EECS 20N: Structure and Interpretation of Signals and Systems MIDTERM 1
Department of Electrical Engineering and Computer Sciences 26 September 2006
UNIVERSITY OF CALIFORNIA BERKELEY

LAST Name Instructor FIRST Name Complexified

Lab Time Midnight !

- **(10 Points)** Print your name and lab time in legible, block lettering above AND on the last page where the grading table appears.
- This exam should take up to 70 minutes to complete. You will be given at least 70 minutes, up to a maximum of 80 minutes, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except one double-sided 8.5" × 11" sheets of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The exam printout consists of pages numbered 1 through 8.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the eight numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

You may use this page for scratch work only.
Without exception, subject matter on this page will *not* be graded.

MT1.1 (25 Points) Consider a function f defined as follows:

$$f : \mathbb{R} \to \mathbb{C}$$
$$\forall t \in \mathbb{R}, \quad f(t) = (-1)^{i\,|t|}.$$

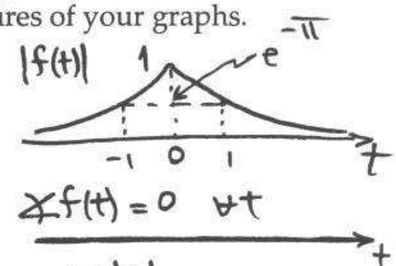
Each of the following parts can be solved independently of the other.

(a) Determine an expression for, and provide a clear sketch of the graphs of, $|f(t)|$ and $\angle f(t)$, the magnitude and angle, respectively, of function f, where $f(t) = |f(t)|\, e^{i\angle f(t)}$. Be sure to label all the salient features of your graphs.

$-1 = e^{i\pi} \Rightarrow (-1)^{i|t|} = (e^{i\pi})^{i|t|} = e^{-\pi|t|}$

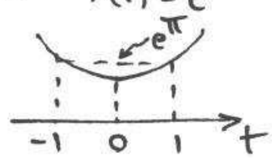
$\Rightarrow f(t) = e^{-\pi|t|} > 0 \Rightarrow |f(t)| = f(t) \quad \forall t$

$\measuredangle f(t) = 0$



If you let $-1 = e^{-i\pi}$, then you'd get $f(t) = e^{\pi|t|} > 0 \Rightarrow$

$|f(t)| = f(t)$, $\measuredangle f(t) = 0 \quad \forall t$



(b) Let f_e and f_o denote the even and odd components of f, respectively, where, $\forall t \in \mathbb{R}$,

$$f(t) = f_e(t) + f_o(t), \quad f_e(t) = \frac{f(t) + f(-t)}{2}, \quad \text{and} \quad f_o(t) = \frac{f(t) - f(-t)}{2}.$$

Determine f_e and f_o. You may do this by showing how each of the components is related to f, or providing the graph of each component f_e and f_o.

Noting that $f(t) = f(-t)$, we conclude that $f_e(t) = f(t)$, $\forall t$ and $f_o(t) = 0 \quad \forall t$

MT1.2 (30 Points) You can tackle the two parts of this problem independently. Explain your responses succinctly, but clearly and convincingly.

(a) Albert attends the concert *only if* Sally attends the concert. If Blake attends the concert, then Sally does *not* attend the concert.

Albert is attending the concert. Is Blake attending the concert? NO!

Let A: Albert attends   S: Sally attends   B: Blake attends

We have: $A \Rightarrow S$ (Albert attends only if Sally attends)

$B \Rightarrow \neg S$ (If Blake attends, then Sally does not atend)

The second assertion is equivalent to $S \Rightarrow \neg B$. Clearly, $A \Rightarrow S \Rightarrow \neg B$

(b) Determine whether the following argument is valid.

> **Monday, 2 October 2006:** If there is no news today of a looming economic depression, nor any revelation of a political scandal in the executive branch, the prime minister will complete the remaining portion of her term in office, *and* the parliament will pass her education overhaul bill into law at the end of this week.
>
> **Tuesday, 3 October 2006:** The prime minister announced her resignation at 8am today.
>
> Therefore, her education overhaul bill will never be passed by the parliament.

Let D: there's news today of economic depression

S: there's revelation today of a political scandal

C: the PM will complete her term in office

E: the Parliament will pass the PM's education bill by week's end.

We're told: $(\neg D \wedge \neg S) \Rightarrow C$, $\Rightarrow E$ (i.e., $\Rightarrow C \wedge E$)

$$\neg C \Rightarrow \neg(\neg D \wedge \neg S) \not\Rightarrow \neg E$$

We cannot use negation 4 of the premise to negate the conclusion.

The argument is invalid.

Aside: Note that $\neg D \wedge \neg S = \neg(D \vee S)$

so $\neg C \Rightarrow D \vee S$

MT1.3 (25 Points) Consider a function G defined as follows:

$$G : \mathbb{R} \to \mathbb{C}$$
$$\forall \omega \in \mathbb{R}, \quad G(\omega) = \frac{1}{1 + i\dfrac{\omega}{\omega_0}},$$

where ω_0 has a fixed positive real value.

Determine an expression for, and provide a clear sketch of the graphs of, the magnitude $|G(\omega)|$ and angle $\angle G(\omega)$ of function G, where $G(\omega) = |G(\omega)|\, e^{i\angle G(\omega)}$.
For what value(s) of ω does the function $|G(\omega)|$ attain a maximum? What is the value of $|G(\omega_0)|$? What are the values of $\angle G(\omega)$ for $\omega = -\omega_0, 0, +\omega_0$? Determine the limits:

$$\lim_{\omega\to-\infty} |G(\omega)|, \quad \lim_{\omega\to+\infty} |G(\omega)|, \quad \lim_{\omega\to-\infty} \angle G(\omega), \quad \text{and} \quad \lim_{\omega\to+\infty} \angle G(\omega).$$

You may express your answers to these questions by placing appropriate labels on your sketches.

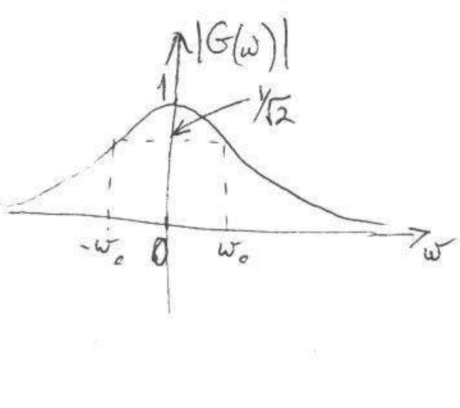
$$|G(\omega)| = \frac{1}{\left|1 + i\frac{\omega}{\omega_0}\right|} = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$

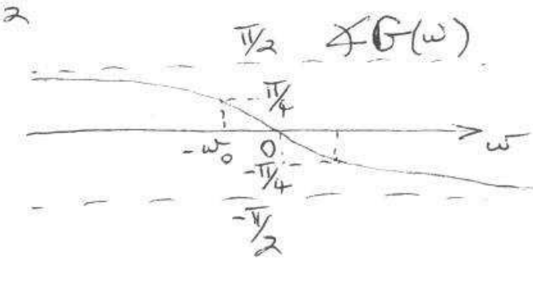
$$\measuredangle G(\omega) = \overset{0}{\measuredangle 1} - \measuredangle\left(1 + i\frac{\omega}{\omega_0}\right) = 0 - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$\lim_{\omega\to\pm\infty} |G(\omega)| = 0 \qquad \lim_{\omega\to+\infty} \measuredangle G(\omega) = -\pi/2 \qquad \lim_{\omega\to-\infty} \measuredangle G(\omega) = \frac{\pi}{2}$$

$$\measuredangle G\left(\frac{\omega_0}{\omega_0}\right) = -\pi/4 \qquad \measuredangle G\left(\frac{-\omega_0}{\omega_0}\right) = +\pi/4 \qquad |G(\omega)|_{\omega=0} = \frac{1}{1} = 1$$

$$|G(\omega)|_{\omega=\pm\omega_0} = \frac{1}{\sqrt{2}}$$





MT1.4 (25 Points) Consider the discrete-time signal $f : \mathbb{Z} \to \mathbb{R}$, characterized as follows:

$$\forall m \in \mathbb{Z}, \quad f(m) = \begin{cases} 1 & m = 0, 1, 2 \\ 0 & \text{elsewhere.} \end{cases}$$

You can tackle the two parts of this problem independently.

(a) A related signal $p : \mathbb{Z} \to \mathbb{R}$ results from *modulating* f, as follows:

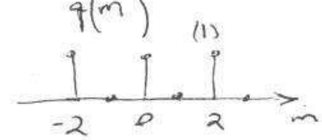
$$\forall m \in \mathbb{Z}, \quad p(m) = \frac{1}{2}\left[1 + (-1)^m\right] f(m).$$

Provide a well-labeled sketch of the signal p.

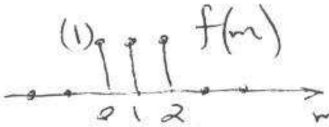
$$f(m) \longrightarrow \otimes \longrightarrow p(m)$$

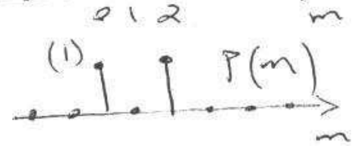
$$q(m) = \frac{1}{2}\left[1+(-1)^m\right]$$

$$q(m) = \begin{cases} 1 & m \text{ even} \\ 0 & m \text{ odd} \end{cases}$$



$$p(m) = \begin{cases} f(m) & m \text{ even} \\ 0 & m \text{ odd} \end{cases}$$





(b) A related signal $q : \mathbb{Z} \to \mathbb{R}$ results from the *convolution* of the signal f with itself; this is written as $q = f * f$, or $q(n) = (f * f)(n), \forall n \in \mathbb{Z}$. In particular, the signal q satisfies the following *convolution sum*:

$$\forall n \in \mathbb{Z}, \quad q(n) = \sum_{m=-\infty}^{\infty} f(m)\, f(n-m).$$

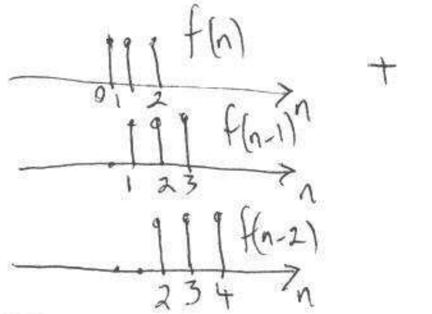
Provide a well-labeled sketch of graph(q). (This would be a stem plot, that is, a "lollypop" plot.)

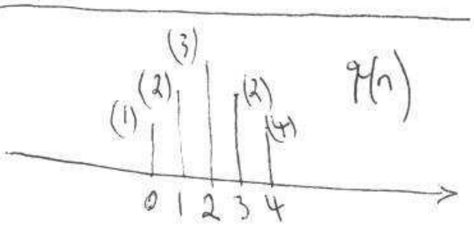
Hint: Discrete-time convolution is generally simpler than continuous-time convolution. Start by sketching the signal f as a function of m. Also, plot the "time-reversed and shifted" version of f (i.e., $f(n-m)$) as a function of m, for various values of n. Then perform point-wise multiplication and summation, as suggested by the convolution sum above (but do it graphically!). Try to determine for what values of n the convolution sum is zero, so you know what values of n to focus your attention on.

Two ways to do this: (a) "Echo" Method; (b) Flip and Shift

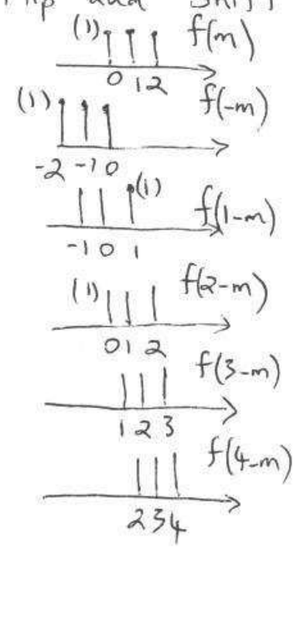
(a) $q(n) = \sum_{m=0}^{2} f(m)\, f(n-m)$ limits of the summation have been changed to account for the fact that $f(m) = 0$ for $m \neq 0, 1, 2$.

$q(n) = f(0) f(n) + f(1) f(n-1) + f(2) f(n-2) = f(n) + f(n-1) + f(n-2)$





(b) Flip and Shift



$f(n-m)$ does not overlap w/ $f(m)$ for $n < 0$ and $n > 4$.

Ptwise multiply and add to get the same result as in (a).

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Problem	Points	Your Score
Name	10	10
1	25	25
2	30	30
3	25	25
4	25	25
Total	**115**	115