University of California, Berkeley **Department of Mechanical Engineering**

ME 104 Spring Semester 2024: Midterm Exam 1 (6 March 2024)

Choosing cylindrical coordinates in an inertial frame of reference, we may write the position vector $\mathbf{r}$ of a particle A of mass m as

$$\mathbf{r} = \boldsymbol{\chi}(A, t) = R\mathbf{e}_R + z\mathbf{k},$$

where

$$\mathbf{e}_R = \cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j}, \qquad \mathbf{e}_\theta = -\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j}.$$

Suppose that A moves along a path $\mathcal{C}$ whose curvature κ is everywhere <u>positive</u>. Let s be the arclength of $\mathcal{C}$. The unit tangent vector $\mathbf{e}_t$ and the principal unit normal vector $\mathbf{e}_n$ are defined by

$$\boldsymbol{e}_t = \frac{d\boldsymbol{r}}{ds}, \qquad \mathbf{e}_n = \frac{1}{\kappa}\frac{d\mathbf{e}_t}{ds},$$

and the unit binormal vector is

$$\mathbf{e}_b = \mathbf{e}_t \times \mathbf{e}_n.$$

<u>Problem 1 (15 points)</u>

(a) Calculate the time derivatives $\dot{\mathbf{e}}_R$ and $\dot{\mathbf{e}}_\theta$ and obtain the expression for the velocity vector of A in the cylindrical coordinate system.

(b) Hence, deduce that the acceleration of A is given by

$$\boldsymbol{a} = \left(\ddot{R} - R\dot{\theta}^2\right)\boldsymbol{e}_R + \left(R\ddot{\theta} + 2\dot{R}\dot{\theta}\right)\boldsymbol{e}_\theta + \ddot{z}\,\boldsymbol{k}.$$

(c) Also, show that the acceleration of A is expressed on the Frenet-Serret basis by

$$\mathbf{a} = \ddot{s}\,\mathbf{e}_t + \kappa\dot{s}^2\mathbf{e}_n.$$

(d) Deduce that

$$\mathbf{v} \times \mathbf{a} = \kappa\,\dot{s}^3\,\boldsymbol{e}_b.$$

<u>Problem 2 (35 points)</u>

Consider a rigid tube ML that is attached to rigid horizontal bars KL and NM which are driven about the vertical z-axis by a motor at K (see Fig. 1). Suppose that a slider of mass m kg is connected to the tube at L by a linear massless spring of stiffness k N/m. The uncompressed length of the spring is

$$l_0 = h + \delta \quad \text{m.}$$

<u>Neglect friction</u>.

(a) Suppose that for $t \leq 0$ the slider is in equilibrium at $z = 0$ and $\theta = 0$. Draw the free-body diagram of the equilibrated slider and calculate the static compression δ.

(b) For $t \geq 0$, let θ be prescribed in radians for some interval of time by

$$\theta = 2t^2.$$

Write the expressions for the velocity and acceleration vectors of the slider in cylindrical coordinates.

(c) For a general position $\boldsymbol{r}$ of the slider at $t \geq 0$, when the length of the spring is

$$l(t) = z + h,$$

determine the force exerted on the slider by the spring.

(d) Draw the free-body diagram of the slider in motion. Label the forces in vector form.

(e) Apply Euler's first law as a <u>vector equation</u>.

(f) Solve for the radial and transverse components of force as functions of time.

(g) Show that the $z(t)$ satisfies the differential equation for simple harmonic motions.

(h) If $\dot{z}(0) = c > 0$ m/s, calculate the unit tangent vector $\boldsymbol{e}_t$ at time $t = 0$.

(i) Evaluate $\ddot{z}$ at time $t = 0$.

(j) Calculate the curvature and the unit binormal vector $\boldsymbol{e}_b$ at time $t = 0$.

(k) Find the principal unit normal vector $\boldsymbol{e}_n$ at time $t = 0$.

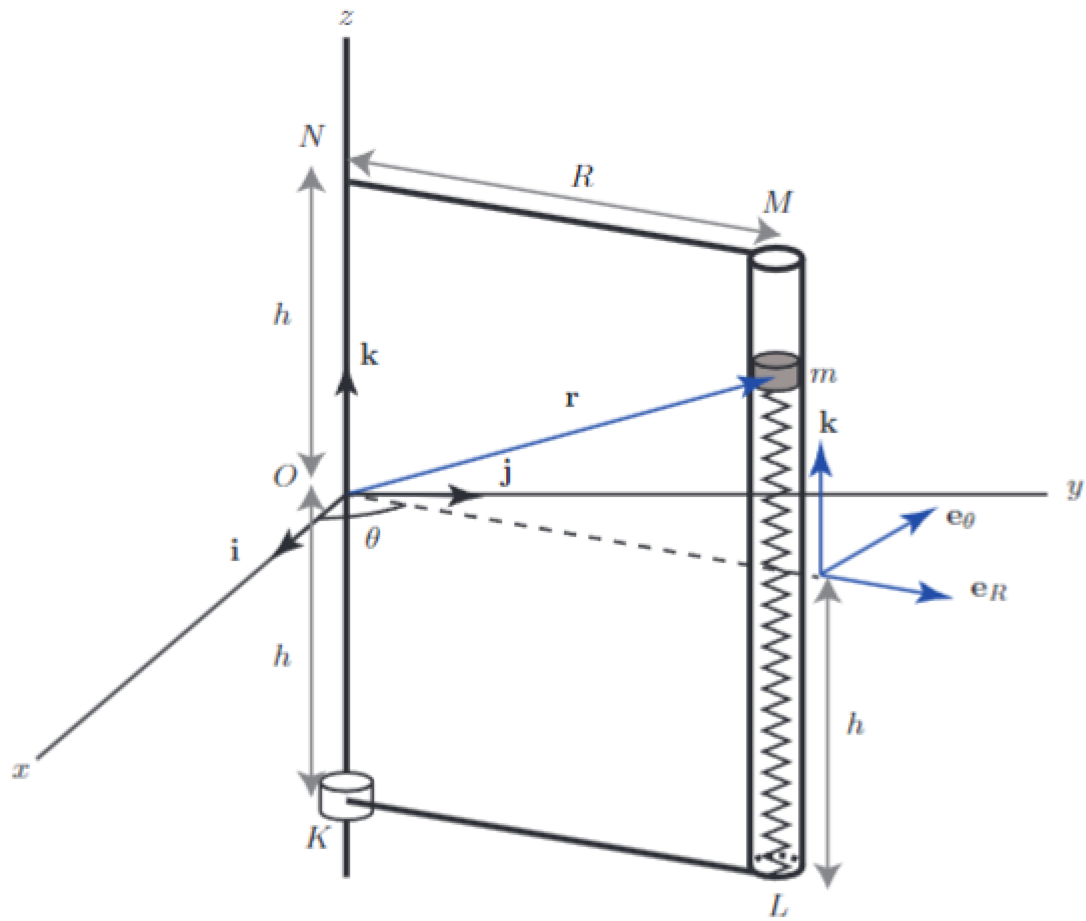

Fig. 1