Name: _

ID #: _

1. (20 Points) Row reduce

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & 6 & 16 \end{bmatrix}$$

into a row echelon form where the top right entry of the matrix is 0.

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2. (20 Points) Solve the boundary value problem

$$y'' - 4y' + 5y = 0$$

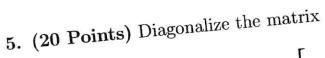
$$y(0) = 1, y\left(\frac{\pi}{2}\right) = 3.$$

3. (20 Points) Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 4 & -1 & 3 \end{bmatrix}$$

- (a) Find a basis for the column space of A.
- (b) Find a basis for the null space of A.

- 4. (20 Points) Take S the vector space of 3×3 matrices over \mathbb{R} under the usual matrix addition (this is well known to be a vector space, you can assume that).
- (a) Consider \mathcal{T} , the subset of \mathcal{S} where every diagonal entry is 0 (i.e. a matrix is in \mathcal{T} only if all of its diagonal entries are 0). Prove that \mathcal{T} is a subspace of \mathcal{S} .
- (b) Determine the dimension of the subspace \mathcal{T} .



$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

over \mathbb{C} , i.e. write it in the form PDP^{-1} where P and D are 2×2 matrices in \mathbb{C} with D a diagonal matrix. You should find all three of P, D, and P^{-1} .

6. (20 Points) Consider W the subspace of \mathbb{R}^4 spanned by

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \right\}$$

(a) Find an orthogonal basis for W.

(b) Find the point in W which is closest to $\begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \end{bmatrix}$

7. (20 Points) Find the general solution to

$$y'' - 2y' + y = \frac{e^t \ln t}{t}$$

using variation of paramters.

8. (30 Points, True/False)

- 3 points for each correct answer
- 1.5 points for blank T/F
- 0 points for incorrect

A statement is "True" if it is true in ALL circumstances of the stated type! Otherwise it is False. Be sure to bubble only one circle, bubbling both will give you 0 points, and if it is unclear, you will also get 0 points - no stray markings!

- (1) For a real square matrix A the singular values of A are the square roots of the eigenvalues of A.
- (2) For A any real $m \times n$ matrix with m > n, for any singular value decomposition of A into $U \sum V^T$, the matrix \sum has a row of all 0s.
- (3) For b, c any real constants, there are infinitely many choices for for them for which the differential equation y'' + by' + cy = 0 will have $y = e^t \sin t$ as a solution.
- (4) If A and B are two real matrices of the same size, all of whose entries are the same, aside from one differing entry, then A and B are never row equivalent.
- (5) In a real inner product space if two nonzero vectors u and v are orthogonal, then the two vectors 2u + v and u + 2v are never orthogonal.

8. (Continued)

- (6) If we have a 3×3 homogeneous linear system of real differential equations with constant coefficients, there is always a solution where for some fixed real number r, each function is of the type Ce^{rt} .
- (7) If A is a real matrix with a non-real eigenvalue, then A^T also has a non-real eigenvalue.
- (8) We can find a particular solution to the differential equation $y'' + y' + y = \sin^9 t$ using undetermined coefficients.
- (9) Two real matrices A and B of the same size with different reduced row echelon forms have different column spaces.
- (10) For W a subspace of \mathbb{R}^n , if some point v in \mathbb{R}^n has w as the point it is closest to in W, then the closest point in W to v + w must be 2w.

9. (Bonus, 10 Points) Find the Fourier sin series on $[0,\pi]$ for

$$f(x) = x \sin x$$

expressing your answer in \sum notation.