Draft

EECS 20N: Structure and Interpretation of Signals and Systems MIDTERM 1
Department of Electrical Engineering and Computer Sciences 5 October 2005
UNIVERSITY OF CALIFORNIA BERKELEY

LAST Name ______________ **FIRST Name** ______________

Lab Time ______________

- **(10 Points) Print your name and lab time in legible, block lettering in the appropriate spaces provided above AND on the back of the last page.**
- **This midterm should take you up to 80 minutes to complete. You will be given at least 80 minutes—up to a maximum of 90 minutes—to work on the midterm.**
- Aside from a two-sided 8.5×11-inch formula sheet for your exclusive use, you may not access, or cause to be used or accessed, any reference in print or electronic form at any time during the quiz. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct.
- **The midterm printout consists of pages numbered 1 through 10.** When you are prompted by the teaching staff to begin work, verify that your copy of the midterm is free of printing anomalies and contains all of the ten numbered pages. If you find a defect in your copy, notify the staff immediately.
- *Please write neatly and legibly, because if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in evaluating your midterm. No exceptions.*
- NO PROBLEM ON THIS MIDTERM REQUIRES AN EXPLANATION. Feel free to use for scratch work the blank spaces provided herein. Note, however, that only your final answers are looked at in evaluating your work.
- We hope you wrestle and beat this midterm to the ground!

Problem 1 (42 Points) Consider the set

$$X = [\{a, b, c\} \to \{1, 2, 3\}] .$$

(a) (14 Points) How many elements are in the set X? Explicitly specify one element in X.

$3^3 = 27$

$f \in X$ s.t. $f(a) = 1$, $f(b) = 1$, $f(c) = 1$

(b) (14 Points) Of the elements in X, how many define, respectively, a graph of an "onto" function $f : \{a, b, c\} \to \{1, 2, 3\}$? Explicitly specify one element in X that defines a graph of an onto function.

Every element in $\{1,2,3\}$ must be mapped to. This can only happen if a, b, c are mapped to distinct elements in $\{1,2,3\}$. This means there are 3 choices for one element in $\{a,b,c\}$, two choices for another one, and one choice for the last one $\therefore$ a total of six choices. Example: $f \in X$, s.t. $f(a)=1$, $f(b)=2$, $f(c)=3$. This function is onto.

(c) (14 Points) Of the elements in X, how many define, respectively, a graph of a one-to-one function $f : \{a, b, c\} \to \{1, 2, 3\}$? Explicitly specify one element in X that defines a graph of a one-to-one function.

For $f \in X$ to be one-to-one, each element in $\{a,b,c\}$ must be mapped to a distinct element in $\{1,2,3\}$. This coincides with the set of onto functions $\Rightarrow$ 6 one-to-one fctns can be define on X. Ex: $f(a)=1$, $f(b)=2$, $f(c)=3$ where $f \in X$.

Problem 2 (20 Points) Consider two assertions P and Q. We know that the following compound assertion is true:

$$(P \Rightarrow Q) \wedge (P \Rightarrow \neg Q)$$

We do not know whether Q is true; it may or may not be true.

Choose the strongest statement below that is most accurately reflective of the truth or falsehood of P. Circle your choice.

(I) P must be true.

(II) P could be true, but does not have to be true.

(III) P must be false.

Feel free to use the blank space below for scratch work. There is no partial credit here, so your work below will not be considered in grading this problem. Only the choice you circle will be considered.

If Q is true $\Longrightarrow$ $\neg Q$ is false

If Q is false $\Longrightarrow$ $\neg Q$ is true

We're told $(P \Rightarrow Q) \wedge (P \Rightarrow \neg Q)$ is true.

Hence, each of $P \Rightarrow Q$ and $P \Rightarrow \neg Q$ must be true:

Case 1: Q is true, $\neg Q$ is false. The only way for $P \Rightarrow \neg Q$ to be true is if P is false.

Case 2: Q is false, $\neg Q$ is true. The only way for $P \Rightarrow Q$ to be true is if P is false.

Hence, P must be false.

Note: This problem highlights the basis of "proof by contradiction." That is, if a premise P lead to two contradictory conclusions, then it must be false.

Problem 3 (14 Points) Consider the following system F:

$$F : [\mathbb{R} \to \mathbb{R}] \to [\mathbb{R} \to \mathbb{R}] \text{ such that } \forall x \in [\mathbb{R} \to \mathbb{R}] \text{ and } \forall t \in \mathbb{R},$$

$$(F(x))(t) = (t+1)x(t)\ ,$$

where $\mathbb{R}$ denotes the set of real numbers.

Choose the strongest statement below that most accurately reflects one or more properties of the system F.

(I) The system F is memoryless and causal.

(II) The system F is memoryless, but it is not causal.

(III) The system F is not memoryless, but it is causal.

(IV) The system F is neither memoryless nor causal.

There is no partial credit here. Accordingly, no explanation will be considered in grading your selection among the statements above.

- F is causal b/c it looks only at $x(t)$ to determine $y(t)$. Alternatively, if two input signals x_1 and x_2 are identical up to an arbitrary time t_0, i.e., if $x_1(t) = x_2(t)$ $\forall t \leq t_0$, then their corresponding outputs $(F(x_1))(t) = (t+1)x_1(t)$ and $(F(x_2))(t) = (t+1)x_2(t)$ must be equal $\forall t \leq t_0$. This clearly is true: simply replace $x_1(t)$ with $x_2(t)$ in the right-hand side of $(F(x_1))(t) = (t+1)x_1(t)$, for example.
- F is not memoryless, however, because of the presence of "$(t+1)$" in the expression for $(F(x))(t)$. For example, consider an input signal x such that $x(t_1) = x(t_2)$ for some $t_1 \neq t_2$, $t_1, t_2 \in \mathbb{R}$.
$(F(x))(t_1) = (t_1+1)\,x(t_1) = (t_1+1)\,x(t_2) \neq (t_2+1)\,x(t_2)$. This violates a necessary condition for memorylessness.

Problem 4 (14 Points) Determine whether the following statement is true or false:

$$\{g | g = \text{graph}(f) \wedge f : X \to Y\} \subset P(X \times Y),$$

where $P(X \times Y)$ denotes the power set of $X \times Y$.

Circle your answer: True False

You will receive 0 credit for an incorrect choice, 8 points for no choice (i.e., for not answering the problem), and 14 points (full credit) for a correct choice. This is to discourage selection based on a coin toss!

Each element in $\{g | g = \text{graph}(f) \wedge f: X \to Y\}$ is of the form: $\{(x,y) | x \in X \wedge y \in Y \wedge y = f(x)\}$

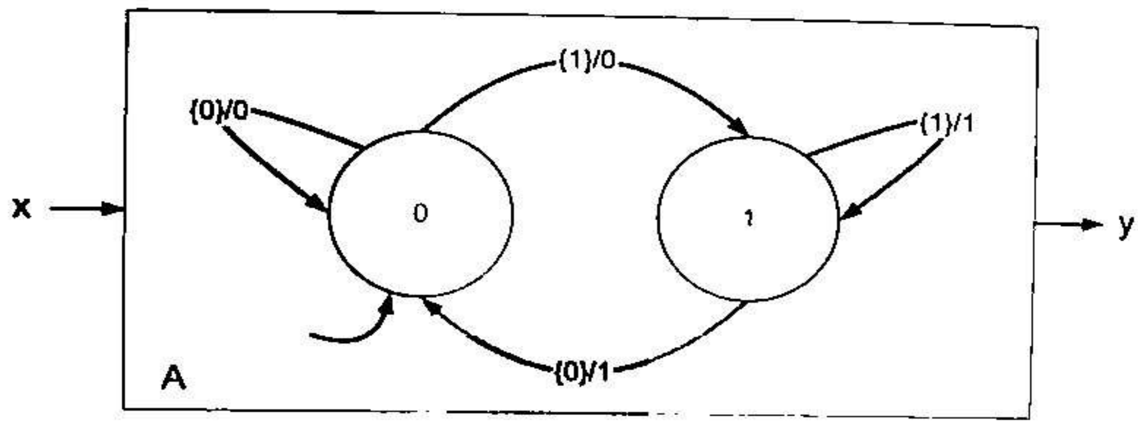
The power set of $X \times Y$, i.e., $P(X \times Y)$, contains elements each of which is a set of ordered pairs. Each set of ordered pairs in $P(X \times Y)$ defines a graph of a relation in $X \times Y$. Some elements within $P(X \times Y)$, however, define graphs of functions $f: X \to Y$.

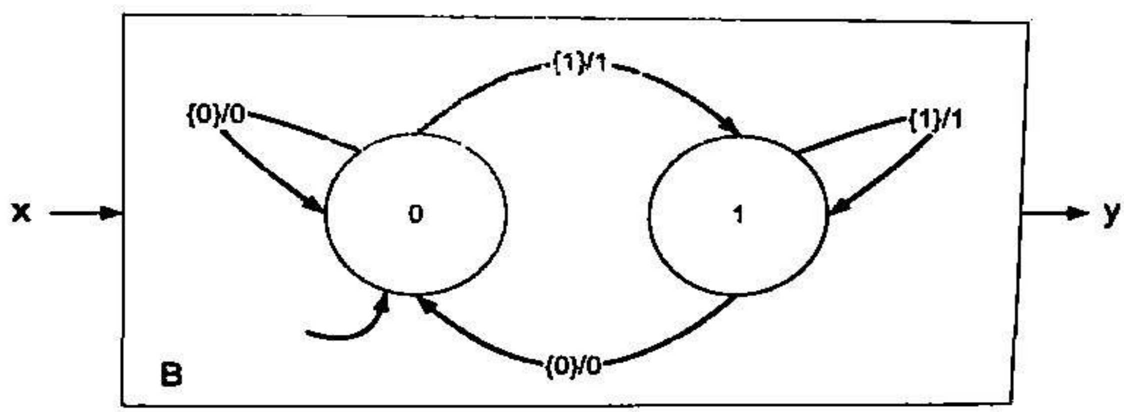
Therefore, the assertion is true.

Note: This problem is a variant of Exercise 3 (b) at the end of Chapter 2 of the textbook, wherein you noted that $\{g | g = \text{graph}(f) \wedge f: X \to Y\} \subset X \times Y$ is false.

Replacing the right-hand side $X \times Y$ with $P(X \times Y)$ makes the difference here.

Problem 5 (30 Points) Consider two finite-state machines A and B having input and output alphabets $I_A = O_A = I_B = O_B = \{0, 1, \text{absent}\}$ and further characterized, respectively, by the state-transition diagrams shown below.





The input and output signal spaces for each machine are identical, i.e., $x, y \in [\mathbb{N}_0 \to \{0, 1, \text{absent}\}]$, where $\mathbb{N}_0 = \{0, 1, 2, \ldots\}$.

(a) (15 Points) Which one of the following four statements is true? Circle your choice.

(I) A is not memoryless, and B is not memoryless.

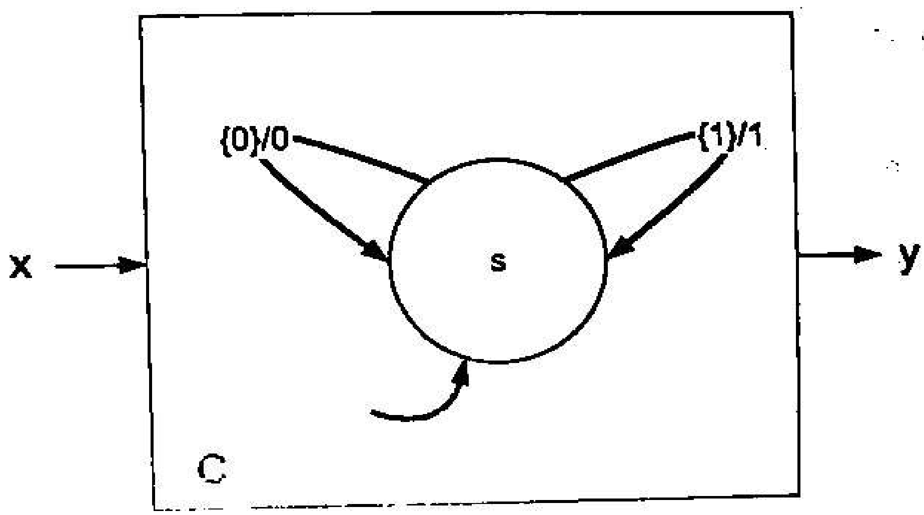
(II) A is not memoryless, but B is memoryless.

A is a unit delay system. B is the identity system.

(III) A is memoryless, but B is not memoryless.

(IV) A is memoryless, and B is memoryless.

(b) (15 Points) Consider a finite-state machine C having input and output alphabets given by $\mathsf{I}_C = \mathsf{O}_C = \{0, 1, \text{absent}\}$. The state-transition diagram for C is shown below:



Which one of the following four statements is true? Circle your choice.

(I) C is bisimilar to A and is bisimilar to B.

(II) C is bisimilar to A but is not bisimilar to B.

(III) C is not bisimilar to A but is bisimilar to B.

(IV) C is bisimilar neither to A nor to B.

C is the identity system, exactly like B.

Problem 6 (20 Points) Consider a system with an input signal space $X = [\mathbb{N}_0 \to \{0, 1, 2, 3\}]$ and an output signal space Y identical to the input signal space.

For any input signal $x \in X$, the system produces an output signal $y \in Y$ as follows:

$$\forall n \in \mathbb{N}_0, y(n) = \max(x(n), x(n-1)) .$$

That is, the output at instant n is the larger of the input signal instant at n and at the prior instant $n - 1$.

The following skeleton finite-state machine, if properly defined, implements the system. Define the finite-state machine by labeling all the salient features, including, for example, drawing the appropriate arcs, specifying all state transition parameters (guards and outputs on the arcs, including on self-loops, if any), etc.

For this problem, ignore stuttering.