Full name:

Student ID:

You must complete this exam on your own. If you need clarification on any question, you may ask one of the instructors. You should state and sign the honor code below and turn in this page, along with your paperwork. Good luck!

I affirm that I have neither given nor received any unauthorized aid on this examination, nor have I concealed any violations of the honor code. As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others.

Name and Signature:

This exam is composed with five independent exercises with a total of 10 questions on 5 separated pages. Please write your name at the top of each of them and write your answers below the statement of the exercise/problem.
Do not write on the back of the page.
Duration: 70 min. The number of point is given for information and may be subjected to modifications.

All the answers have to be clearly justified. The grade for each question take into account the justification

All binomial coefficients, integral, quotient ratio have to be simplified. All your computations as to be simplified as much as possible

Full name:

Exercise 1 A discrete random variable X represents the number of heads obtained when flipping a fair coin four times.

1. (3pt) Write down the probability mass function (PMF) of X.
2. (3pt) Calculate $P(X = 3)$.

Full name:

Exercise 2 The BART orange line from West Oakland to Downtown Berkeley arrives every 10 minutes starting at 7am. That is, they arrive at 7, 7:10, 7:20, 7:30 and so on. A passenger arrives at the Downtown Berkeley station at a time that is uniformly distributed between 7 and 7:30. The time unit is the minute.

1. (3pt) Let X be the arrival time of the passenger, recall the density of X and its expectation.

2. (3pt) what is the probability that the passenger wait less than 3 minutes for a BART;

Full name:

Exercise 3

1. (3pt) Let X be an exponential distribution with parameter λ. We recall that $\mathbb{E}[X] = \frac{1}{\lambda}$ and $Var(X) = \frac{1}{\lambda^2}$. Let $a \geq 0$. Compute $\mathbb{E}[(X-a)^2]$.

2. (3pt) Which value of a minimize $\mathbb{E}[(X-a)^2]$.

Full name:

Exercise 4

From past experience, a college knows that on the average 20% of students accepted for admission will attend a master program. The college approves the application of 100 students for attending this program.

1. (3pt) Let X be the number of student attending the class among the 100 selected. What is the distribution of X? Precise the parameters.

2. (3pt) The ideal size of a class is 30 students. Approach the probability that between 28 and 32 students (included, that is compute $\mathbb{P}(28 \leq X \leq 32)$) will attend the class (numerical value expected to get full points).

   Hint: we give you the normal table in the appendix

Full name:

Exercise 5 We consider the function

$$f(x) = \begin{cases} \lambda / x^2 & x > 2 \\ 0 & x \leq 2 \end{cases}$$

1. (3pt) Find the value of λ such that f defines a density function.
2. (3pt) Compute $P(X > 10)$.

We recall $\Phi(x) = \mathbb{P}(N(0,1) \leq x)$.

TABLE 5.1: AREA $\Phi(x)$ UNDER THE STANDARD NORMAL CURVE TO THE LEFT OF X

X	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998