00315

Name: _

ID #: _

1. (20 Points) Row reduce

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & 6 & 16 \end{bmatrix}$$

into a row echelon form where the top right entry of the matrix is 0.

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2. (20 Points) Solve the boundary value problem

$$y'' - 4y' + 5y = 0$$

$y(0) = 1,\ y\left(\dfrac{\pi}{2}\right) = 3.$

3. (20 Points) Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 4 & -1 & 3 \end{bmatrix}$$

(a) Find a basis for the column space of A.

(b) Find a basis for the null space of A.

4. (20 Points) Take $\mathcal{S}$ the vector space of 3×3 matrices over $\mathbb{R}$ under the usual matrix addition (this is well known to be a vector space, you can assume that).

(a) Consider $\mathcal{T}$, the subset of $\mathcal{S}$ where every diagonal entry is 0 (i.e. a matrix is in $\mathcal{T}$ only if all of its diagonal entries are 0). Prove that $\mathcal{T}$ is a subspace of $\mathcal{S}$.

(b) Determine the dimension of the subspace $\mathcal{T}$.

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5. (20 Points) Diagonalize the matrix

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

over $\mathbb{C}$, i.e. write it in the form PDP^{-1} where P and D are 2×2 matrices in $\mathbb{C}$ with D a diagonal matrix. You should find all three of P, D, and P^{-1}.

6. (20 Points) Consider W the subspace of $\mathbb{R}^4$ spanned by

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \right\}$$

(a) Find an orthogonal basis for W.

(b) Find the point in W which is closest to $\begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \end{bmatrix}$

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7. (20 Points) Find the general solution to

$$y'' - 2y' + y = \frac{e^t \ln t}{t}$$

using variation of paramters.

8. **(30 Points, True/False)**

3 points for each correct answer
1.5 points for blank T/F
0 points for incorrect

A statement is "True" if it is true in ALL circumstances of the stated type! Otherwise it is False. Be sure to bubble only one circle, bubbling both will give you 0 points, and if it is unclear, you will also get 0 points - no stray markings!

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(1) For a real square matrix A the singular values of A are the square roots of the eigenvalues of A.

(2) For A any real $m \times n$ matrix with $m > n$, for any singular value decomposition of A into $U \sum V^T$, the matrix $\sum$ has a row of all 0s.

(3) For b, c any real constants, there are infinitely many choices for for them for which the differential equation $y'' + by' + cy = 0$ will have $y = e^t \sin t$ as a solution.

(4) If A and B are two real matrices of the same size, all of whose entries are the same, aside from one differing entry, then A and B are never row equivalent.

(5) In a real inner product space if two nonzero vectors u and v are orthogonal, then the two vectors $2u + v$ and $u + 2v$ are never orthogonal.

8. **(Continued)**

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(6) If we have a 3×3 homogeneous linear system of real differential equations with constant coefficients, there is always a solution where for some fixed real number r, each function is of the type Ce^{rt}.

(7) If A is a real matrix with a non-real eigenvalue, then A^T also has a non-real eigenvalue.

(8) We can find a particular solution to the differential equation $y'' + y' + y = \sin^9 t$ using undetermined coefficients.

(9) Two real matrices A and B of the same size with different reduced row echelon forms have different column spaces.

(10) For W a subspace of $\mathbb{R}^n$, if some point v in $\mathbb{R}^n$ has w as the point it is closest to in W, then the closest point in W to $v + w$ must be $2w$.

9. (Bonus, 10 Points) Find the Fourier sin series on $[0, \pi]$ for

$$f(x) = x \sin x$$

expressing your answer in $\sum$ notation.