NKT_2 \tag{2}$$
$$P_2 V_2 = NKT_1 \tag{3}$$
$$P_2 V_2 = P_1 V_1 \tag{4}$$
$$P_2(3V_1) = P_1 V_1 \tag{5}$$
$$P_2 = P_1/3 \tag{6}$$

3. Since $T_1 = T_2$, $\Delta T_{1\to2} = T_2 - T_1 = T_1 - T_1 = 0$,

$$\Delta U = Q - W = U_2 - U_1 = d/2NK\Delta T_{1\to2} = d/2NK(T_2 - T_1) = 0 \tag{7}$$

4. Along the path $1 \to 2$, $T$ is a constant, $V_2 = 3V_1$,

$$W = \int PdV = \int \frac{NKT_1}{V} dV \tag{8}$$
$$= NKT_1 \int_{V_1}^{V_2} \frac{dV}{V} \tag{9}$$
$$= NKT_1 \int_{V_1}^{3V_1} \frac{dV}{V} \tag{10}$$
$$= NKT_1 \ln(3V_1/V_1) \tag{11}$$
$$= NKT_1 \ln(3) \tag{12}$$
$$= (P_1 V_1 \ln(3)) \tag{13}$$

Since the gas is expanding, it is doing work and work is positive in the $\Delta U = Q - W$ convention.

5. Since the pressure is constant from $2 \to 3$,

$$W = \int PdV = P_2 \int_{V_2}^{V_3} dV = P_2 \int_{3V_1}^{V_1} dV = P_1/3(V_1 - 3V_1) = 2/3P_1V_1 \tag{14}$$

6.

$$\Delta U = Q - W \tag{15}$$
$$\implies Q = \Delta U + W \tag{16}$$
$$Q = -d/3P_1V_1 + -2/3P_1V_1 \tag{17}$$
$$= -(d+2)/3P_1V_1 = -5/3P_1V_1 \tag{18}$$

7. From $2 \to 3$, the change in temperature can be found using the ideal gas law, where

$$P_2V_2 = NKT_2 = NKT_1 = (P_1/3)(3V_1) \tag{19}$$

and

$$P_3V_3 = NKT_3 = P_1/3V_1 \tag{20}$$

This leads to

$$\Delta U_{2\to3} = d/2NK\delta T_{2\to3} \tag{21}$$
$$= d/2\Delta(PV) = d/2(P_3V_3 - P_1V_1) \tag{22}$$
$$= d/2(P_1/3V_1 - (P_1/3)(3V_1)) \tag{23}$$
$$= -d/3P_1V_1 = -P_1V_1 \tag{24}$$

8. Since the area under the graph is zero, $W = \int PdV = 0$

9. From the first law

$$\Delta U_{3\to1} = Q - W = Q \tag{25}$$

and we can find the change in internal energy similar to part 6.

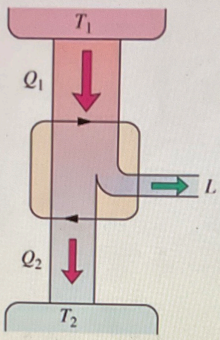
$$\begin{aligned}\Delta U_{3\to1} &= d/2\Delta(PV) &(26)\\ &= d/2(P_1V_1 - P_1/3V_1) &(27)\\ &= (d/2)(2/3)P_1V_1 &(28)\\ &= P_1V_1 &(29)\end{aligned}$$

10. Since internal energy is a state variable $\Delta U = 0$. This can be seen by $\Delta U = U_{1\to2} + U_{2\to3} + U_{3\to1} = 0 - P_1V_1 + P_1V_1 = 0$

11. $W_{tot} = W_{12} + W_{23} + W_{31} = P_1V_1\ln(3) - 2/3P_1V_1 + 0 = P_1V_1(\ln(3) - 2/3)$

# Solution:

**Problem 4 (15pts)**
The Carnot engine shown in the figure below operates between two temperature reservoirs, $T_1$=850 K and $T_2$=300 K.
For each cycle it delivers 1200 J of work.
1) (5pts) What is the efficiency of the engine?
2) (5pts) What is the heat $Q_1$ supplied by the hot reservoir during the cycle?
3) (5pts) What is the heat $Q_2$ rejected to the cold reservoir?




1) Carnot efficiency $\eta = 1 - \frac{T_c}{T_h} = 1 - \frac{300\,K}{850\,K} = \boxed{0.647}$

5 = fully correct

4 = evaluation error / algebra mistake

1 point - swapping means they have not understood that the efficience can never be negative and/ or bigger than one.

~~3~~ = swapped $T_c/T_h$

2 = didn't use $T_c$ and $T_h$

1 = didn't use $T_c$ or $T_h$

0 = no wor

★ could use $W_{out}/Q_{in}$ instead of $1 - T_c/T_h$

b) $\eta = \frac{W_{out}}{Q_{in}}$ $\quad Q_{in} = \frac{W_{out}}{\eta} = \frac{1200\,J}{0.647} = \boxed{1855\,J}$

5 = fully correct (based on a)

4 = algebra mistake / no units

~~3~~ 1 point = swapped Wout and Qin

2 = used other formula

1 = effort, but not on the right track at all

0 = no work

W = |Q1| - |Q2|

3) $0 = W + Q_1 + Q_2$ $Q_2 = -W - Q_1$

|Q2| = |Q1| - W

~~$Q_2 = 1200\text{ J} - 1855\text{ J} =$~~ $\boxed{-655\text{ J}}$

Q2 is positive as it is the heat added to the cold reservoir.

~~↑ negative sign means out~~

5 = fully correct (based on a and b)

4 = algebra mistake / no units

3 = small conceptual misunderstanding

2 = large conceptual misunderstanding

1 = effort, but not on the right track at all

0 = no work

# 7B – Lanzara Midterm 1 Question 5 Answer Key

Fall 2024

1. (2 pts) Expression for heat needed to increase temperature.

$$\Delta Q_{\text{heating}} = M c_{\text{Pb}} \Delta T \qquad \text{(constant mass)}$$

2. (2 pts) Expression for heat needed to melt metal.

$$\Delta Q_{\text{melting}} = L_f \Delta m \qquad \text{(constant temperature)}$$

Here $\Delta m$ is the amount of lead that is melted.

3. (4 pts; partial credit is given for each process) Differential expressions for entropy.

$$\delta Q = T dS; \qquad \Delta S = \int_{\text{initial}}^{\text{final}} \frac{\delta Q}{T}$$

You cannot just integrate $\int \delta Q = \Delta Q$, as $T$ may change as you are adding heat. A lot of students did this, and happened to get the correct answer for $\Delta S_{\text{melting}}$ as this process is at constant temperature. I still gave credit for $\Delta S_{\text{melting}}$ for that.

$$\delta Q_{\text{heating}} = M c_{\text{Pb}}\, dT; \qquad \Delta S_{\text{heating}} = \int_{T_0}^{T_1} \frac{M c_{\text{Pb}}\, dT}{T}$$

$$\delta Q_{\text{melting}} = L_f\, dm; \qquad \Delta S_{\text{melting}} = \int_0^M \frac{L_f\, dm}{T_1}$$

4. (4 pts; credit is still given if bounds are applied incorrectly) Integrated expressions for entropy.

$$\Delta S_{\text{heating}} = M c_{\text{Pb}} \log(T)\Big|_{T=T_0}^{T=T_1} = M c_{\text{Pb}} \log\left(\frac{T_1}{T_0}\right)$$

$$\Delta S_{\text{melting}} = \frac{L_f m}{T_1}\Big|_{m=0}^{m=M} = \frac{L_f M}{T_1}$$

5. (3 pts) Final result.

$$T_0 = \frac{T_1}{2}; \qquad \Delta S = M\left(c_{\text{Pb}} \log(2) + \frac{L_f}{T_1}\right)$$