Full name:

Student ID:

You must complete this exam on your own. If you need clarification on any question, you may ask one of the instructors. You should state and sign the honor code below and turn in this page, along with your paperwork. Good luck!

I affirm that I have neither given nor received any unauthorized aid on this examination, nor have I concealed any violations of the honor code. As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others.

Name and Signature:

This exam is composed with five independent exercises with a total of 10 questions on 5 separated pages. We also give an annex page for more info regarding Exercise 1, question 2. Please write your name at the top of each of them and write your answers below the statement of the exercise/problem.
Do not write on the back of the page.
Duration: 70 min. The number of point is given for information and may be subjected to modifications.

All the answers have to be clearly justified. The grade for each question take into account the justification

All binomial coefficients have to be simplified. All your computations as to be simplified as much as possible

Full name:

Exercise 1: combinatorics

1. (3 points) A student is to answer only 8 out of 10 questions in an examination to pass the exam. The student decides to choose to respond to only 8 questions among the 10 possible. However, the student must answer at least 3 of the first 5 questions, How many choices as this student? (exact numeric answer expected)

2. (3 points) We consider a standard card deck with 52 card composed with four suits (heart, spades, club, diamonds). Each suit includes three court cards: King, Queen and Jack, and also includes ten numeral cards, from one (Ace) to ten. A hand of 5 cards which have distinct consecutive values and **are not all of the same suit** is a straight. For instance, a hand consisting of the five of spades, six of spades, seven of spades, eight of spades, and nine of hearts is a straight.[1] What is the probability that one is dealt a straight?

[1] See the annex page for more details if needed

Full name:

Exercise 2

It has been determined that 90% of the population brush their teeth twice a day, that is a person randomly chosen in the population has a probability 0.9 to bruch his/her/they teeth twice a day. We consider a sample of 20 people. We denote by X the random variable representing the number of people brushing their teeth twice a day in the sample.

1. (3 pts) What is the distribution of X? Explain the assumption you made.
2. (3 pts) Give the probability that at least 18 people in the sample brush their teeth twice a day? (answer given as a polynom of $p = 0.9$)

Full name:

Exercise 3

An automobile manufacturer has three factories: A, B, and C. They produce 50%, 30%, and 20% respectively, of a specific model of car. 30% of the cars produced in factory A are white, 40% of those produced in factory B are white, and 25% produced in factory C are white.

1. (3 points) If an automobile produced by the company is selected at random, find the probability that it is white. (exact numeric answer expected)
2. (3 points) Given that an automobile selected at random is white, find the probability that it came from factory B. (exact numeric answer expected)

Full name:

Exercise 4 A certain typing agency employs 2 typists. The average number of errors per article is 3 when typed by the first typist and 4 when typed by the second. Your article is equally likely to be typed by either typist.

1. (3 points) Let X be the number of error per article made by the the first typist and Y be the number of error made by the second typists. What is the distribution of X and the distribution of Y? Justify and precise the characteristic parameter of this distribution.

2. (3 points) Give the probability that it will have no errors in your article after being typed by the two typist. (explicit answer expected with respect to e^{-n} for different values of n)

Full name:

Exercise 5

We have two instruments that measure the percentage of red blood cells in a blood sample. The measurements given by the Instrument 1 is a random variable X_1 and the one given by by Instrument 2 is $X_2 := 1 - X_1$. We assume that the expected measurement of red cells for Instrument 1 is $\frac{1}{2}$. We assume that X_1 has a variance of 0.1.
From two measurements, we estimate μ by a weighted average M of the measurements X_1 and X_2

$$M = \alpha X_1 + (1 - \alpha) X_2,$$

where α is a weight between 0 and 1.

1. (3 points) Compute $\mathbb{E}[M]$.

2. (3 points) Compute $Var(M)$ and give the value of α minimizing this variance.

Annex page: Straight in poker.

We recall that a straight in poker is a hand with 5 consecutive cards of different suits.

- The Ace (one) is both the highest or the lowest value.
- The order of the card is Ace (lowest), two, three, four, five, six,seven, eight, nine, ten, Jack, Queen,King, Ace (highest).
- It does not loop, that is 10,Jack, Queen,King,Ace of different suits is a straight while Jack, Queen,King, Ace, two of different suits is not.
- It has to be composed with different suit.